Influence of gear topology discontinuities on the nonlinear dynamic response of a gear train subjected to multiharmonic parametric excitation

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Summary. This work investigates the influence of holed gear blanks on the nonlinear dynamic behaviour of a flexible gear train. The system is excited by a multiharmonic internal excitation, namely a time-varying mesh stiffness and the static transmission error. This presentation will summarise the numerical procedure that has been developed to study such systems. A reference configuration without holes is used to give insight into the underlying dynamics and to highlight the effects of holed gear blanks. A thorough parametric study then addresses the robustness of the forced response curves and bifurcation structure to changes in gear blank topology.

Introduction

Weight reduction is a recurring concern in the design of modern mechanical systems. One of the most widespread solution to design lightweight gears is to resort to adding holes in the gear blanks. An accurate prediction of the dynamic behaviour of gears remains challenging due to the functional backlash, necessary to allow for assembly and operation, which can lead to contact loss and a strongly nonlinear response [1]. We herein propose an algorithm based on the harmonic balance method to investigate the dynamics of such systems. The proposed approach is able to take into account the internal excitation associated to geared systems. This excitation consists of the static transmission error (STE), whose origin lies in the teeth deflection under load, manufacturing defects and potential tooth profile modifications, and the time-varying mesh stiffness expressed as the derivative of the transmitted load relative to the STE [2].

Dynamic model

The proposed dynamic model [3] considers a reverse spur gear pair (same number of teeth on the input and ouput gears) modelled as lumped inertias and masses denoted I_1 and m_1 for the input gear and I_2 and m_2 for the output gear. Because both gears have the same number of teeth the shafts rotate at a fundamental frequency Ω. The shafts are modelled by torsional stiffnesses K_1 and K_2 and are supported by bearings of stiffness K_b. The input and ouput are modelled by two lumped inertias I_{in} and I_{out}, respectively. The gears are connected by a nonlinear element consisting of a time-varying

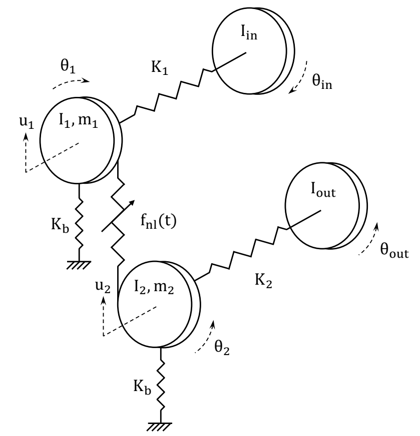


Figure 1: Dynamic model of the flexible transmission considered in this study.

piecewise linear stiffness. It includes the backlash *2b* and the static transmission error as a gap function $g(t)$:

$$\mathbf{f}_{nl}(\mathbf{q}) = \mathbf{G}k_m(t)\left(\mathbf{G}^T\mathbf{q} - g(t)\right)\mathcal{H}\left(\mathbf{G}^T\mathbf{q} - g(t)\right) + \mathbf{G}k_m(t)\left(\mathbf{G}^T\mathbf{q} + g(t)\right)\mathcal{H}\left(-\mathbf{G}^T\mathbf{q} - g(t)\right) \tag{1}$$

Here, $\mathcal{H}$ is the Heaviside step function and $\mathbf{G}$ is a 6×1 column vector allowing for the projection of the displacements in the global reference frame on the line of action. The gap function $g(t)$ is expressed as

$$g(t) = b + q_s(t) - \frac{F_s}{k_m(t)} \tag{2}$$

where $q_s(t)$ is the static transmission error, $k_m(t)$ the mesh stiffness and F_s corresponds to the transmitted load. There exists a number of methods to compute the static transmission error. However this work considers gears with holes which warrants the use of a multibody analysis.

Numerical methods

Due to the functional backlash, the equations of motions of the above described systems are strongly nonlinear. The harmonic balance offers an efficient framework to compute the nonlinear response in the frequency domain. The equation of motion takes the general form

