Tuned Mass Systems with a Hybrid Hysteresis Model

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Summary. Tuned-mass-dampers are used in mechanical systems to reduce the dynamic response of the primary structures. The damping plays a significant role in a tuned-mass-damper system. In many engineering systems, damping is hysteretic in nature. Here, we study tuned mass systems with a hybrid version of hysteresis model with the Bouc-Wen model and a recently developed scalar hysteresis model. The sine sweep responses show two resonance peaks. The sine sweep responses indicate the amplitude vs. frequency of the systems. We compare the numerical results obtained for two different hysteresis models.

Introduction

Tuned mass dampers (TMD) are often used in practical systems to reduce the high amplitude vibrations. There have been extensive researches on TMDs and their applications to engineering problems. TMD devices are extensively used for vibration control, including long span cable bridges, tall buildings, tall water tanks, etc. Much researches have been done over the last several decades on the passive and active control of structures using TMD, see e.g., [1,2].

The damper of the TMD device plays a very important role. Both linear and nonlinear dampers are used in the TMD systems, see e.g., [3,4]. TMD systems with linear viscous damping are often analytically tractable. On the contrary, TMD systems with nonlinear damping models are mathematically challenging.

In many engineering systems, energy dissipation occurs in the form of rate-independent hysteresis. In general, rate-independent hysteresis introduces *signum* nonlinearities in systems. Several researchers used hysteretic dampers in the TMD systems, e.g., [5]. In [6], a hysteretic tuned mass damper is used for structural vibration reduction. In [7], the Bouc-Wen hysteresis model [8,9] is used as the damper in the TMD.

With the above motivation, we will study TMD systems with the Bouc-Wen model and a rate independent scalar hysteresis model developed in [10]. The hysteresis model of [10] is motivated by a study of an elastic plate with several frictional microcracks.

The above two models are briefly discussed below.

The Bouc-Wen hysteresis model is given by

$$\dot f(t) = \dot z(t)\left\{A - \left[\beta\,\mathrm{sgn}\left(\dot z(t)\, f(t)\right) + \gamma\right]\left|f(t)\right|^n\right\} \tag{1}$$

where $A > 0$, $\beta > 0$, $\gamma \in [-\beta, \beta]$ and $n > 0$ are the model parameters. Here, $z(t)$ is the given input and $f(t)$ is the corresponding hysteretic output of the system.

The rate-independent scalar hysteresis model of [10] is given by

$$\dot\theta(t) = \frac{\kappa}{|z(t)| + \in}\left\{\theta_a + \beta_0\,\mathrm{sgn}\,(z(t)\,\dot z(t)) - \theta(t)\right\}\cdot\left|\dot z(t)\right| \tag{2}$$

where $z(t)$ is the input displacement to the system and θ is an internal variable. Here, θ_a, κ, β_0 are model parameters and $\in$ is a small regularizing parameter. The hysteretic force is given by

$$f(t) = \theta(t)\cdot z(t) \tag{3}$$

We use the Bouc-Wen model and the scalar hysteresis model of [10] in parallel in TMD systems. We compare the numerical results obtained for both cases.

TMD with the Bouc-Wen hysteresis and the hysteresis model of [10] in parallel

Figure 1(left) shows a TMD system with the Bouc-Wen hysteresis model and the hysteresis model of [10] in parallel. Here, m_1 is the mass of the primary system, k_1 is the stiffness of the spring on which m_1 is mounted, m_2 is the mass of the secondary structure which is attached to the primary mass with a spring of stiffness k_2 and two hysteretic dampers in parallel indicated by h_1 and h_2. Here, the damper h_1 is governed by the hysteresis model of [10], and the damper h_2 is governed by the Bouc-Wen model. A harmonic force $F_0 \sin(\omega t)$ is applied to the primary mass.

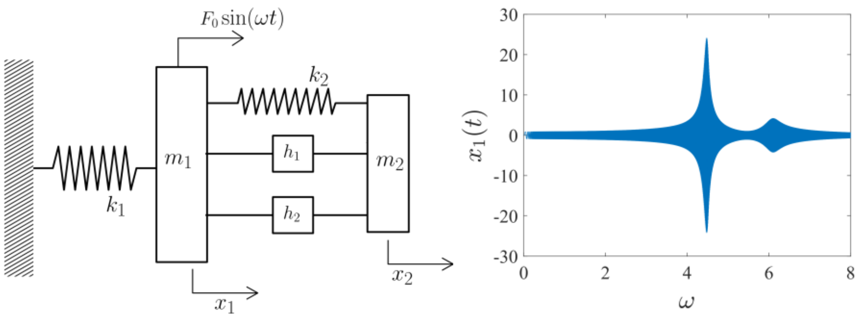

Fig. 1 Left: A tuned mass system with the Bouc-Wen hysteresis and the hysteresis model of [10] in parallel. Right: Response of the TMD with the Bouc-Wen and the model of [10] in parallel. Parameters used: $m_1 = 1$, $m_2 = 1/10$, $k_1 = 1$, $k_2 = 1/10$, $A = 1$, $n = 1$, $\beta = 0.5$, $\gamma = 0.1$, $\kappa = 4$, $\theta_a = 2$, $\beta_0 = 1.8$, $\in = 10^{-6}$, $F_0 = 1.2$, $\alpha = 0.01$, $\alpha_1 = 0.7$ and $\alpha_2 = 0.9$.

