## Stability transitions of flexible nano-swimmer under rotating magnetic field

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<u>Summary</u>. Micro-nano-robotic swimmers have a promising potential for future biomedical tasks such as targeted drug delivery and minimally-invasive diagnosis. An efficient method for controlled actuation of such nano-swimmers is applying a time-varying external magnetic field. While rigid helical nano-structures that move in corkscrew motion under a rotating magnetic field are hard to fabricate, we recently found that two nano-rods (magnetized nickel and a rhodium tail) connected by a soft polysaccharide hinge may also exhibit helical motion under a rotating magnetic field. We also discovered interesting mode transitions depending on the actuation frequency. In low frequency regime, the nano-swimmer tumbles in plane, while larger frequencies lead to out-of-plane helical motion, optimum of peak speed, followed by step-out loss of synchronization. In this work, we analyzed these effects by formulating the spatial dynamics of 3D rotating nano-swimmer compose of two links connected by a passive rotary joint with a torsion spring representing the flexible hinge. Assuming quasi-steady Stokes drag acting on the links, numerical simulations and analysis of the nano-swimmer's nonlinear dynamics reveal the stability transitions of possible synchronized solution, as well as bifurcations with respect to actuation frequency. The results highlight the importance of simple low-dimensional models of nonlinear dynamics and their utility in predicting and optimizing motion of magnetically-actuated nano-swimmers.

## Introduction

Inspired by the motion of bacteria and other microorganisms, researchers have developed artificial helix-shaped microand nano-structures that can perform corkscrew motion and helical-path swimming upon a suitable stimulation with external energy sources. These small-scale helical devices attract much interest because of their great potential for disease diagnosis, minimally invasive surgery, telemetry, targeted therapies, or plasmonic-based nanorheology. The most common strategy consists of actuating magnetically responsive structures comprising helical components that revolve around their long axis when they are actuated using rotating magnetic fields resulting in corkscrew locomotion.

In this work (recent paper [1]), we show that incorporating a complex helical body shape in swimmers' architectures is not necessary to enable helical swimming. This is an advantageous feature of the presented hinged swimmers, especially in terms of fabrication, adjusting the length of hinges and rods becomes easier. In this contribution, we show for the first time that a highly integrated multifunctional nonhelical nano-swimmer is capable of helical klinotactic swimming when stimulated by purely rotating magnetic fields.



Figure 1: (A) Simplified Theoretical model; (B) A SEM image of the finalized structure of nano-swimmer

The nano-swimmer consists of two cylindrical rigid metallic links (nickel and rhodium) joined by a soft polymeric hinge (Figure 1B). The ferromagnetic nickel link serves as the magnetically responsive motile head component. The non-ferromagnetic rhodium segment acts as a tail, while the polymeric hinge functions as a flexible joint to promote the swimmer's motion by deformation. In the experiments [1], we observed different motion phases, as shown in previous helical nanobots' papers [2,3], at low frequency tumbling, low-speed movements, at a higher frequency, moving forward in a helical path, in further frequency increase we obtain asynchronous swimming.

In order to systematically understand the theoretical mechanism of the spatial dynamic motion of the swimmers, a theoretical model was developed. The theoretical model of the two-link nano-swimmer comprises two rigid slender cylinders connected by a point-size revolute joint, with a relative angle  $\delta$  of rotation about a body-fixed axis perpendicular to the link's longitudinal direction (Figure 1 A). The flexibility of the revolute joint represented by a linear torsion spring with stiffness k, the torque at the joint is given by  $\tau_k = -k\delta$ . The head link made of ferromagnetic material, its magnetization vector **m** directed along the link's longitudinal axis  $\hat{\mathbf{t}}_1$ . While being actuated by a rotating magnetic field. The external magnetic field B(t) is rotating in the Y-Z plane by  $B(t) = \begin{bmatrix} 0 & \sin(\omega t) & \cos(\omega t) \end{bmatrix} b$ , where b is the magnetic field's intensity and  $\omega$  is its rotational frequency.