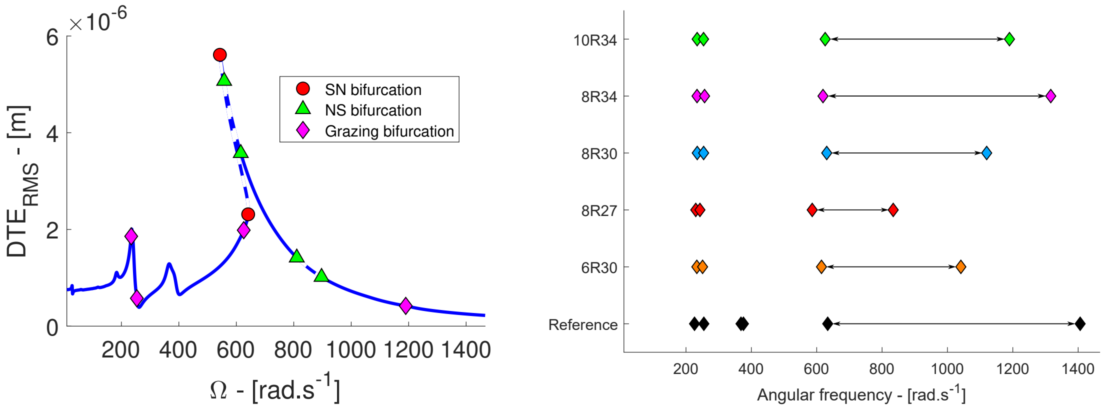
$$[\mathbf{M}]\,\ddot{\mathbf{q}} + [\mathbf{C}]\,\dot{\mathbf{q}} + [\mathbf{K}]\,\mathbf{q} + \mathbf{f}_{nl}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{f}_{ex} \tag{3}$$

where $\mathbf{q}$ contains the generalised displacement of each DoF and $[\mathbf{M}]$, $[\mathbf{C}]$, $[\mathbf{K}]$ are respectively the mass, damping and stiffness matrices. $\mathbf{f}_{ex}$ is the vector of external periodic forcing and $\mathbf{f}_{nl}$ the vector of nonlinear forces, i.e. the mesh force caused by contact, or lack thereof, between gear teeth. The terms in equation (3) are thus expanded as truncated Fourier series. More specifically, the generalised displacements are expressed as

$$\mathbf{q} \approx \sum_{k=0}^{H} \mathbf{a}_k cos(k\Omega t) + \mathbf{b}_k sin(k\Omega t) = [\mathbf{T} \otimes [I_n]]\tilde{\mathbf{q}} \tag{4}$$

where $\mathbf{T}$ contains the harmonic base functions up to the truncation order H, $\otimes$ is the Kronecker tensor product, $[\mathbf{I}_n]$ the identity matrix of size n and $\tilde{\mathbf{q}}$ is the vector in which the Fourier coefficients are stored. The nonlinear forces are treated by the well-known alternating frequency/time procedure [4]. An arc-length continuation method is coupled with the HBM to build the frequency response curves. Hill's method [5] is used to assess the stability of the computed points and a number of test functions are defined to locate smooth [6, 7] and grazing bifurcations.

Numerical example



(a) (b)

Figure 2: Forced response curve (a) and grazing bifurcation diagram (b). Dashed lines indicate unstable regions and solid lines indicate stable regions. Saddle-node, Neimark-Sacker and grazing bifurcations are represented with circle, triangle and diamond markers, respectively.

Conclusion

A numerical methodology has been developed to study the influence of gear topology discontinuities on the dynamics of geared systems. Results show that holes have a beneficial effect in curtailing the frequency range where the system exhibits vibro-impacts, both at low and high torques.

References

[1] Rigaud E., Perret-Liaudet J. (2020) Investigation of gear rattle noise including visualization of vibro-impact regimes. Journal of Sound and Vibration 467, 115026.

[2] Garambois, P., Donnard G., Rigaud E., Perret-Liaudet J. (2017) Multiphysics coupling between periodic gear mesh excitation and input/output fluctuating torques: Application to a roots vacuum pump. Journal of Sound and Vibration 405:158 - 174.

[3] Nevzat Özgüven H., Houser D.R. (1988) Mathematical models used in gear dynamics—A review. Journal of Sound and Vibration 121(3):383-411.

[4] Cameron T. M., Griffin J.H. (1989) An Alternating Frequency/Time Domain Method for Calculating the Steady-State Response of Nonlinear Dynamic Systems. Journal of Applied Mechanics, American Society of Mechanical Engineers.

[5] Peletan L., Baguet S., Torkhani M., Jacquet-Richardet G. (2013) A comparison of stability computational methods for periodic solution of nonlinear problems with application to rotordynamics. Nonlinear Dynamics 72(3):671-682.

[6] Govaerts W. (2000) Numerical bifurcation analysis for odes. Journal of Computational and Applied Mathematics 125:5 –68.

[7] Govaerts W., Sijnave B. (1999) Matrix manifolds and the jordan structure of the bialternate matrix product. Linear Algebra and its Applications 292:245–266.