Reconfigurable Feedback Control of a Flexible Structure with a Nonstationary Backlash via a Digital-Twin Framework

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Summary. The following work portrays a method to adaptively reconfigure a nonlinear controller to ensure the asymptotical stability of a position control mechanism that exhibits nonlinear and nonstationary backlash. The presented approach utilises a remote-based digital twin to perform the reconfiguration. Thus, the digital controller can be implemented locally using simple hardware. At the same time, the reconfiguration process takes place on a remote server that uses the acquired signal and dynamical models to perform simulations and calculations at a high rate. The proposed approach is demonstrated based on a numerical simulation of a three degrees-of-freedom structure coupled to an electrical DC motor via a lead screw. The lead screw exhibit backlashes whose characteristic width grows during the normal operation of the mechanism. Such system loses their closed-loop stability when the backlash becomes large. A nonlinear dead zone is used to restabilise the closed-loop system. The dead zone parameter is tuned based on a digital twin to ensure the closed-loop asymptotical stability and to avoid over-conservative design. Finally, the successful use of the proposed methodology is presented based on the simulation results.

Introduction

When a feedback-controlled mechanical system characteristic changes over time, the original design of the controller can lead to instabilities [1]. Wear and tear lead to such changes by introducing, for example, a nonlinear backlash. Although backlash is present in most mechanical position control systems [2], its effect on the linear feedback law is frequently neglected. However, even when the backlash is considered when formulating the identification and control problem [2, 3], its characteristics are usually modelled as a time-invariant. However, these time-invariant models are not applicable when wear and tear processes occur [4], in which case the characteristics of the backlash change over time.
To account for the time dependence of the backlash, an adaptive controller [3] can be used. However, the downside of doing so is that the controller's structure is complex, and its implementation is not straightforward. Therefore, in this work, a PID controller [5] is considered, with the addition of a dead zone to overcome the limit cycles introduced by the backlash and maintain minimal control effort minimal in the backlash gap [3]. The dead zone parameter is estimated asynchronously using a remotely-based digital twin [6] to monitor the changing backlash.
The core purpose of the digital twin is to create a fusion between physical and data-based models. Most research on digital twins is concerned with utilising them for decision-making and structural health monitoring, with only a few remarks on the use of digital twins in control, using classical ideas like controller scheduling [7]. This is in contrast to the past developments in digital signal processing, which have led to a more profound understanding of advanced control methodologies [8, 9]. Therefore, this work tries to pave the way and demonstrate the capabilities of incorporating digital twins into system control by employing the digital twin as an asynchronous estimator, resulting in a simple structure of the digital controller. This idea is not, by any means, a demonstration of the full capabilities of digital twins but only acts as a stepping stone towards a fuller understanding of digital twins' potential use in feedback control.

Control problem formulation

PID Feedback controllers are widely used for position control of flexible structures [5]. The schematics of a position controller of a flexible mechanism are presented in Figure 1(a). The mechanism is composed of 3 lumped masses con-

