Dynamics and stability of a planar three-link swimmer with passive visco-elastic joint in Ideal fluid

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Summary. We study the nonlinear dynamics of a three-link swimmer model in ideal fluid, where inertial forces due to added mass are dominating while viscous drag forces are negligible. We consider an underactuated swimmer where one joint is periodically actuated while the other joint is passive and viscoelastic, with torsional spring and damper. The swimmer's motion depends significantly on the amplitude and frequency of the actuated joint angle. Optimal frequency is found where the swimmer's net displacement per cycle is maximized, under symmetric periodic oscillations of the passive joint. In addition, upon crossing critical values of amplitude and frequency, the system undergoes a bifurcation where the symmetric solution loses stability and asymmetric solutions evolve, for which the swimmer moves along an arc. We analyze these phenomena using numerical simulations and analytical methods of *Floquet* theory and *Hill's determinant*. The results demonstrate the important role of *parametric excitation* on stability of motion for flexible underactuated locomotion.

Introduction

Autonomous swimming robots have a promising potential for various applications such as surveillance and protection in marine environment, search and rescue missions, and maintenance operations within pipe systems of complex infrastructures [1], [2]. A common model assumes ideal fluid [3], [4], where the viscosity is negligible and the swimmer-fluid interaction is induced by reactive forces that represent added mass effect. Our previous work [4] used this model to study multi-link swimmers under kinematic input prescribing all joint angles, numerically, analytically and experimentally (Figure 1b). Inspired by biological swimmers in nature that utilize body flexibility, the recent work [5] studied a modified model of planar three-link swimmer having one passive viscoelastic joint (torsional spring + damper) and one actuated joint with oscillating angle $\theta_2(t) = \varepsilon\cos(\omega t)$, see Figure 1a. Unlike [4], in the semi-passive model [5] the excitation frequency ω and amplitude ε of the active joint have a significant effect on the response of the passive elastic joint and the resulting motion.

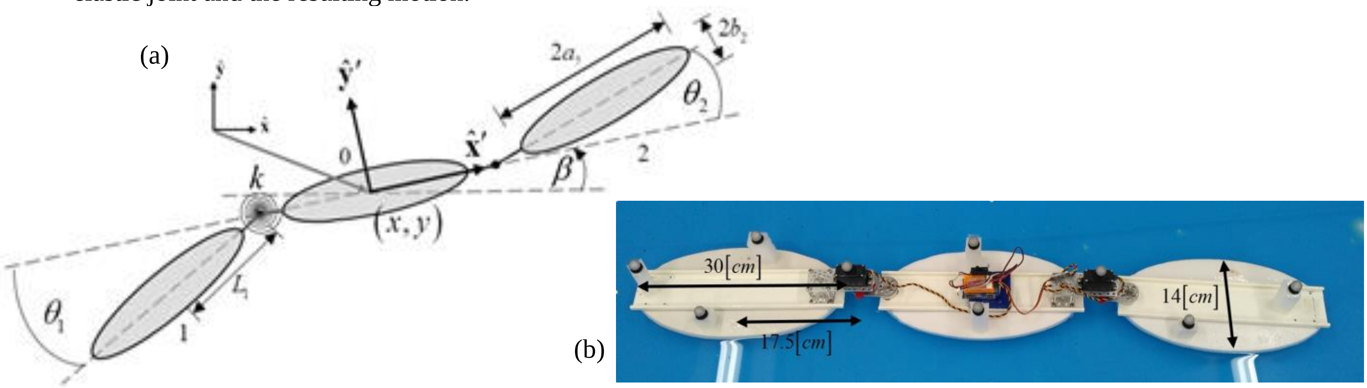


Figure 1: (**a**). Swimmer model – (x, y) are the position of the body-fixed reference frame origin. β is the rotation angle of the body-fixed reference frame. a_i and b_i are the major and minor radii of the elliptic links. θ_i are the relative angles between links. (**b**). Our previous experimental robotic swimmer with two actuated joint angles [4]

Results

In this work, we revisit the model in [5] and study its nonlinear dynamics and stability, both numerically and analytically. Numerical simulations of the system's nonlinear dynamics result in symmetric periodic motion, in which the passive joint angle $\theta_1(t)$ is oscillating symmetrically about zero while the swimmer's net motion is translation along a straight line (Figure 2, blue curves). For a fixed amplitude ε, An optimal frequency ω is found where the net displacement per cycle is maximized (Figure 3, top left). Analyzing stability of periodic solutions reveals a bifurcation point depending on input's amplitude and frequency, where the symmetric periodic solution loses stability and a pair of stable asymmetric solutions evolve, which involve oscillations of $\theta_1(t)$ about nonzero mean angle, resulting in net rotation such that the swimmer moves along an arc (Figure 2, red curves). Asymptotic analysis of the symmetric solution under small-angle assumption $\varepsilon \ll 1$ enables obtaining explicit expressions for the optimal frequency and displacement. Analyzing small variations about the symmetric periodic solution gives a Hill-type equation (linear time-periodic 2nd- order ODE) whose stability can be approximated using truncated Hill's determinant [6]. We obtain analytic conditions for the stability transitions depending on input's amplitude and frequency, which agree with the numerical simulations (Figure 4). Finally, we conduct additional numerical simulations in order to analyze added effects of nonzero initial momentum, drag forces, and tension spring mechanism at the passive joint.

