When friction and vibro-impact makes music: physical model of the tromba marina

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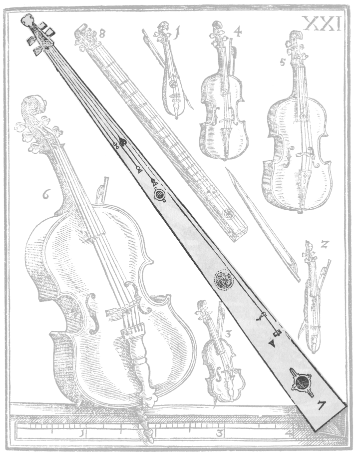

Figure 1: Representation of a tromba marina (adapted from [1]).

The tromba marina (Fig.1) is a bowed string ancient instrument able to imitate the sound of a brass instrument [2, 3]. The functioning of the instrument takes advantage of two non-linear phenomena: dry friction, which allows the emergence of self-sustained oscillations, and vibro-impact mechanism, which makes the "brassy" sound. In this study, a physical model of tromba marina is developed to investigate the phenomenon of spectral enrichment caused by the collisions between one foot of the bridge and the soundboard.

The model consists of a perfectly flexible string fixed at both extremity, vibrating in the plane parallel to the soundboard (Fig. 2a). The string is coupled to a soundboard through the bridge (Fig. 2b). The foot of the bridge located under the string is considered rigidly linked to the soundboard, whereas the other foot is free to move independently of the soundboard, except when they are in contact. The dynamic behavior of the assembly is represented by a set of $N_s + N_b$ modal equations. The N_s modes of the string are known analytically, whereas the N_s modes of the body (soundboard and bridge) are obtained with a finite element model.

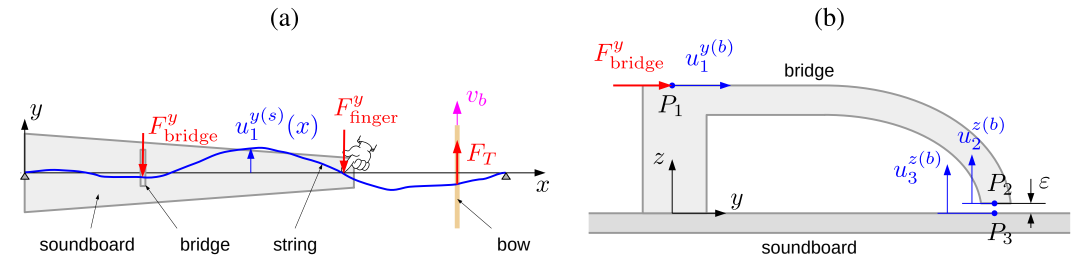

Figure 2: (a) Top view of the simplified instrument. (b) Front view of the bridge. P_1 is the coupling point with the string. P_2 and P_3 are the points likely to enter into contact during playing.

The contact law used to produce the vibro-impact mechanism is the Hunt & Crossley model [4]. The model contains three parameters (stiffness coefficient k, non-linear power exponent n and damping coefficient λ) and expresses the contact force as

$$F_c = \begin{cases} k\delta^p + \lambda\delta^p\dot{\delta} & \text{if } \delta > 0 \\ 0 & \text{if } \delta \leq 0\,, \end{cases} \tag{1}$$

where $\delta = u_3^{z(b)} - u_2^{z(b)} - \varepsilon$ is the indentation between the two bodies in contact (ε is a control parameter which represents the initial gap between the bridge foot and the soundboard, see Fig. 2b). The friction law describing the interaction between the bow and the string assumes that the string perfectly sticks to the bow during sticking phases and that the friction coefficient during sliding phases depends on the relative velocity as

$$\mu(\Delta v) = \mu_d + \frac{\mu_s - \mu_d}{1 - \Delta v/v_0}\,, \tag{2}$$

where Δv is the relative velocity between the bow and the string, μ_s is the coefficient of static friction, μ_d the asymptotic coefficient of dynamic friction and v_0 a parameter controlling the shape of the friction curve.

Time-domain simulations are performed using an explicit numerical scheme of the form

$$\mathbf{x}(t_{i+1}) = \mathbf{A}\mathbf{x}(t_i) + \mathbf{B}\mathbf{f}(t_i)\,, \tag{3}$$

where $\mathbf{x}$ is a vector containing all modal coordinates and their time derivatives and $\mathbf{f}$ is a vector containing the modal forces. The coefficients in $\mathbf{A}$ and $\mathbf{B}$ are obtained using a piecewise constant approximation of the right-hand side of modal equations. At each time step, the contact force F_c is calculated using the available solution at t_i. The other unknown forces (coupling force F^y_{bridge} between the string and the body, reaction force F^y_{finger} of the finger stopping the string, friction force F_T exerted by the bow on the string) are obtained by enforcing constraints: continuity of displacement between the string and the body, zero displacement at finger location, sticking or sliding condition. Finally, the pressure radiated by the soundboard is calculated using Rayleigh integral, assuming that it is surrounded by an infinite baffle.

