Non-periodic dynamics in a delay model of flute-like musical instruments

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Summary. We investigate the emergence of quasiperiodic sound regimes in a model of flute-like musical instruments. The model itself is a system of delay differential equations of neutral type (NDDEs). We employ advanced numerical continuation methods to compute bifurcation diagrams in the space of relevant playing and making parameters. Our results show the role played by the detuning between the instrument resonance frequencies in the emergence of quasiperiodic regimes.

Self-sustained musical instruments are complex nonlinear dynamical systems and show a wealth of dynamical regimes. This includes equilibrium solutions where no sound is produced and periodic oscillations which most often correspond to the notes produced by the instrument, but also non-periodic oscillation regimes [1]. The exisence and stability of these different sound regimes depend sensitively on making parameters which are fixed by the instrument maker and on playing parameters which are continuously tuned by experienced players. In the context of western classical music, non-periodic sound regimes are most of the time avoided and can be considered as a defect of the musical instrument or as a lack of control from the musician. On the other hand, some non periodic sounds, often referred to as *multiphonics*, are often played on purpose in jazz, contemporary or traditional music. Flute-like instruments in particular, show a diversity of non-periodic sound regimes (see Figure 1). This includes *multiphonic sounds* played on purpose in transverse flutes and recorders, *rolling notes* which instrument makers try to avoid, modulated regimes due to wall vibrations [2], but also, *e.g.*, *sonidos rajados*, a highly-modulated sound produced by traditional pan-like flutes from Central Chile [3]. Here we consider a model of flute-like musical instrument, and investigate the physical mechanism responsible for the emergence of non-periodic regimes.

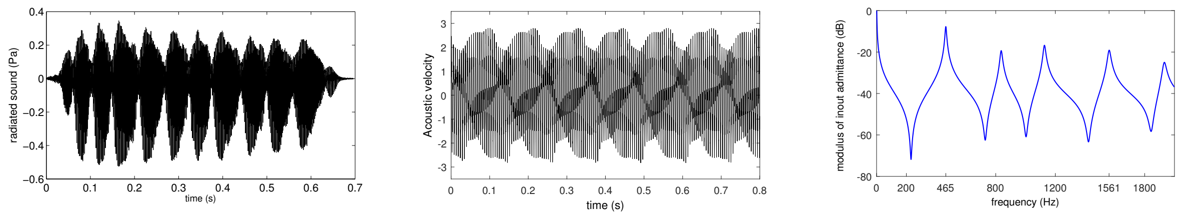

Figure 1: Left: Non periodic sound played by a traditional chilean pan-like flute. Middle: Non periodic simulated sound obtained using the NDDE model of a recorder. Right: Modulus of the input admittance of an alto recorder, for the fingering used to play the note B flat. The admittance is modelled as a sum of 6 resonant modes.

In flute-like musical instruments such as transverse flutes, recorders, quenas and organ pipes, the sound production results from the coupling between an intrinsically unstable air jet blown by the musician and an acoustic resonator which is composed of the air column contained in the pipe of the instrument [4]. Depending on the instrument, the pipe can include several tone holes: different *fingerings*, corresponding to different combinations of open and closed tone holes, are used by the musician to change the properties of the acoustical resonator, including its resonance frequencies and damping factors. More precisely, the air pressure in the musician mouth results in the emergence of an air jet at the output of a channel which is a part of the instrument (for recorders) or formed by the musicians lips (in transverse flutes, pan-like flutes ...). This air jet is intrinsically unstable and oscillates around a sharp edge called *labium*. This oscillation results in an alternate flow injection inside and outside the instrument pipe, which constitutes a dipolar pressure source for the acoustic resonator. The acoustic waves thus created propagate in the acoustic resonator and perturb back the air jet at the channel exit. This perturbation is amplified while convected along the jet, and sustains the air jet oscillation and, as such, the sound production. Importantly, the convection time of the perturbation along the air jet introduces a delay τ in the system, whose value is directly related to the pressure P_m in the musician's mouth, which is one of the main control parameter. Overall, flute-like instruments are modelled by a system of $2n$ delay differential equations of neutral type (NDDE), with n the number of acoustic modes taken into account to model the resonator, as shown in Figure 1 (right).