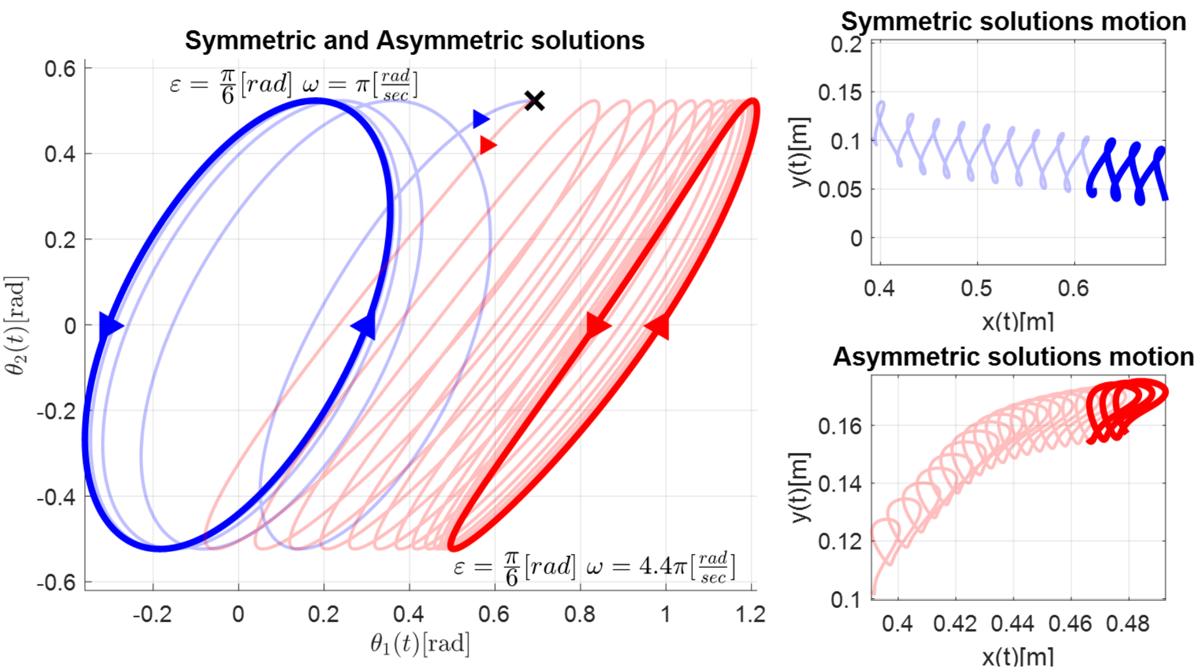
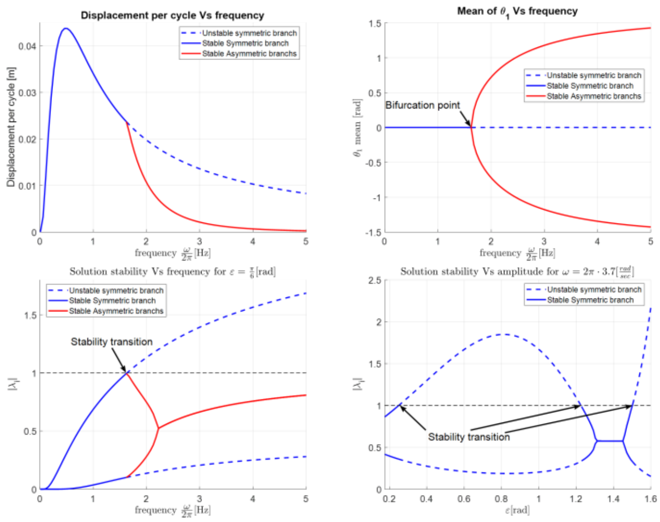


Figure 2: Transient simulations – Left: trajectories in joint-angles plane. Right: trajectories in x-y plane. Blue: symmetric solution, Red: asymmetric solution. Simulations with the same amplitude and initial condition and different frequencies result in significantly different solutions trajectories.



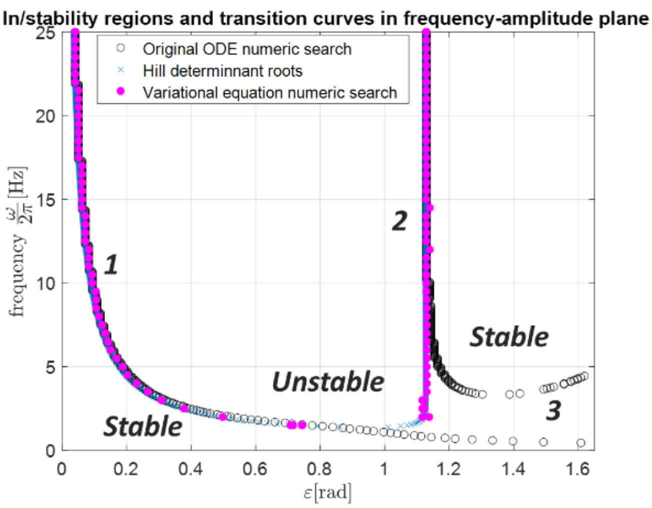


Figure 3: Steady state solution parameters in ω and ε – Optimal frequency is found where the net displacement per cycle is maximized. A bifurcation point depending on input's amplitude and frequency occurs, where the symmetric solution loses stability and a pair of stable asymmetric solutions evolve. Stability transitions involve Floquet multipliers crossing $|\lambda_i|$=1.

Figure 4: Stability and instability regions and transition curves in frequency-amplitude plane – The analytic stability transition condition of Hill's determinant predicts the first and second stability transitions. A third transition at higher amplitudes is not captured by the asymptotic analysis.

References

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