To highlight the phenomenon of spectral enrichment, a *crescendo* is simulated by bowing the open string with linear increase of the bow speed, after a short phase intended to establish a periodic regime, the Helmholtz motion [5, 6]. Figure 3 illustrates the results of the simulation. During a first phase of the simulation, no collision occurs. After $t = 1.6$ s, the body response has sufficient amplitude so that the free foot of the bridge enter into contact with the soundboard. Figure 4a shows the sound pressure radiated by the instrument for the same gesture and Figure 4b the corresponding spectrogram. As expected, the occurrence of collisions from $t = 1.6$ s is accompanied by a strong spectral enrichment. To highlight this, the evolution of the spectral centroid is shown on the same figure. The emergence of high rank harmonics during the *crescendo*, making the sound perceived as "brassy", is a common feature with brass instruments. However, the effect appears suddenly in the case of the tromba marina, as seen by the abrupt increase in spectral centroid at $t = 1.6$ s, whereas a progressive spectral enrichment is typically observed when a *crescendo* is played on a brass instrument. This significant difference is due to the fact that different physical phenomena are at the origin of the spectral enrichment: non-linear wave propagation in the air column contained in the brass instrument [7], vibro-impact mechanism for the tromba marina.

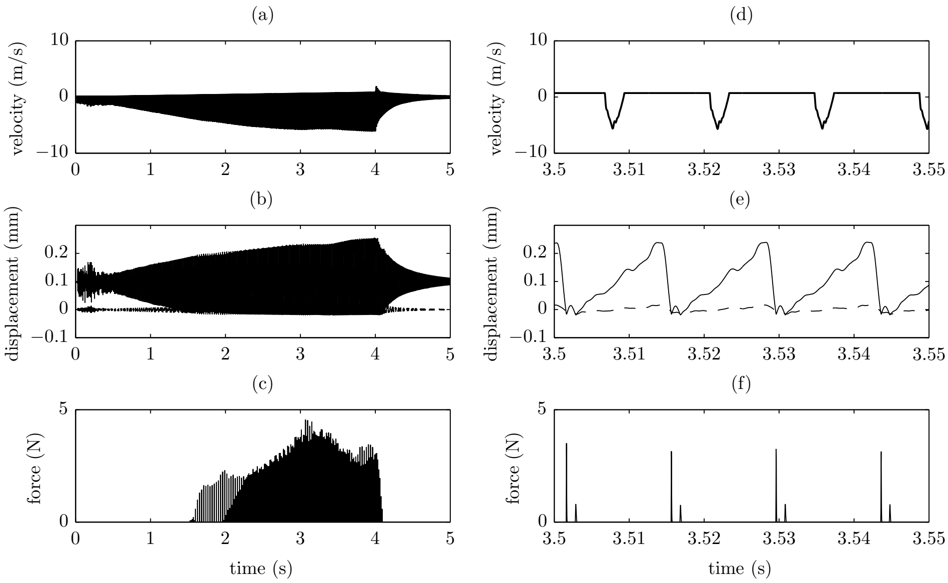

Figure 3: (a)-(d) Velocity of the string at bowing point, (b)-(e) displacement of point P_2 and P_3 (see Fig. 2b), (c)-(f) contact force F_c.

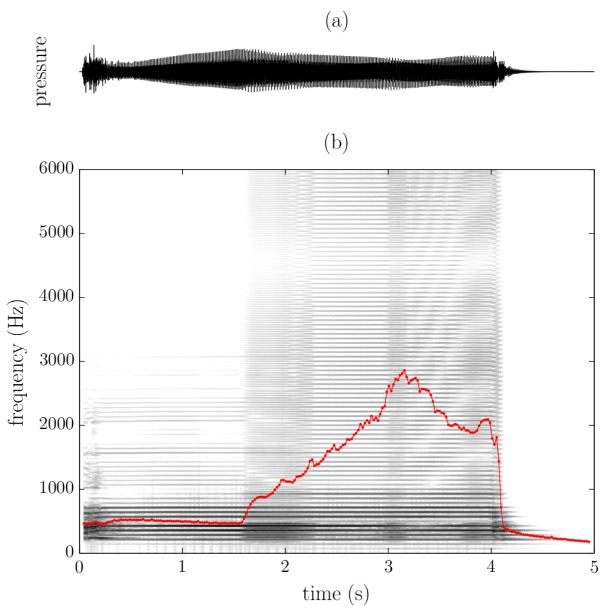

Figure 4: (a) Sound pressure (normalized) radiated during a *crescendo* and (b) its spectrogram. The superimposed red curve is the spectral centroid.

References

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