# Optimization Process for Ride Quality of a Nonlinear Suspension Model Based on Newton-Euler's Augmented Formulation

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*Summary*. This paper addresses modeling a Double A-Arm suspension, a three-dimensional nonlinear model has been developed using the multibody systems formalism. Dynamical study of the different components responses was done, particularly for the wheel assembly including the tire body and the knuckle. To validate those results, a similar model was constructed and simulated by RecurDyn; a professional multibody dynamics simulation software. The model has been used as an Objectif function that generate an optimization algorithm for ride quality improvement.

## Introduction

Modern systems are often very complex and consist of many components interconnected by joints and force elements. These systems are commonly called multibody systems (MBS). Vehicles can be considered as MBS and the dynamic equations that govern their motion are highly nonlinear which in most cases cannot be solved analytically. One must resort to the numerical solution of the resulting dynamic equations. Nikravesh work in [1] can be considered as an interesting reference about the Newton-Euler equations and its most suitable forms.
The aim of this study is the implementation of the augmented formulation in a dynamical model of the double A-Arms suspension. This system contains two lateral control arms to hold the wheel where the length between the upper and lower arms is not the same [2]. The upcoming sections represent a suspension mathematical model witch can be used as an Objectif function that generate an optimization algorithm for ride quality improvement.

## Dynamic Equations of motion

From [1], The equation of motion for the mass center of a rigid body is directly obtained from Newton's second law as

$$\mathbf{M}.\ddot{\mathbf{q}} = \mathbf{h} \tag{1}$$

For a system of $n_b$ constrained bodies, we must revise Equation 1 by including the reaction (constraint) forces

$$\mathbf{M}.\ddot{\mathbf{q}} = \mathbf{h} \quad \rightarrow \quad \mathbf{M}.\ddot{\mathbf{q}} = \mathbf{h} + {}^{(c)}\mathbf{h} \tag{2}$$

Assume that the position constraints between the $n_b$ bodies form $n_c$ constraint equations that are expressed in general form as

$$\mathbf{\Phi}(\mathbf{q}) = 0 \tag{3}$$

The velocity and acceleration constraints are expressed as

$$\dot{\mathbf{\Phi}} = \mathbf{\Phi}_q.\dot{\mathbf{q}} = 0 \tag{4}$$

$$\ddot{\mathbf{\Phi}} = \mathbf{\Phi}_q.\ddot{\mathbf{q}} - \boldsymbol{\alpha} = 0 \tag{5}$$

Where $\mathbf{\Phi}_q$ is the Jacobian matrix. According to the method of Lagrange multipliers, the array of reaction forces ${}^{(c)}\mathbf{h}$ can be presented as

$${}^{(c)}\mathbf{h} = \mathbf{\Phi}_q^T.\boldsymbol{\lambda} \tag{6}$$

$\boldsymbol{\lambda}$ is a vector of $n_c$ Lagrange multipliers. Then Equation 2 is rewritten as

$$\mathbf{M}.\ddot{\mathbf{q}} = \mathbf{h} + \mathbf{\Phi}_q^T.\boldsymbol{\lambda} \tag{7}$$

This system contains $n_b$ equations and $n_b$+$n_c$ unknowns, accelerations and Lagrange multipliers.

## THREE-DIMENSIONAL NONLINEAR MODEL OF DOUBLE A-ARM SUSPENSION

To well understand the selected approach, three-dimensional nonlinear model of double A-Arm suspension will be presented. The system contains four (04) moving bodies that are connected with each other by kinematic joints and driven by a set of forces, Figure 1. With this formulation the number of unknowns is $n + m$, since not only $\boldsymbol{q}$ but also $\boldsymbol{\lambda}$ needs to be calculated.

$$\mathbf{q} = (\mathbf{q}_1 \;\; \mathbf{q}_2 \;\; \mathbf{q}_3 \;\; \mathbf{q}_4)^T \tag{8}$$

where

$$\mathbf{q}_i = (x_i \;\; y_i \;\; z_i \;\; \gamma_i \;\; \beta_i \;\; \varphi_i)^T \tag{9}$$

For validation, RecurDyn software is used. It provides realistic simulation of multibody dynamics.

