Investigation of bifurcations in a nonlinear rotor system using numerical continuation

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Summary. Nonlinearities in rotating systems have been seen to cause a wide variety of rich phenomena, however the understanding of these phenomena has been limited because numerical approaches typically rely on 'brute force' time simulation, which are inefficient due to issues of step size and settling time, cannot locate unstable solution families and may miss key responses if the correct initial conditions are not used. This work uses numerical continuation to explore the responses of such systems in a more systematic way. A simple isotropic rotor system with a smooth nonlinearity is studied, and the rotating frame is used to obtain periodic solutions. Asynchronous responses with oscillating amplitude are seen to initiate at certain drive speeds due to internal resonance, in a manner similar to that observed for non-smooth rotor stator contact systems in previous literature. These responses are isolated, in the sense that they will only meet the more trivial synchronous responses in the limit of zero damping and out of balance forcing. In addition to increasing our understanding of the responses of these systems, the work establishes the potential of numerical continuation as a tool to systematically explore the responses of nonlinear rotor systems.

Introduction

Nonlinear dynamic system's response can show bifurcations with small changes in its parameters. Therefore, systematically locating these bifurcation points is vital. Rotating machinery makes this more challenging with gyroscopic coupling that ties the whirl frequencies to the rotor speed. As a result, the internal resonance phenomenon can occur in a large range of operating rotor speeds, resulting in the bouncing orbits involving rotor-stator contact. The contacting interaction can be defined in various ways in the literature, ranging from rigid impact [1] to soft penalty contact [2].
For the solution of nonlinear problems within rotordynamics field, time simulations [3] and analytical and semi-analytical approaches based on harmonic balance method [4] have been used, aside from experimental investigations. However, this work uses the numerical continuation method applied directly to the systems of ODE, as a contrast to Refs. [5], due to its advantages over time simulations in computational cost and over algebraic methods in setup simplicity.
The analysed system is a 2-dof overhung rotor with isotropic cubic nonlinearity, shown in Figure 1 in the dimensional form. The equation of motion is nondimensionalised and transformed into the autonomous rotating coordinate frame equations given in Eq. (1).

$$U'' + \left(-\hat{\Omega}J\left(\hat{I_p}-2\right)+2\zeta\right)U' + \left(\hat{\Omega}^2\left(\hat{I_p}-1\right)+1+2\zeta\hat{\Omega}J\right)U + \gamma\hat{r}^2U = \hat{m}\hat{e}\hat{\Omega}^2\begin{Bmatrix}1\\0\end{Bmatrix}, \qquad J=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \tag{1}$$

where hat indicates the nondimensional form, $U=[\hat{u},\hat{v}]$ is the rotating coordinate vector with coordinates $\hat{u}$ and $\hat{v}$, $\hat{\Omega}$ is the rotor speed, $\hat{I}_p$ is polar moment of inertia, ζ is damping ratio, J is a skew symmetric matrix, γ is the cubic stiffness, $\hat{r}$ is the distance of disk geometric centre to the stationary equilibrium, $\hat{m}$ is the disk mass, $\hat{e}$ is the eccentricity of the disk, $(.)'$ is the derivative with respect to time, all in nondimensional forms.
The main focus of this work is on asynchronous periodic orbits that are caused by internal resonance, and we address similarities and differences to the case of contacting nonlinearity. The solutions found are validated with time simulation. AUTO open-source ODE numerical continuation software and MATLAB ode45 explicit integrator were used in the study.

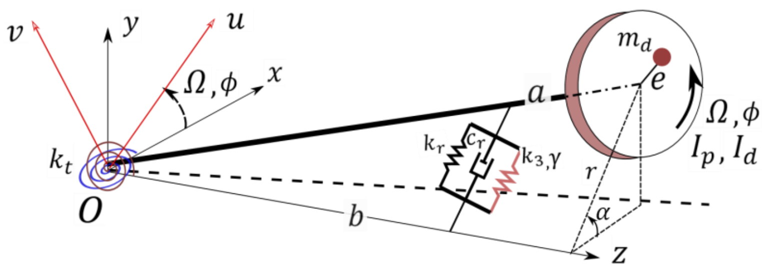

Figure 1: The 2-dof overhung rotor system with isotropic cubic stiffness.

Results

The bifurcation diagram resulting from the numerical continuation procedure is plotted in Figure 1(a). In this plot, the responses of the system with different damping levels are given together to see the effect of damping. The solution includes the synchronous response, which is skewed highly towards right due to stiffening, and asynchronous response that has two distinct sets of solution families, named here as double- and single-loop solution families after their apparent shapes in Figures 1(b) and 1(c). These orbits are periodic due to being viewed in the rotating frame (otherwise quasi-periodic in the stationary frame). Figures 1(d) and 1(e) show close-ups of asynchronous response to the synchronous solution. The double-loop and single-loop solutions were linked to the internal resonance conditions 3:1 and 2:1 in the rotating coordinates, respectively, by comparing them to the Campbell diagram with signed frequencies. These clearly show the closeness of the periodic responses to the stationary ones in the case of very low damping, ζ=1e-5. The periodic solutions were shown to be present only below a certain damping value that is peculiar to the solution family. This limit was found to be 0.0116 for double-loop solutions and 0.082 for single-loop solutions, as illustrated in Figure 2.