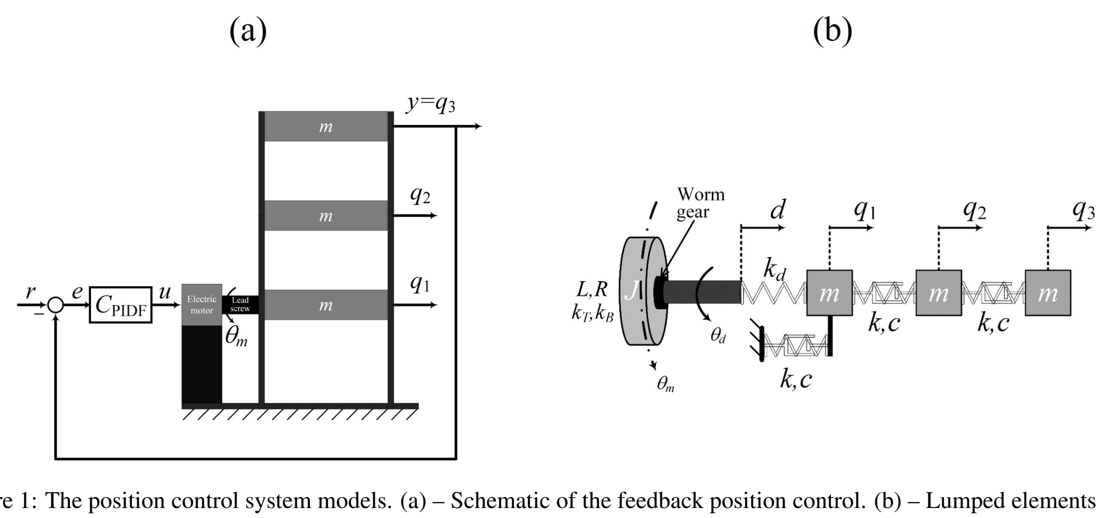

Figure 1: The position control system models. (a) – Schematic of the feedback position control. (b) – Lumped elements model.

nected via flexible beams. An electrical motor that is connected via a lead screw to the bottom mass is used to drive the system. The system's lumped model shown in Figure 1(b) is used for the numerical simulation. In the lumped model of Figure 1(b), L and R denote the electrical motor's inductance and resistance, respectively, J its lumped inertia, k_B and k_T are the torque and back-emf motor's constants, θ_m the angular position, and ω_m the angular velocity.
The motor is connected to the lower mass via a lead screw, whose angle is denoted as θ_d, and its end position, d, is related to its angular position via

$$d = p\theta_d, \tag{1}$$

where p denotes the lead screw's pitch. The lead screw elasticity is modelled using a linear spring k_d, which in the following simulation is assumed to be infinitely rigid. m, k, and c denote the lumped mass of each platform and the connecting beams' elasticity and dissipation, which are assumed to be linear. Finally, q_i $i = 1, 2, 3$, denote the horizontal position of each platform. The physical values used throughout the paper are given in Table 1.

Table 1: Physical properties and values used in the numerical simulation of the three DOF with the DC electrical motor

Symbol	Value, units	Symbol	Value, units	Symbol	Value, units
m	5.2, kg	J	$5 \cdot 10^{-6}$, kg $\cdot$ m^2	p	$10^{-3}/2\pi$, m/rad
k	10^4, N/m	L	10^{-3}, H	k_T	0.8, (N·m)/A
c	36.9, N/(m·s)	R	12, Ohm	k_B	0.1, V/(m/s)

Linear models and controller

Assuming that the inertial loading of the structure is negligible compared to the motor's torque, a single direction coupling model can be used to represent the motor and platform dynamics. The motor's input-output relation under these assumptions can be written as

$$\theta_m = \frac{1}{s}\frac{k_T}{JLs^2 + JRs + k_B k_T}u, \tag{2}$$

where u denotes the input voltage to the motor. The top platform and the lead screw positions are related via:

$$q_3 = \frac{c^2s^2 + 2cks + k^2}{m^2s^4 + 3cms^3 + (c^2 + 3km)s^2 + 2cks + k^2}d. \tag{3}$$

When the lead screw backlash is neglected, i.e., $\theta_d = \theta_m$, the static relation of Eq. (1) can be used to couple the two systems' dynamics.
To control the motion and position of the top platform such that it follows a stepwise constant reference signal, r, a feedback PID controller of the following form

$$C_{PID}(s) = K_p + K_i\frac{1}{s} + K_d\frac{s}{T_f s + 1}, \tag{4}$$

is introduced, where K_p, K_i, and K_d are the proportional, integral, and differential gains, respectively, and T_f is the differential filter time constant. Such that the motor's input voltage u is obtained from the tracking error $e = r - q_3$ as

$$u = C_{PID}(s)e. \tag{5}$$

The PID was designed to achieve a zero steady-state error, a bandwidth frequency of 10 rad/s, and a minimum phase margin of 10 deg.

