Suppression of synchronous spiking in two interacting populations of excitatory and inhibitory quadratic integrate-and-fire neurons

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Summary. Collective oscillations and their suppression by external stimulation are analyzed in a large-scale neural network consisting of two interacting populations of excitatory and inhibitory quadratic integrate-and-fire neurons. In the limit of an infinite number of neurons, the microscopic model of this network can be reduced to an exact low-dimensional system of mean-field equations. Bifurcation analysis of these equations reveals three different dynamic modes in a free network: a stable resting state, a stable limit cycle, and bistability with a coexisting resting state and a limit cycle. We show that in the limit cycle mode, high-frequency stimulation of an inhibitory population can stabilize an unstable resting state and effectively suppress collective oscillations. We also show that in the bistable mode, the dynamics of the network can be switched from a stable limit cycle to a stable resting state by applying an inhibitory pulse to the excitatory population. The results are confirmed by numerical simulation of the microscopic model.

Introduction

Synchronization processes in large populations of interacting dynamical units are the focus of intense research in physical, technological and biological systems. In neural networks, synchronization can play a dual role. Under normal conditions, synchronization is responsible for cognition and learning, while excessive synchronization can cause abnormal brain rhythms associated with neurological diseases such as Parkinson's disease, epilepsy, and others. Various algorithms have been developed to suppress unwanted synchronized network oscillations. A therapeutic procedure clinically approved for the treatment of Parkinson's disease is a high-frequency (HF) deep brain stimulation (DBS). The mechanism of action of DBS is still poorly understood. Clinical observations show that the effects of lesions and DBS of the same target area are similar. This suggests that HF stimulation suppresses neuronal activity in the target area. In this context, the effect of HF stimulation can be explained in terms of stabilization of neuron's resting state [1]. However, there is no clear theoretical understanding of how HF stimulation affects synchronization processes in neural networks.

Recent advances in dynamical systems theory have allowed us to better understand the effects of synchronization in large-scale oscillatory networks. A major breakthrough in these studies was achieved by Ott and Antonsen [2], who showed that the microscopic model equations of globally coupled heterogeneous phase oscillators (Kuramoto model) can be reduced to a low-dimensional system of ordinary differential equations that accurately describe the macroscopic evolution of the system in the infinite-size (thermodynamic) limit. Later this approach was extended to a particular class of heterogeneous neural networks composed of all-to-all pulse-coupled quadratic integrate-and-fire (QIF) neurons [3]. In thermodynamic limit, a low-dimensional system of mean-field equations was derived for biophysically relevant macroscopic quantities: the firing rate and the mean membrane potential. The approach has been further developed in recent publications to analyze the occurrence of synchronized macroscopic oscillations in networks of QIF neurons with a realistic synaptic coupling [4], in the presence of a delay in couplings [5] and in the presence of noise [6].

Here, we demonstrate that mean-field equations are useful not only for understanding the occurrence of collective oscillations in large-scale neural networks, but also for understanding the effect of stimulation on synchronization processes [7]. As an example, we consider a network of two interacting populations of excitatory and inhibitory QIF neurons.

Microscopic model and low-dimensional mean-field equations in the thermodynamic limit

The microscopic state of the QIF network is determined by the set of $2N$ neurons' membrane potentials $\{V_j^{(E,I)}\}_{j=1,\dots,N}$, which satisfy the following system of $2N$ ordinary differential equations:

$$\tau \dot{V}_j^{(E,I)} = (V_j^{(E,I)})^2 + \eta_j^{(E,I)} + \mathcal{I}_j^{(E,I)}, \quad \text{if } V_j^{(E,I)} \geq V_p \text{ then } V_j^{(E,I)} \leftarrow V_r. \tag{1}$$

Here, τ is the membrane time constant and $V_j^{(E,I)}$ is the membrane potential of neuron j in either the excitatory (E) or the inhibitory (I) population. For simplicity, we set the number of neurons N and the time constant τ the same for both populations. The heterogeneous parameter of excitability $\eta_j^{(E,I)}$ is a current that specifies the behavior of each isolated neuron and the term $\mathcal{I}_j^{(E,I)}$ defines the synaptic coupling between neurons as well as external stimulation. The isolated neurons ($\mathcal{I}_j^{(E,I)} = 0$) with the negative value of the parameter $\eta_j^{(E,I)} < 0$ are at rest, while the neurons with the positive value of the parameter $\eta_j^{(E,I)} > 0$ generate instantaneous spikes, which are approximated by the Dirac delta function. The spikes are emitted at the moments when the membrane potential $V_j^{(E,I)}$ reaches a peak value V_p. Immediately after the spike emission the membrane potential is reset to a value V_r. We assume $V_p = -V_r \to \infty$. The values of the heterogeneous parameter $\eta_j^{(E,I)}$ for both populations are independently taken from the Lorentzian distributions: $g_{E,I}(\eta) = \Delta_{E,I}/\{\pi[(\eta - \bar{\eta}_{E,I})^2 + \Delta_{E,I}^2]\}$, where $\Delta_{E,I}$ and $\bar{\eta}_{E,I}$ are respectively the width and the center of the

distribution for the excitatory (E) and inhibitory (I) populations. The last term $\mathcal{I}_j^{(E,I)}$ in Eqs. (1) describes synaptic coupling and an external stimulation. For the excitatory and inhibitory populations, this term respectively is

$$\mathcal{I}_j^{(E)} = -J_{IE}S_I(t) + I_E(t), \quad \mathcal{I}_j^{(I)} = J_{EI}S_E(t) - J_{II}S_I(t) + I_I(t). \tag{2}$$

