Fluid dynamics effect in large array

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Summary. Fluid dynamics investigation of coupled oscillators and arrays of beams are important for several applications in biology, medicine, and especially and increasingly for MEMS and NEMS sensors. The most available literature on oscillators is either a single beam or small-sized arrays. Limited literature is available concerning the dynamics of large-sized arrays of beams. Our group has dedicated its research goals to understanding coupled MEMS oscillators and array dynamics to enhance technologies such as fast-scan AFM, nanometrology, and precision lithography. Some of these applications require knowledge of arrays in a fluidic environment. Before investigating fluid-structure interactions of such large-array a sound understanding of the fluid motion is required, which is the focus of this work. We investigate a large array of beams using the boundary integral technique, where the flow is governed by unsteady Stokes and Continuity equation. The analysis is performed for all beams that are equally excited in phase for different gap sizes between the beams at different Reynolds numbers, including the comparison of an increasing array size with the same and varying array lengths. Results include the onset of array effects, added mass, and viscous dissipation for a critical number of beams M, the gap between the beams and Reynolds numbers. The analysis suggests an increase in interactions between the neighbor and non-neighbor beams with an increase in array size at the same array length. The work guides for the design of arrays in fluids for different spacing and Reynolds numbers.

Background

Tuck [1] and Green [2] present an extensive work of dynamics for an individual beam, whereas Basak [3] and Manickavasagam [4] conducted investigations on small arrays of beams (number of beams less than five). Much lesser work can be found on a large array in general. No details are available for the large array of beams oscillating in fluids. In this work, we focus on the fluid dynamics of the large arrays [5].

Methodology

We base our analysis on the well established Boundary Integral Method (BIM) applied to unsteady Stokes and continuity equations. By applying the BIM, we have deduced the partial differential equations into integral form to calculate the fluid properties over the beams within the array using the analytical equation 1.

$$f(Re,x) = \frac{i}{Re}\left(\frac{1}{x} + (1i \cdot \sqrt{iRe} \cdot \mathrm{sgn}(x) \cdot K_1(\sqrt{iRe} \cdot \mathrm{abs}(x) \cdot (-1i)))\right) \tag{1}$$

The closed-form analytical equation is extended for M number of beams by following the technique used by Basak [3] and Manickavasagam [4]. We investigate the velocity configuration of all beams active in-phase, where x = (ξ'_j - ξ_k), ξ'_j is an node on the beam surface, ξ_k is the midpoint between any two nodes, $\xi_k = \frac{1}{2}(\xi_{j'} + \xi_{j'+1})$, Re is the corresponding Reynolds number and K_1 is the modified bessel function of first kind.

$$\xi'_j = (2+2\bar{g})m - (3+3\bar{g}) - \cos\left(\frac{\pi j}{N}\right) \qquad [m = 1,2,3,...,M], \tag{2}$$

To mitigate the singularities at the edges, the beam is discretized into unequal segments by using a generalized equation 2, which is applicable to all array of beams if the number of beams M in an array is greater than or equal to two as shown in Figure 1a and $\bar{g}$ is the nondimensional gap.

Analysis and Results

The following investigations are considered for a velocity configuration of all beams active with a gap between the beams $\bar{g}$ of 0.1 as shown in Figure 1a, which is below the critical gap theoretically [6] at Reynolds number 0.1. The input velocity of the beams and the matrix obtained from the analytical equation yields the hydrodynamic force over the array. Absolute, real, and imaginary hydrodynamic forces give us new insights into the large-array in fluids [5]. In Figures 1b to 1d horizontal axis presents the normalized beam number, the absolute A(F), imaginary I(F) (added mass), and real part R(F) (viscous dissipation) of the hydrodynamic force [7] are normalized corresponding to its respective width of the beam and plotted on the vertical axis.

