Nonlinear vibration control of smart plates using nonlinear modified positive position feedback approach

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Summary. In this paper, nonlinear vibration control of a plate is investigated using Nonlinear Modified Positive Position Feedback (NMPPF) method that is applied through a piezoelectric layer on the plate. NMPPF controller consists of a resonant second-order nonlinear compensator for vibration suppression at targeted resonance. In this model, transverse vibrations are studied and stimulations are performed for the primary resonance. Using time-space separation of the differential equations of the model and Galerkin method the temporal nonlinear equations governing the system have been found. Then, the free and forced vibrations of the structure with the NMPPF controller has been solved using Method of Multiple Scales to obtain an analytical solution. Results show that the NMPPF controller reduces the amplitude of the vibration by inducing an increase to the damping coefficient. In addition, the NMPPF controller provides a higher level of suppression in the overall frequency domain response by increasing the compensator gain. Finally, the results of the analytical solution for the closed-loop NMPPF controller are presented and compared with the result of the conventional Positive Position Feedback (PPF) controller and nonlinear integral resonant controller (NIRC).

Introduction

Controllers are one of the most effective ways to control linear and nonlinear vibrations. Hence, different control strategies have been presented and utilized [1]. One of the methods for nonlinear vibration control is to use of active control. The advantage of using active control is its real-time adjustment according to the condition of the system and alternations in the input disturbance force on the system. In order to have the highest level of suppression in the vibration control process, it is essential to design a controller compatible with the nonlinear characteristics of the system oscillations. Linear and nonlinear active vibration controllers typically employ piezoelectric actuators. Active vibration control is usually applied using piezoelectric ceramics as actuators and sensors, as an example, piezoelectric actuators are used in Atomic Force Microscopes to produce high-frequency vibrations. The purpose of this paper is to obtain the equations of nonlinear vibrations and controller system for elastic plate with a piezoelectric layer that follows the classical theory of Kirchhoff. In addition, Van Karman's nonlinear strains have been used to investigate geometrical nonlinear effects. This plate has a piezoelectric layer at its upper surface. This layer actually is utilized to actively suppress the vibrations of the plate. The external force applied to the piezoelectric layer is divided into two groups: 1) the control force ($F_c(t)$), the control force via the controller's compensator will be logged; and 2) the harmonic excitation force distributed uniformly on the plate as the disturbance. The constitutive equations for piezoelectric layer is utilized to implement the effect of applied voltage into the electromechanical model. Method of Multiple Scales is utilized for calculation of the frequency response of the system and the controller. The resulting modulation equations are, then, used to verify the effectiveness of the proposed controller. Having the solution for the controllers, results are graphically demonstrated and discussed. In order to understand the performance of the controllers in more detail, sensitivity analysis on the closed loop system responses is performed and the influence of each parameter on the control output have been investigated.

MATHEMATICAL MODELING OF THE STRUCTURE

In this section, the nonlinear dynamic model of the structure is investigated. The structure is composed of two layers of square plates, a substructure layer and a piezoelectric layer with the different thickness on top of the substructure, as shown in Fig. 1. It is assumed that the h_p and h_s are thicknesses of the piezoelectric layer and substructure, respectively. Also, as shown in Fig. 1, a is the side length of the plate. The origin of the coordinate system is placed on the corner of the middle plane of the substructure layer. The boundary conditions of the plate are considered as simply support and $u.v$ and w are the displacement of the plate in the $x.y$ and z directions respectively.

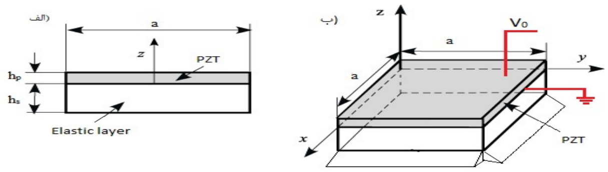

Figure 1. The symmetric unimorph piezoelectric plate.

Using Hamilton's principle, equations of motion of the plate based on classical theory and von Karman strain and then applying Galerkin method for simply suppotted B. C. in all edge of plate, nonlinear temporal equations can be expressed as:

$$\ddot{W}(t) + \omega_w^2 W(t) + \tilde{\alpha} W^3(t) + \tilde{\eta}_w \dot{W}(t) = \tilde{F}\cos(\tilde{\Omega} t) \qquad (1)$$