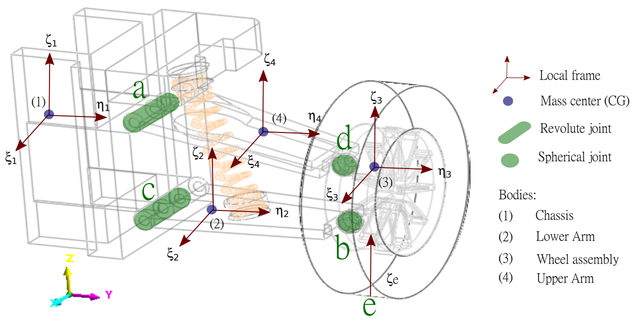


Figure 1: Three-Dimensional Double A-Arm Suspension system

| Type of joint | Notation | body$_i$-body$_j$ | Constraints Eqts = 0 |
|---|---|---|---|
| Translational | $\Phi^{(2tr,3rot)}_{CoG_1}$ | GRF-1 along Z-axis | $\begin{pmatrix} x_1 & y_1 & \gamma_1 & \beta_1 & \varphi_1 \end{pmatrix}^T$ |
| Revolute in $c$ | $\Phi^{(3tr,2rot)}_{c}$ | 1-2 about x-axis | $\Phi^{3tr}(c_1, c_2)$ $\Phi^{2rot}_1(\vec{u}_1, \vec{u}_2)$ or $\Phi^{2rot}_2(\vec{u}_2, \vec{u}_1)$ |
| Spherical in $b$ | $\Phi^{(3tr)}_{b}$ | 2-3 | $\Phi^{3tr}(b_2, b_3)$ |
| Spherical in $d$ | $\Phi^{(3tr)}_{d}$ | 3-4 | $\Phi^{3tr}(d_3, d_4)$ |
| Cylindrical in $a$ | $\Phi^{(2tr,2rot)}_{a}$ | 4-1 $\begin{cases} \text{along x} - \text{axis} \\ \text{about x} - \text{axis} \end{cases}$ | $\Phi^{2tr}(a_4, a_1) = 0$ $\Phi^{2rot}_4(\vec{u}_4, \vec{u}_1)$ or $\Phi^{2rot}_1(\vec{u}_1, \vec{u}_4)$ |
| Driving in $e$ | $\Phi^{1D}_{A}$ | 3-A in Z-direction | $z_A - f(t)$ |

Table 1: Constraints configuration of the double A-arm system.

## OPTIMIZATION PROCESS

The chassis vertical acceleration is minimized, while the design constraints on the suspension working space and dynamic tire load should be satisfied. The RMS of the acceleration of a sprung mass $\ddot{Z}1$ is frequently used to evaluate the riding quality of a vehicle. A rider's comfort improves as the acceleration decreases. Ride comfort is chosen to be the design criterion. The design optimization problem can be described as: Minimize $\ddot{Z}1(m_1, m_2, m_3, m_4, K_s, N_s, K_t, N_t)$.
We choose the *fmincon* function in MATLAB to execute this simple optimization process which .

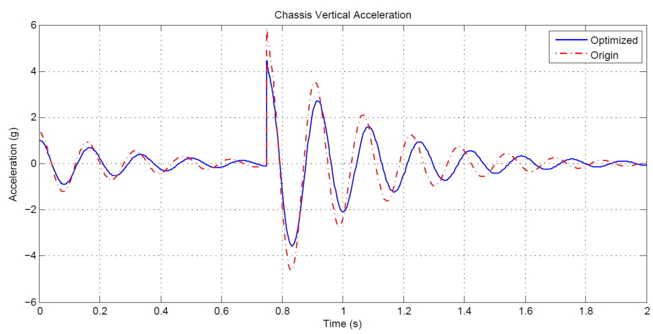


Figure 2: Optimized and origin vertical accelerations.

## Conclusions

This paper has focused on the vertical motion of the Nonlinear planar model of the Double A-Arm suspension. The comparison between the responses of the three-Dimensional nonlinear model and the one in the simulation software shows that a precise setting in the multibody modeling of mechanical systems can offer great results in short period of time with low processing capacity requirements.
As a final step in the process, this model was transformed to an Objectif function. The use of this function allowed us to generate an optimization algorithm capable of calculating the optimal suspension to improve vehicle ride quality. The vertical acceleration of the chassis was reduced but the process has some difficulties because of the set of the differential-algebraic equations and the constraints violation

## References

[1] Nikravesh, P.E. (1988) Computer-Aided Analysis of Mechanical Systems. Prentice-Hall, New Jersey.
[2] Reza, N.J. (2014) Vehicle Dynamics, Theory and Application. Second Edition. Springer, New York.