From Figure 1b, it is observed that at the same array length and different array size there is an increment in the added mass and viscous dissipation which is due to the array effect which can be observed with an increase in the number of beams in an array, whereas with varying array length and increase in array size there is little effect on the dissipation as shown in the Figure 1c and drastic increment in the added mass due to increase in the array length as shown in the Figure 1d. The interaction between neighbor and non-neighbor increases with an increase in array size and at the same array length. If the distance between the beams is below a critical gap where the interaction of beams in an array is high due

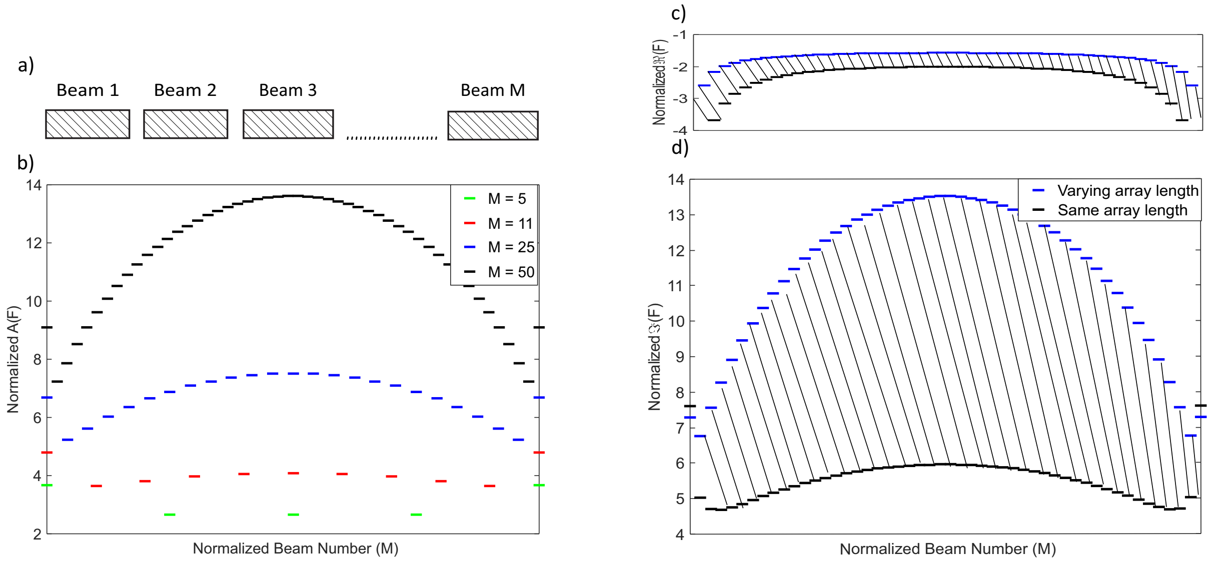

Figure 1: (a) Array of beams in non-viscous fluid (b) Absolute pressure for different array sizes (c) Real pressure and (d) Imaginary pressure for 50 beams array at $\bar{g} = 0.1$ and $Re = 0.1$.

to the overlapping of viscous layers, we can observe the array effect for varying the gaps between the beams for different Reynolds numbers. The hatched area in Figures 1c and 1d represents the increase in viscous dissipation and decrease in added mass with an increase in array size at the same array length.

Conclusions

It is important to understand the fluid dynamics of a large array of beams oscillating in the unbounded fluid environment to be able to accurately predict the full dynamics of such systems, and in turn to effectively develop future array technologies like AFM, lithography, and nano-metrology. In this work, we have extended the BIM of 2 beams by Basak [3] and small-sized arrays of beams by Manickavasagam [4] for any arbitrary number of beams to understand the associated array effects originating from the fluid environment. We have analyzed M equal to $5, 11, 25, 50$ beams exciting in-phase in a non-viscous fluid to calculate the hydrodynamic forces of large arrays of beams for different gaps between the beams and Reynolds numbers. New insights from A(F), I(F), and R(F) reveal critical design-relevant parameters for the development of future array technology. In addition, this work can be extended by oscillating the large array of beams close to a rigid surface and analyzing the influence of rigid surface on viscous damping and added mass.

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