Cristal Baschet: minimal model to predict the emergence of self-sustained oscillations

Audrey Couineaux, François Gautier and Frédéric Ablitzer
Laboratoire d'Acoustique de l'Université du Mans, CNRS UMR 6613, Le Mans, France

Context

The Cristal Baschet (Fig. 1a) is a contemporary musical instrument composed of a large number of glass rods arranged in chromatic scale. The sound is produced by rubbing glass rods with wet fingers, which causes the occurrence of stick-slip phenomenon. Each rod is connected to an assembly of threaded shafts, whose mechanical properties determine the pitch of the note. The vibrations are then transmitted to large metal sheets or cones that act as radiating elements. The manufacturing and tuning of the instrument is essentially based on empirical know-how and involves many parameters. Their influence on the sound and playability of the instrument is not clearly understood. One of the problems encountered is the difficulty to produce sounds in the high register of the instrument. In this study, a minimal model of Cristal Baschet is developed to analyze the emergence of self-sustained oscillations by means of linear stability analysis, with the aim of proposing design rules to improve the playability of the instrument.

Model

The minimal model focuses on the interaction between the wet finger and an isolated resonator (Fig. 1b). The resonator consists of a glass rod connected to a threaded shaft. Its dynamic behavior is represented by a set of modes. The modal parameters, which depends on design parameters (geometry, material properties...), can be obtained from a finite element model or from experimental modal analysis of the instrument. To model the occurrence of self-sustained oscillations from the frictional interaction with the finger, the knowledge of mode shapes at the interaction point is sufficient. Two gesture parameters, i.e. parameters that are controlled by the musician, are considered: the velocity of the finger v_f along the rod and the normal force F_N exerted by the finger on the rod. To describe the interaction between the finger and the resonator, the friction law considered in this study assumes that the glass rod perfectly sticks to the finger during sticking phases ($\Delta v = 0$) and that the friction force during sliding phases depends on the relative velocity $\Delta v = \dot{u} - v_f$ between the finger and the glass rod [2]. The sliding friction force is therefore expressed as $F_T = \mu(\Delta v) F_N$ with

$$\mu(\Delta v) = \mu_d + \frac{\mu_s - \mu_d}{1 - \Delta v / v_0} \,, \tag{1}$$

where μ_s is the coefficient of static friction, μ_d the asymptotic coefficient of dynamic friction and v_0 a parameter controlling the shape of the friction curve (Fig. 1c).

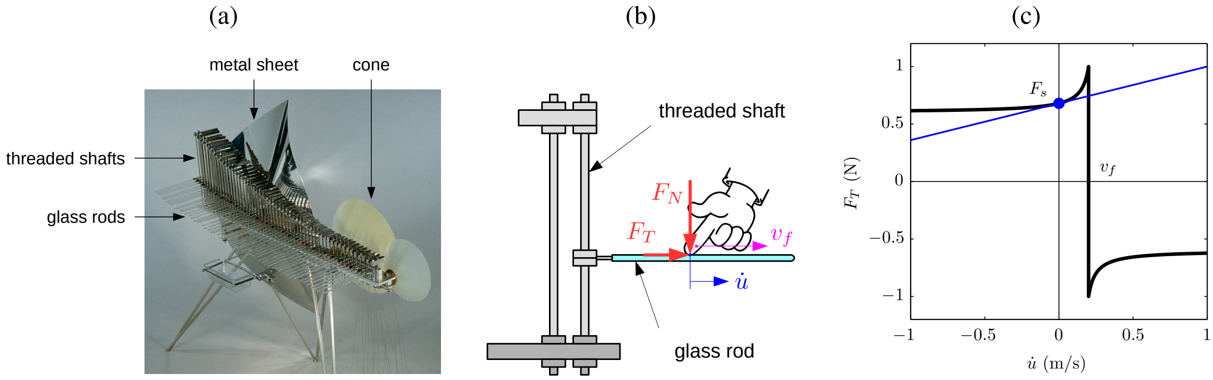

Figure 1: (a) Cristal Baschet (adapted from [1]). (b) Interaction between wet finger and resonator. (c) Friction curve and its linearization around static equilibrium in sliding situation.

Linear stability analysis

As a first step to determine the conditions at which the self-sustained oscillations can occur, the stability of the static solution in a sliding state is examined [3]. For this purpose, it is considered that the glass rod is in static equilibrium (i.e. $\dot{u} = 0$) under the action of a constant friction force F_s resulting from the motion of the finger at constant speed. Assuming small fluctuations of the glass rod velocity around its equilibrium position, a linearization of the friction curve around the static solution is performed to express the corresponding variations in friction force as $F_T \approx F_s + C\dot{u}$, where coefficient C represent the local slope of the friction curve (Fig. 1b). Inserting this linearized expression into the modal equations yields