Equations of motion of the system are as follows:

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 - f = F_0 \sin(\omega t) \tag{7}$$

$$m_2\ddot{x}_2 - k_2 x_1 + k_2 x_2 + f = 0 \tag{8}$$

$$\dot{\theta}(t) = \frac{\kappa}{|x_2(t) - x_1(t)| + \in}\left\{\theta_a + \beta_0 \operatorname{sgn}[(x_2(t) - x_1(t))(\dot{x}_2(t) - \dot{x}_1(t))] - \theta(t)\right\}\left|\dot{x}_2(t) - \dot{x}_1(t)\right| \tag{9}$$

$$\dot{h}_2 = \alpha_2\left[\left|\dot{x}_2(t) - \dot{x}_1(t)\right|\left\{A - \left[\beta \operatorname{sgn}\left(h_2(t)(\dot{x}_2(t) - \dot{x}_1(t))\right) + \gamma\right]\left|h_2(t)\right|^n\right\}\right] \tag{10}$$

Here, $h_1 = \alpha_1 \cdot \theta \cdot (x_2(t) - x_1(t))$

and

$f = \alpha\left(h_1 + h_2\right)$

where α_1 and α_2 are scalar multipliers that control the level of damping. Note that, the input displacement to the dampers is $(x_2(t) - x_1(t))$.

We use a slowly time varying frequency $\omega = 10^{-5}t$ for the frequency sweep calculations. Figure 1(right) shows a frequency sweep response of the primary mass for the TMD with the Bouc-Wen hysteresis and the hysteresis model of [10] in parallel. Resonance peaks are seen in the frequency sweep response. In Figure 1(right), the frequency sweep responses indicate the amplitude vs. frequency of the primary mass. We can see the primary and secondary resonances of the TMDs. By tuning the parameters of the TMDs, we can get the desired resonant amplitudes for our systems.

Conclusions

In this paper, we have numerically studied tuned mass systems with the Bouc-Wen hysteresis model and the hysteresis model of [10] in parallel as the dampers. The net damping force is numerically controlled by two parameters. The goal of the paper was to numerically study the sine-sweep frequency responses of the TMDs. The sine-sweep response gives a clear idea of how the amplitude of the system varies with the frequency. The study helps to develop the idea to tune the model parameters in order to achieve desired resonant responses.

References

[1] Jangid R.S., Datta T. K. (1995) Seismic Behaviour of Base-Isolated Buildings: A State-of-the Art Review. *Proc. Institution of Civil Engineers-Structures and Buildings*. 110(2):186-203.
[2] Soto M.G., Adeli H. (2013) Tuned mass dampers. *Archives of Computational Methods in Engineering*. 20(4):419-31.
[3] Randall S.E. Halsted D.M., III, Taylor D.L. (1981) Optimum vibration absorbers for linear damped systems. *J. Mech. Des*. 103(4): 908-913.
[4] Starosvetsky Y, Gendelman O.V. (2009) Vibration absorption in systems with a nonlinear energy sink: nonlinear damping. *J. Sound Vib*. 324(3-5): 916-39.
[5] Gerges R.R., Vickery B.J. (2005) Design of tuned mass dampers incorporating wire rope springs: Part I&II: Dynamic representation of wire rope springs. Eng. Struct. 27(5):653-674.
[6] Carpineto N., Lacarbonara W., Vestroni F. (2014) Hysteretic tuned mass dampers for structural vibration mitigation. In: *J. Sound Vib*, 333, 1302-1318(2014).
[7] Lacarbonara W., Vestroni F. (2002) Feasibility of a vibration absorber based on hysteresis, In: *Proceedings of the Third World Congress on Structural Control*, Como, Italy.
[8] Bouc R. (1967) Forced vibration of mechanical systems with hysteresis. In: *Proceedings of the 4th Conference on Nonlinear Oscillation*, p. 315, Prague.
[9] Wen Y. (1976) Method for random vibration of hysteretic systems. *J. Eng. Mech. Div*. 102(2), 249–263.
[10] Biswas S., Jana P., Chatterjee A. (2016) Hysteretic damping in an elastic body with frictional microcracks. *Int. J. Mech. Sci*. 108, 61–71.