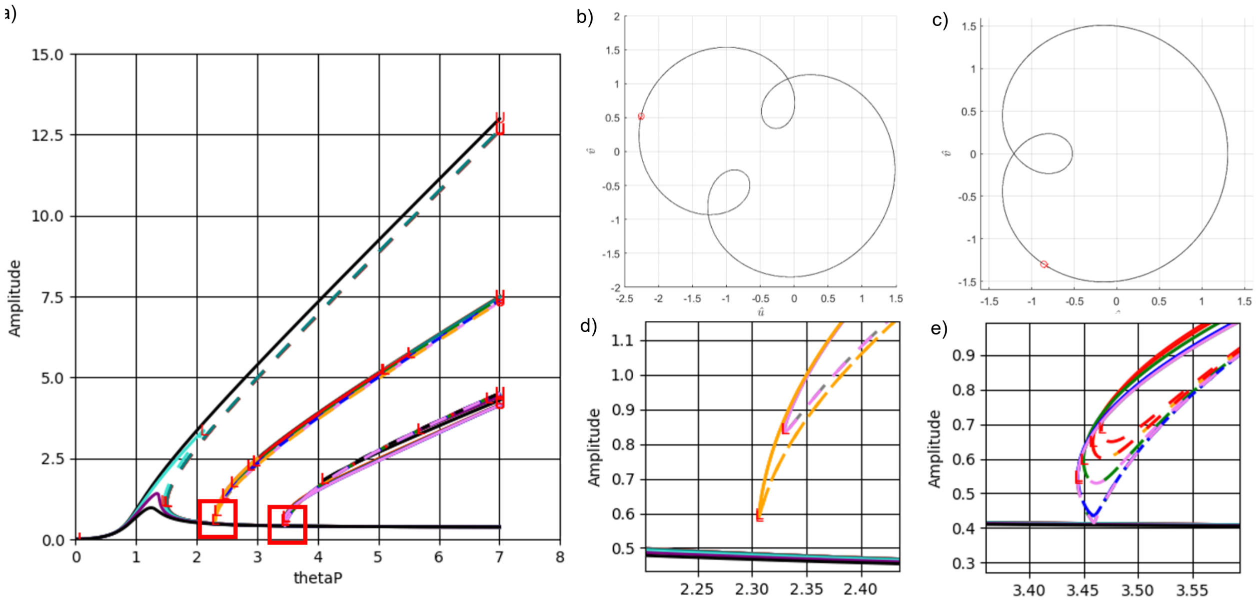

Figure 1: a) Continuation results as bifurcation diagram, b-c) double- and single-loop periodic orbits in rotating frame, d-e) and their low damping tips' close-up. Red "L" signs locate the folds of the solution families. Dashed lines show unstable solutions.

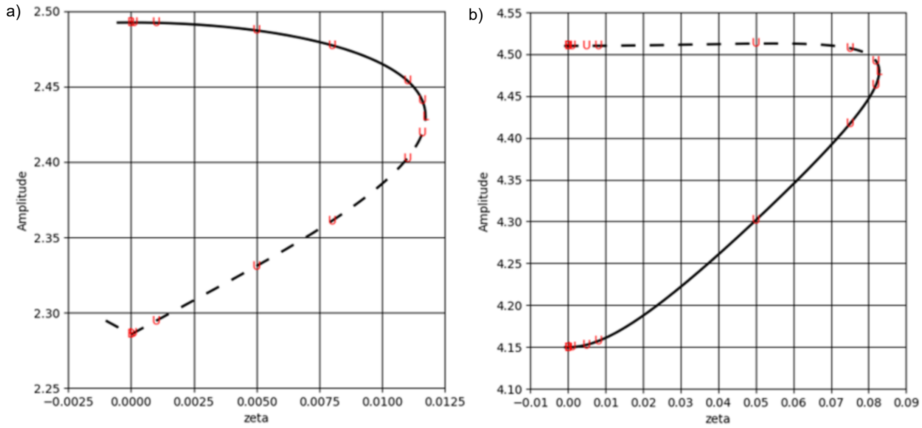

Figure 2: Damping limits for a) double- and b) single sloop solution families were 0.0116 and 0.082, respectively. The "U" signs locate the orbits corresponding to the sampled damping values. Dashed lines show unstable solutions.

Conclusions

The softer extreme of rotor-stator contact interaction was investigated by replacing the contact with a cubic stiffness nonlinearity. Direct application of continuation on the equations of motion was easy to setup and the solution procedure can be validated with time simulations easily. The following conclusions were drawn. The previously reported intermittent contact patterns for the models of hard impact and compliant snubber rings can also be observed in the case of the geometric nonlinearity of cubic stiffness. The level of damping changes the region where periodic motions was expected. Higher damping can cause the periodic solutions to disappear from the bifurcation diagrams. However, for low damping there is a limit to the growth of the periodic orbit region towards the lower rotor speeds; for very low values of damping (e.g. ζ=1e-4 and 1e-5) the periodic solution family approaches the synchronous solutions. It might be inferred that the gap closes asymptotically as ζ and the oscillation amplitude approach zero. This indicates that this region requires very little disturbance for synchronous solutions to jump to the periodic orbits.

References

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