The control force, $F_c(t)$, will be added to the equation. Also, ω_w^2 consists of two parts of the natural frequency created in the sheet with the piezoelectric layer and a coefficient of the voltage created from the piezoelectric layer to the elastic layer. The equation is created for the natural frequency of $\omega_w^2 = \omega^2 + s_1 V_e$ where s_1 is the constant coefficient of the voltage applied to the plate. The method of Multiple Time Scales is used to find a uniform nonlinear structure. The nonlinear system of Equation (1) under NMPPF controllers are defined as follows

$$\ddot{\bar{r}}(t)+\tilde{\eta}_r\dot{\bar{r}}(t)+\omega_r^2\bar{r}(t)+\tilde{\delta}\overline{r^3}(t)=\tilde{k}_r U(t) \tag{2}$$

$$\dot{\bar{s}}(t)+\omega_s\bar{s}(t)=\tilde{k}_s U(t)$$

where $\bar{r}(t)$ *and* $\bar{s}(t)$ are compensatory first and second order state variables, respectively. k_r and k_s are the inputs of the control. The control law pertains to the modified positive position feedback controls system in the main system of Equation (1) as $F_c(t)=\tau_r r(t)+\tau_s s(t)$; that are τ_r and τ_s respectively, are first order compensator and second in a closed loop system. Using Multiple time scale method, the coupled equations obtained for the frequency are the response of the main system and the controller domain.

$$\left(\frac{\hat{f}}{2}\right)^2=\left[\left(\frac{\hat{\eta}_u}{2}+\frac{\hat{\tau}_s\hat{k}_s}{2}\right)a+\frac{\hat{\tau}_r\hat{\eta}_r b^2}{2k_r a}\right]^2+\left[\frac{3\hat{\alpha}}{8}a^3+\frac{\hat{\tau}_s\hat{k}_s}{2}a-\frac{3\hat{\delta}\hat{\tau}_r}{8\hat{k}_r}\frac{b^4}{a}-\frac{\hat{\tau}_r}{\hat{k}_r}(\sigma_r-\sigma_f)\frac{b^2}{a}-\sigma_f a\right]^2 \tag{3}$$

$$\left[1-\frac{\hat{\eta}_r^2}{\hat{k}_r^2}\frac{b^2}{a^2}\right]^{\frac{1}{2}}=\frac{3\hat{\delta}}{4\hat{k}_r}\frac{b^3}{a}+\frac{2}{\hat{k}_r}(\sigma_r-\sigma_f)\frac{b}{a}. \tag{4}$$

Results and discussions

This part has discussed the vibration range in the resonant frequency region and the performance of the controllers in controlling the vibration amplitude in the resonance region. The frequency response of main system is given in Fig. 2. Fig. 3 indicates the frequency response of the closed loop system with the PPF controller. Implementation of PPF controller reduces vibration range in the resonant frequency region $(\sigma_f = 0)$. This controller amplifies the production of two-peak amplitudes with relatively large amplitude around the frequency. Fig. 4 shows the frequency response of NMPPF controlled systems. In this Fig., the right peak has higher amplitude. Also, Fig. 5, shows the comparison of the effects of these nonlinear controllers on the frequency response of smart plate.

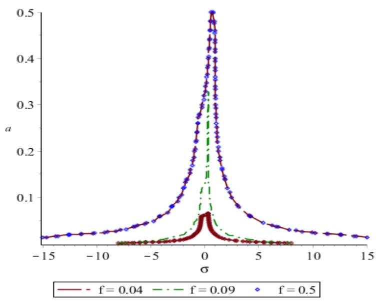

Fig 2. Frequency response of the uncontrolled system

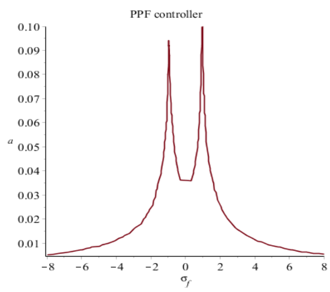

Fig 3. Frequency response of the PPF controlled

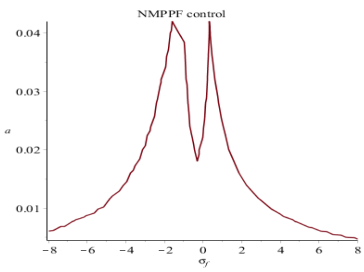

Fig 4. Frequency response of the NMPPF controlled system

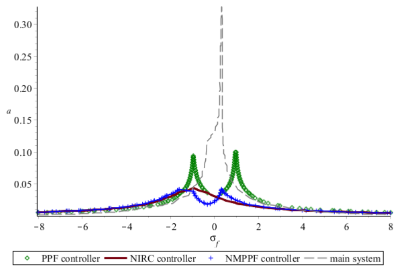

Fig 1. Comparison of the PPF, NIRC, and NMPPF controller performances

Conclusion

In this article, active nonlinear vibrations control of a simply supported smart plate using the NMPPF controller introduced. The system response also studied under NIRC and PPF control approaches. It is shown that the PPF controller had a weaker control effect than the other two controllers. Also, the NMPPF controller reduced the vibration amplitude on a large bandwidth in the frequency domain better than the other two methods.

References

[1] Omidi E and Mahmoodi S.N (2014) October. Nonlinear vibration control of flexible structures using nonlinear modified positive position feedback approach. In ASME (2014) *Dynamic Systems and Control Conference* (pp. V003T52A002-V003T52A002). American Society of Mechanical Engineers.