Assuming a spatially uniform magnetic field, it induces a pure torque  $\tau^m$  on the magnetized head link, which depends on its orientation and the time-varying field B(t) as  $\tau^m = \mathbf{m} \times \mathbf{B}$ . The nano-swimmer submerged in a viscous fluid. The Stokes drag forces and torques acting on each of the two links  $f_i$ ,  $\tau_i$  depend linearly on their angular velocity vectors  $\mathbf{v}_i$ ,  $\Omega_i$ . Neglecting hydrodynamic interactions between the links implies linear drag resistance relations.

The vector of generalized coordinates describing the swimmer's spatial pose is  $\mathbf{q} = (x, y, z, \phi, \theta, \psi, \delta)^T$ , where (x, y, z) denote the position of the head link's center,  $(\phi, \theta, \psi)$  are Euler angles describing its spatial orientation (ZXZ convention), and  $\delta$  is the joint angle. Formulating the links' linear and angular velocities  $\mathbf{v}_i, \Omega_i$  in terms of orientation and generalized velocities,  $\mathbf{q}$  and  $\dot{\mathbf{q}}$ . Static equilibrium balance of forces and torques on the two links, including viscous drag terms  $\begin{pmatrix} f_i \\ \tau_i \end{pmatrix} = -\begin{pmatrix} \xi_i & \mathcal{B}_i \\ \mathcal{B}_i^T & \kappa_i \end{pmatrix} \begin{pmatrix} \mathbf{v}_i \\ \Omega_i \end{pmatrix}$ , where  $\xi_i, \mathcal{B}_i, \kappa_i$  are the translational, rotational and coupling viscous resistance tensor as notated in [2], magnetic torque  $\mathbf{\tau}^m$  and elastic joint torque  $\mathbf{\tau}_k$ , leads to a system of first-order nonlinearly coupled ordinary differential equations of the form  $\dot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, t)$ .

## Results

This system is integrated numerically using MATLAB's ODE15s function, under initial conditions  $\mathbf{q}(0) = 0$  of straightened links aligned with X-axis. The swimmer's motion is extracted from solutions of  $\mathbf{q}(t)$  after convergence to steady-state synchronized periodic motion, whenever it exists. Beyond the step-out frequency, such synchronized motion no longer exists, and we observe quasi-periodic oscillations.



Figure 2: (A) schematic diagram of nano-swimmer illustrating precession angles and XYZ coordinate. (B) Simulation's values of the average speed of a flexible hinged nano-swimmer as a function of rotating magnetic frequency. (C) Mean precession cone angles  $\alpha$  and  $\beta$  as a function of the external rotating magnetic field frequency in the experiment.

In order to study the locomotion behavior of nano-swimmers, we define the following parameters shown in Figure 2A. YZ plane corresponds to the plane of the magnetic field rotation.  $\alpha$  is the maximum angle between the magnetic head and the X-axis, and  $\beta$  is the maximum angle between the tail link and the X-axis. For the analysis of nano-swimmers' locomotion characteristics, magnetic fields of 15 mT and rotational frequencies of up to 50 Hz with steps of 5 Hz investigated. At low frequency below 10 Hz, the nano-swimmer's movement speed was relatively low at around 0.5 µm/s. The nano-swimmer was bent a little and rotated in YZ plane. From 10 Hz to 30 Hz, it started to move forward in a helical path. The speed increased almost linearly up to 27.49 µm/s in the frequency region of 10 - 25 Hz. At the frequency range of 25 - 30 Hz, the speed curve shows an inflection point corresponding to a maximum speed of around 28 µm/s. A further increase in the frequency results in a decrease in the speed towards zero.

We classify the precession behavior into three different motion phases: tumbling, helical and asynchronous swimming, based on the speed and precession angles of the swimmer, as shown in Figures 2B and 2C. The transformation of the nano-swimmer's locomotion from tumbling to helical motion is supported by the fact that the precession angle  $\alpha$  and  $\beta$  dramatically decrease as the rotational magnetic frequency increases. The simulation results in [1] were in good agreement with the experimental results.

The different dynamic behavior by in-plane tumbling or spatial helical klinotactic swimming can be switched by changing the magnetic field frequency and strength. The effects of the magnetic field strength, the Ni head length, and the hinge length on locomotion behavior were investigated by numerical simulation and experiments in [1], were in good fit. We will also present new analytical results of the system's periodic solutions, their existence and stability conditions, for specific limit cases.

## References

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