Nonlinear backlash model

Since the positioning mechanism is based on a lead screw, it might exhibit backlash in practice, mainly as it wears. A simple model for backlash, which is a nonlinear dynamic element, is by introducing a dynamic dead zone of known width whenever the velocity changes sign [3].
The following nonlinear model can be used to relate the motor and lead screw angular velocities:

$$\omega_d = \begin{cases} \omega_m & (\mathrm{sign}(\omega_m) > 0 \text{ and } \theta_m - \theta_d \geq \theta_b/2) \text{ or } (\mathrm{sign}(\omega_m) < 0 \text{ and } \theta_m - \theta_d \leq -\theta_b/2) \\ 0 & \text{Otherwise} \end{cases}, \tag{6}$$

where θ_b denotes the backlash gap angle.
A time-invariant backlash model, with a constant backlash gap angle θ_b, is readily available with many simulation software. However, since the backlash gap angle is time-varying here, a separate procedure for the state transition was used, based on the model of Eq. (6). To illustrate its response, Figure 2 shows the output of the backlash model for a gap angle of $\theta_b = 40$ degrees, and for an amplitude changing sinusoidal input of the following form:

$$\theta_m(t) = (1 + t/10)\sin(2\pi t).$$

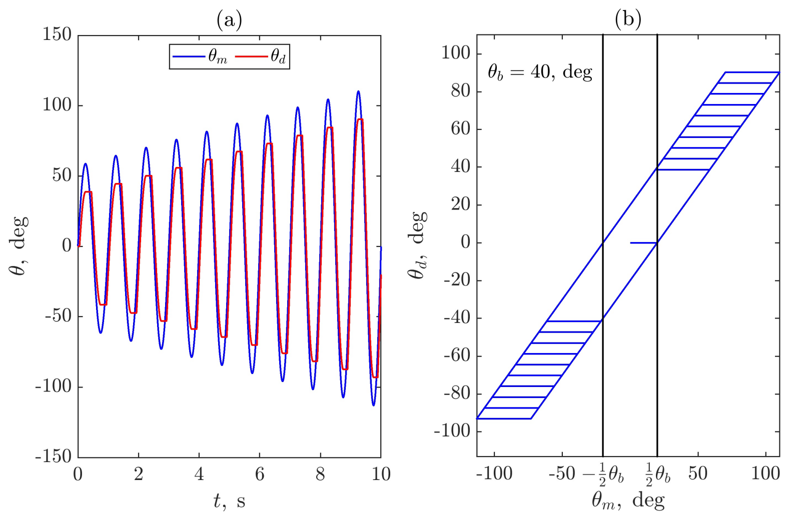

Figure 2: Simulation results for the backlash model for an amplitude-modulated sinusoidal input.

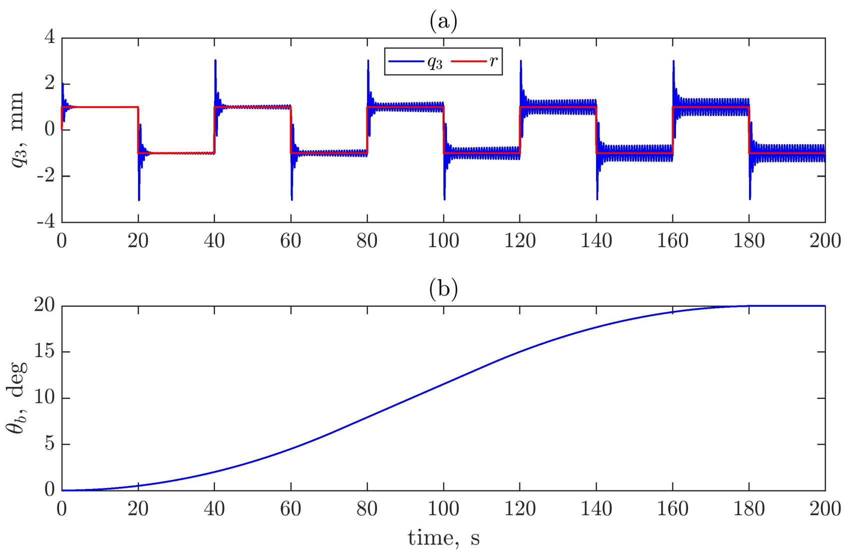

Figure 3: Simulation results for the linear controller with a nonstationary backlash.