We focus here on the theoretical investigation of this model, and use advanced numerical methods to perform a bifurcation analysis. Because delay differential equations (DDEs) have an infinite dimensional phase space, they are more complicated to solve numerically than ordinary differential equations (ODEs). In particular, specific numerical methods have to

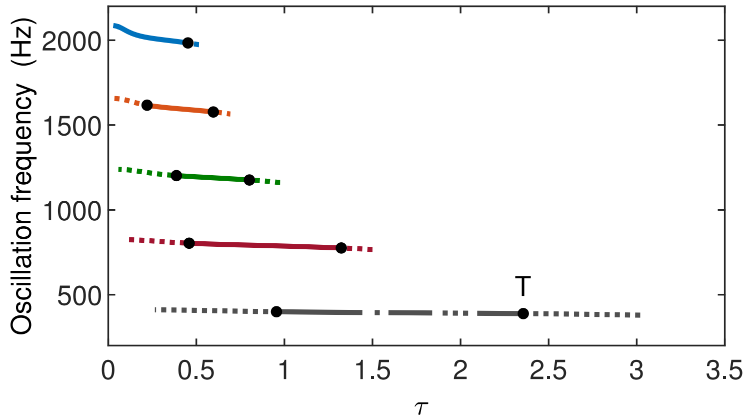

Figure 2: Bifurcation diagram of the NDDE flute model, showing the frequency along branches of stable (solid lines) and unstable (dotted lines) periodic solutions, with respect to the delay τ. Black dots correspond to torus bifurcation points T, where a periodic solution changes stability, with two complex conjugate Floquet multipliers crossing the unit circle.

be considered for the calculation and continuation of their solutions. We use here the continuation toolbox DDE-Biftool [5, 6]. This allows for the calculation and continuation of equilibrium solutions, periodic solutions and their respective bifurcations in system of DDEs. More precisely, a collocation method is used to compute periodic solutions. This is coupled to a predictor-corrector algorithm to continue the branches of solutions with respect to a parameter of interest of the physical model. Here, these numerical methods have been adapted so that the same calculations are possible for NDDEs [7, 8].

The bifurcation analysis of the NDDE model of flute-like instruments shows that multiple branches of periodic solutions emerge in Hopf bifurcations when the main control parameter P_m increases, which corresponds to a decreasing value of the delay τ (see Figure 2). From a musical point of view, the different branches of periodic solutions correspond to the different *registers* of the instruments, that is to say to periodic regimes associated with the different acoustic modes of the resonator. These periodic regimes have oscillation (*playing*) frequencies close to the resonance frequencies of the instrument, and can be interpreted as the different notes played by a musician for a given fingering. The stability analysis of these periodic solution show that they destabilise through torus bifurcations when the mouth pressure P_m increases further (*i.e.* when τ decreases). These bifurcations can lead to stable quasiperiodic oscillations. We investigate further the emergence of quasiperiodic regimes by performing the numerical continuation of curves of torus bifurcation in the (P_m, ξ)-plane. Here, ξ is an inharmonicity parameter, which models the detuning between the different acoustic resonant modes. The obtained bifurcation set demonstrates the major role played by the resonator inharmonicity on the existence and stability of quasiperiodic regimes.

Overall, our results provide a better understanding of the physical mechanism responsible for the emergence of non periodic sound regimes in flute-like musical instruments. They strongly suggest that experimentally-observed modulated sound regimes can be interpreted as quasiperiodic oscillations on a stable torus, resulting from the loss of stability of one of the *registers* of the instrument. The bifurcation analysis unveils the major role played by the inharmonicity of the acoustic resonator in this process. This paves the way towards a better experimental control of non-periodic sound regimes, which instrument makers and musicians either try to avoid or to enhance depending on the musical and cultural context.

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