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\mathbf{C} - C\boldsymbol{\Phi}_f\boldsymbol{\Phi}_f^{\mathrm{T}}\right)\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \boldsymbol{\Phi}_f F_s \,, \tag{2}$$

where $\mathbf{M} = \mathrm{diag}(m_n)$, $\mathbf{C} = \mathrm{diag}(2m_n\xi_n\omega_n)$, $\mathbf{K} = \mathrm{diag}(m_n\omega_n^2)$ are the modal mass, damping and stiffness matrices, and $\mathbf{\Phi}_f$ is a column vector containing the value of each mode shape at the location of the finger. Looking for a solution to the homogeneous equation in the form $\mathbf{Q}e^{\lambda t}$, one obtains an eigenvalue problem

$$\left(\lambda^2\mathbf{M} + \lambda\big(\mathbf{C} - C\mathbf{\Phi}_f\mathbf{\Phi}_f^{\mathrm{T}}\big) + \mathbf{K}\right)\mathbf{Q} = \mathbf{0}\,, \tag{3}$$

where each eigenvalue λ characterizes an oscillating component whose amplitude either decreases (if $\mathcal{R}e(\lambda) < 0$) of increases (if $\mathcal{R}e(\lambda) > 0$) over time, corresponding respectively to stability or instability of the static equilibrium. Self-sustained oscillations occurs when the smallest modal damping ratio $\xi_{\mathit{eff}} = -\mathcal{R}e(\lambda)|\lambda|^{-1}$ is negative (Fig. 2). The threshold depends on gesture parameters and friction law (both included in coefficient C) and design parameters (through the value of mode shapes $\mathbf{\Phi}_f$ at the interaction point).

Time-domain simulations

In order to verify the criterion calculated from linear stability analysis, time-domain simulations are performed using an explicit numerical scheme of the form

$$\mathbf{x}(t_{i+1}) = \mathbf{A}\mathbf{x}(t_i) + \mathbf{B}\mathbf{f}(t_i)\,, \tag{4}$$

where $\mathbf{x}$ is a vector containing all modal coordinates and their time derivatives and $\mathbf{f}$ is a vector containing the modal forces. The coefficients in $\mathbf{A}$ and $\mathbf{B}$ are obtained using a piecewise constant approximation of the right-hand side of modal equations. At each time step, the unknown friction force F_T exerted by the finger on the glass rod is obtained by enforcing a sticking or sliding condition. The results of time-domain simulations (Fig. 3) are in agreement with the predictions of linear stability analysis. The amplitude grow rate at various levels of normal force follows the same trend as the evolution of effective modal damping ratio shown in Fig. 2.

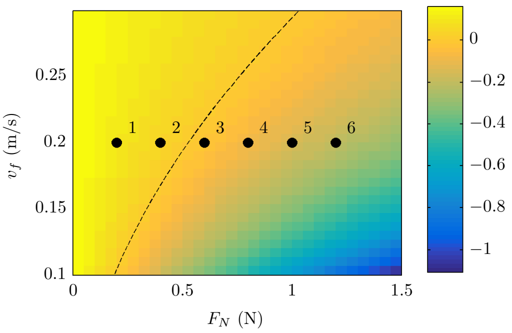

Figure 2: Smallest effective damping ratio ξ_{eff} as a function of the normal force F_N exerted by the finger and its velocity v_f. The dashed line indicates the limit between positive and negative values of the damping ratio, corresponding to stability or instability of the static equilibrium in sliding situation.

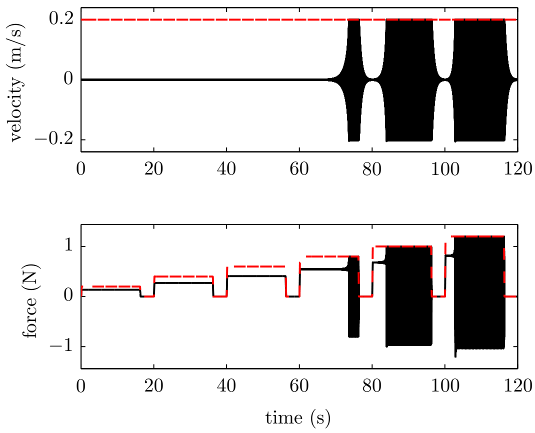

Figure 3: Velocity of the glass rod $\dot{u}$ and friction force F_T obtained from time-domain simulation. The red dashed lines are respectively the finger velocity v_f and the normal force F_N (which also corresponds here to the maximum static friction force since $\mu_s = 1$ in this simulation). The different levels of force correspond to points 1 to 6 in Fig. 2. Although masked by the scale fixed for the velocity, the third level of normal force leads to oscillations with increasing amplitude but low growing rate, not allowing to see the permanent periodic regime.

Conclusion

The minimal model of the Cristal Baschet describes the dependence of the amplitude grow rate of instabilities on the physical parameters: it is shown that the law of friction plays an essential role but its influence depends also on the mode shapes of the resonator at the connection point. These modal parameters are directly adjusted by the instrument maker when he tunes the mobility of these resonators. The model shows that a too low mobility can be responsible for the difficulties in obtaining the sound in the upper register.

References

[1] *Musical Instrument Museums Online (MIMO)* database, `https://mimo-international.com`, Cristal by Bernard Baschet and François Baschet, 1980, inventory number E.983.14.1, Musée de la Musique, Philharmonie de Paris (photo J.M. Anglès).

[2] S. Derler, G.-M. Rotaru, *Stick–slip phenomena in the friction of human skin*, Wear 301(1-2), 324-329, 2013.

[3] V. Debut, J. Antunes, and O. Inacio, *Linear modal stability analysis of bowed-strings*, J. Acoust. Soc. Am. 141(3), 2107–2120, 2017.