The lost motion between θ_m and θ_d can be seen in Figure 2(a), which shows their two waveforms, whereas the nonlinear characteristics of the backlash are more transparent in Figure 2(b), which plot one against the other.
The simulation results of the feedback position control mechanism with a predetermine backlash's gap angle profile are shown in Figure 3. When the width is small, the position control is applied successfully to the system, and the top mass position does converge to the required reference position. However, a limit cycle is observed when the width becomes more significant, and the system can no longer be considered asymptotically stable.

Switched dead zone controller

A dead zone of a prechosen width is incorporated into the controller to capture and maintain the backlash's effect and restabilise the system [3]. The controller input-output relation can now be read as:

$$u_{dz} = \begin{cases} C_{PID}(s)e_{dz} & |e| > \varepsilon_{dz} \\ 0 & \text{Otherwise} \end{cases}, \tag{7}$$

where

$$e_{dz} = \begin{cases} e & |e| > \varepsilon_{dz} \\ 0 & \text{Otherwise} \end{cases}, \tag{8}$$

and the dead zone tolerance, ε_{dz}, is given by:

$$\varepsilon_{dz} = p\hat{\theta}_b, \tag{9}$$

$\hat{\theta}_b$ being the backlash gap angle estimation.

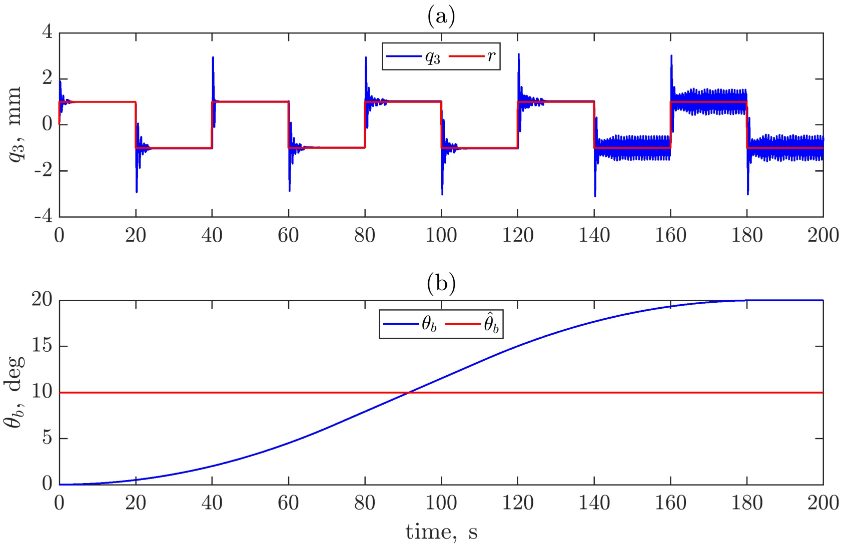

Figure 4: Simulation results for the dead zone controller with a nonstationary backlash and a constant backlash gap angle estimation.

If the controller's dead zone width is tuned too conservatively, i.e., $\hat{\theta}_b$ is chosen large, a significant bias is introduced between the required and actual position. Moreover, no force is applied to the structure inside the dead zone, and its response decays similarly to its initial condition response. Thus, the desired damping levels of the designed controller are lost, and the settling time can become significantly longer.