Here, $S_E(t)$ and $S_I(t)$ determine the mean synaptic activation of E and I populations:
$S_{E,I}(t) = \frac{\tau}{N}\sum_{j=1}^{N}\sum_{k\setminus(t_j^k)_{E,I}<t}\delta(t-(t_j^k)_{E,I})$, where $(t_j^k)_{E,I}$ is the time of the kth spike of the jth neuron in either E or I population and $\delta(t)$ is the Dirac delta function. The positive parameters J_{EI}, J_{IE} and J_{II} define synaptic weights. The current $-J_{IE}S_I(t)$ inhibits E neurons due to synaptic activity of I population, while the current $J_{EI}S_E(t)$ excites I neurons due to synaptic activity of E population. The term $-J_{II}S_I(t)$ determines recurrent inhibition of neurons within I population. The currents $I_E(t)$ and $I_I(t)$ represent external homogeneous stimulation of the excitatory and the inhibitory populations, respectively.
In the thermodynamic limit $N \to \infty$, the microscopic model (1) can be reduced to the exact system of four ODEs [3]:

$$\tau\dot{r}_E = \Delta_E/\pi + 2r_Ev_E, \quad \tau\dot{v}_E = \bar{\eta}_E + v_E^2 - \pi^2r_E^2 - J_{IE}r_I + I_E(t), \tag{3a}$$

$$\tau\dot{r}_I = \Delta_I/\pi + 2r_Iv_I, \quad \tau\dot{v}_I = \bar{\eta}_I + v_I^2 - \pi^2r_I^2 + J_{EI}r_E - J_{II}r_I + I_I(t), \tag{3b}$$

where $r_{E,I}(t)$ and $v_{E,I}(t)$ are respectively the spiking rates and the mean membrane potentials of E and I populations.

Results and conclusions

Relatively simple mean field Eqs. (3) make it possible to conduct a thorough bifurcation analysis of various dynamic modes of a free network and to reveal the mechanisms of action of various stimulation algorithms. We performed a bifurcation analysis of a free network depending on the coupling strengths J_{EI} and J_{IE} of the bidirectional interaction between excitatory and inhibitory populations and the coupling strength J_{II}, which determines the interaction within the inhibitory population. We also built a bifurcation diagram in the plane of the parameters $(\bar{\eta}_I, \bar{\eta}_E)$, which determine the centers of the distributions $g_{E,I}(\eta)$ of the excitability parameter η for I and E populations. As a result of this analysis, three different modes were established. Depending on the values of the parameters, the system can have a single stable fixed point, a single stable limit cycle, or be in a bistable mode with these two coexisting attractors.
As the next step in our analysis, we looked at the problem of controlling network synchronization. Some neurological diseases are successfully treated with high-frequency stimulation. Here, we tested the effectiveness of the HF algorithm for suppressing synchronous spiking in the network of excitatory and inhibitory QIF neurons. We have shown that HF stimulation of the inhibitory population is very effective, whereas HF stimulation of the excitatory population cannot suppress the oscillations. The mechanism of action of HF stimulation is explained using mean-field equations averaged over the stimulation period. The averaged mean-field equations are equivalent to the free mean-field equations, but with a modified parameter $\bar{\eta}_I$ or $\bar{\eta}_E$, depending on which inhibitory or excitatory population is stimulated. When HF stimulation is applied to the inhibitory population, changing the $\bar{\eta}_I$ parameter increases the proportion of spiking neurons in that population. This leads to the stabilization of the state of rest of the network. The averaged mean-field equations made it possible to obtain an analytical expression for the threshold amplitude of HF stimulation, which stabilizes the resting state. This amplitude is proportional to the frequency of stimulation.
HF stimulation of the excitatory population is ineffective, since changing the $\bar{\eta}_E$ parameter increases the proportion of spiking neurons in the excitatory population and cannot stabilize the resting state of the network. Nevertheless, stopping the network oscillation by controlling the excitatory population can still be achieved if the system parameters are in the bistable area. By applying a rectangular inhibitory pulse to this population, the network state can be switched from the stable limit cycle to the stable state of rest.
To test the performance of the above stimulation algorithms for finite-size networks, we numerically simulated the equations of the microscopic model. Modeling networks with 2000 excitatory and 2000 inhibitory QIF neurons gave results that are in good agreement with the results obtained from the mean-field equations. Based on our research, we believe that mean-field equations derived from the microscopic dynamics of interacting QIF neurons can serve as an effective tool for developing various stimulation algorithms to control synchronization processes in large-scale neural networks.

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