A second simulation was carried out using the same backlash's width profile with the dead zone switched controller. In the simulation, the backlash gap angle estimator was chosen as the mean value, i.e., $\hat{\theta}_b = 10$ deg. The simulation's results are shown in Figure 4. It is clear that the system again loses stability when the real backlash's gap angle is significantly larger than the modelled one. Moreover, when the actual backlash's gap angle is smaller, the settling time does get longer as anticipated. An estimation of the backlash is proposed to ensure the closed-loop stability and reduce the amount of conservative used for choosing the dead zone width. The backlash's gap angle estimation is done via a remote digital twin to avoid overflowing the controller and to enable, in the future, the use of more sophisticated estimation and designing methods.

Digital twin reconfigurable controller

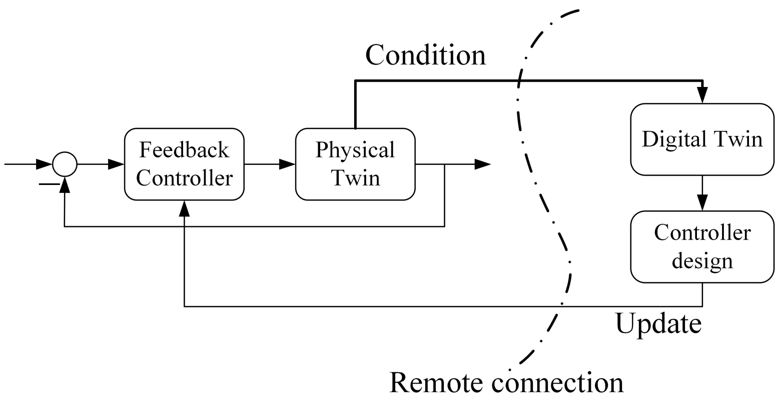

Figure 5: Schematics of the digital twin based reconfigurable controller.

The digital twin is a modern concept in engineering. At its core, the digital twin aims to create a fusion between meaningful physical models of a system, combined with measured data collected from different sensors throughout the physical system's operation [6]. The digital twin as a concept has so far been used during several stages of a product's life-cycle: designing, manufacturing, and condition monitoring. However, the use of a digital twin is not as common in control theory [7], specifically motion control. Although many systems for which digital twins are considered, like wind turbines and aircraft [6], depend on motion controllers for their smooth operation. The idea put forward in this work is to use a simple model of a digit twin to monitor the backlash's width and reconfigure the dead zone feedback controller. The block diagram of Figure 5 represents the physical system (physical twin and feedback controller) on the left-hand side and the simulated system (digital twin and controller design) on the right-hand side. The main difference between the use of a digital twin framework and adaptive control [1, 8], is the asynchronous nature of the communication and the ability to use highly detailed simulations. Therefore, ensuring that the digital feedback controller will operate properly, where the states' and controller's estimation and reconfiguration occur on a remote server.

Backlash gap angle estimation

It is assumed that the control signal, u, and the top platform position, q_3, are measured and are sent to the digital twin asynchronous. After each data transmission, the digital twin can be used to simulate the internal signals of the feedback system. It is desired to estimate both the output and input of the backlash and compare them to estimate the backlash angle. The input of the backlash, the motor's angle, is estimated using the measured control signal, which is fed to the discretised dynamical model of Eq. (2), assuming a zero-order hold and an ideal sampler. The output of the backlash, the lead screw's angle, is estimated using the measured top platform position and a digital-based delayed inverse of the dynamical model of Eq. (3). An impulse invariant discretisation algorithm is used [9] to ensure that the sampled system remains minimum phase, up to the addition of zeros at the origin. Then the inverse filter of the sampled system can be calculated. A discrete modelling delay of N samples is added to the sampled system inverse filter to ensure causality. The minimal value of N required for causality is equal to the pole excess of the sampled system. Since N step delay is added to the backlash output, a similar delay is added to the digital-sampled motor's dynamics. The estimators of the motor and lead screw's angles, $\hat{\theta}_m$ and $\hat{\theta}_d$, are thus the output of the following two sampled time linear systems:

$$\hat{\theta}_m = z^{-N}\hat{G}_1(z)u, \tag{10}$$

$$\hat{\theta}_d = z^{-N}\hat{G}_2^{-1}(z)y, \tag{11}$$

where $\hat{G}_1(z)$ and $\hat{G}_2(z)$ are the discretised dynamical model of Eq. (2) and Eq. (3), respectively.
To estimate the backlash angle, θ_b, the difference between the angles is calculated. It follows from the backlash definition that if $\omega_d \neq 0$, then $\theta_m - \theta_d = \Delta\theta_\pm$, where the plus or minus are chosen based on the sign of ω_m. Therefore, it follows that

$$\theta_b = \Delta\theta_+ - \Delta\theta_-, \tag{12}$$

Equation (12) is accurate up to measurement and estimation errors. Therefore, a threshold is used for the lead screw's angular velocity criteria to overcome these errors. In addition, based on the previous simulation of Figure 4, it seems safer to use a slight overestimation of the backlash gap angle. Consequently, the estimated values which correspond to the angular velocity threshold are rounded up to the nearest scaled-integer degree, and the mean is taken as the estimator. If degree units are used, then the estimator is:

$$\hat{\theta}_b = \frac{1}{N_{\Delta\theta_+}}\sum_{t_s\in t_+}\lceil\Delta\theta_+\rceil_{l/2} - \frac{1}{N_{\Delta\theta_-}}\sum_{t_s\in t_-}\lfloor\Delta\theta_-\rfloor_{l/2}, \tag{13}$$

where t_+ represents the sampling times when $\omega_d \neq 0$ and $\omega_m > 0$, similarly, t_- represents the sampling times when $\omega_d \neq 0$ and $\omega_m < 0$, $N_{\Delta\theta_+}$ and $N_{\Delta\theta_-}$ denote the number of elements in each time subset, and the ceil l and floor l operators are defined as:

$$\lceil x\rceil_l := l\cdot\mathrm{ceil}(x/l)\ ,\ \lfloor x\rfloor_l := l\cdot\mathrm{floor}(x/l). \tag{14}$$

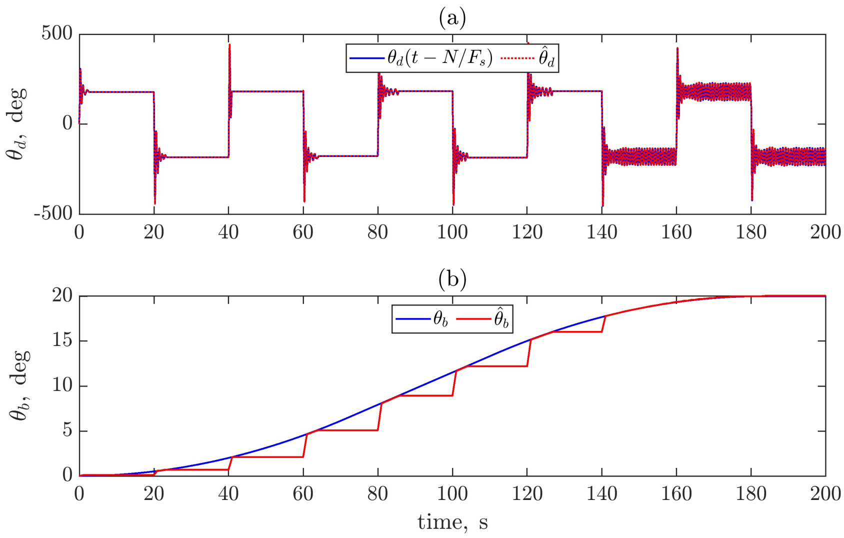

Figure 6: Post-processing identification of the time-varying backlash angle.

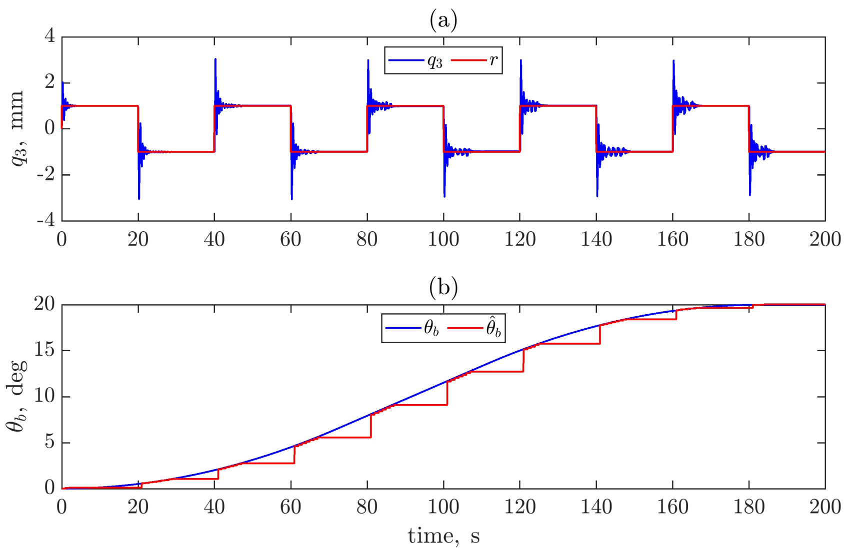

Figure 7: Simulation results for the digital twin reconfigurable controller with a nonstationary backlash.

To check the estimation algorithm and help tune the angular velocity threshold, the simulation data in Figure 4 was post-processed. A 5000 Hz sampling frequency was used, the angular velocity threshold was set to $\varepsilon_b = 0.1$ rad/s, the number of discrete delays added to the digital sampled inverse system $N = 2$, and the rounding factor was chosen as $l = 1/10$.
Figure 6(a) shows the command and output signals, and Figure 6(b) shows the actual and estimated backlash angles obtained using the proposed estimation algorithm. When the input to the digital sampled inverse system is constant (The top mass is at rest following a step change), the estimated lead screw's angle matches the measured one. However, if the input is not constant, a small phase and gain delay are present. This is due to the dynamics of the inverse filter. Consequently, the actual and estimated angles differ whenever the lead screw angular velocity is nonzero. However, as seen in Figure 6(b), the estimation algorithm successfully identifies the backlash angle, even though the two angles differ slightly. The results of Figure 6 were obtained based on noncausal signal post-processing, so the backlash angle estimation has no delay. In the following subsection, the real-time use of a digital twin is simulated, including this processing delay, to include its effects in restabilising the system.

Real-time simulation

A third simulation was carried out based on the block diagram of Figure 5, the dead zone PID controller, and the backlash estimation method. The backlash gap angle estimator was initialised to be zero. To account for the communication delay between the physical and digital twin, a 1-second delay was added.
Figure 7 shows the results of the digital twin reconfigurable dead zone controller. Note that unlike the previous simulation results (Figure 4), the reconfigurable controller now restabilises the nonlinear feedback system at all times. Even though a bias in the final position of each step is present, the worst bias is under 2% of the required reference amplitude. The lead screw's angle estimation procedure yields a sufficiently accurate estimation for stable control, and the delay due to the data transfer does not significantly affect the estimation and control. These results show that the backlash's estimation is accurate and that the closed-loop system remains stable throughout the entire manoeuvre.

Conclusions

This paper puts forward the idea of utilising a digital twin to estimate the nonstationary characteristics of a nonlinear mechanism and, in turn, reconfigure a feedback controller to stabilise the system. The advantage of using a digital twin over classical adaptive control is using asynchronous communication and utilising the server's high computation power. Doing so ensures that the digital controller will operate without introducing additional delays or overflows. Moreover, elaborate models can be used for the inverse estimation problems associated with the signal estimation procedure. Finally, the numerical simulation results are an indication of future possibilities. Mainly, by employing the digital twin, evolved nonlinear mechanisms characteristic can be identified, and a simulation-based design of a reconfigurable controller.

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