Targeted energy transfer between a linear oscillator and a time-dependent nonlinear systems

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Summary. Targeted energy transfer between two weakly and linearly coupled oscillators is studied. One of oscillators possesses time-dependent damping and nonlinear restoring forcing terms. Thanks to the detection of different system dynamics, its phase-dependent characteristics are clarified leading to having finer vision about energy channelling between such oscillators. The developments prepare design tools for tuning parameters of the time-dependent oscillator leading to the design of energy channelling between two oscillators.

We would like to design the targeted energy transfer [1] between a linear and a time-dependent nonlinear oscillator. Our considered system is composed by a linear oscillator weakly coupled to an absorber with a time-dependent cubic rigidity [2] and damping (see Fig. 1). The considered system can correspond to an acoustical mode which is linearly coupled to an adaptative acoustical resonator in nonlinear domains.The studying of such resonators with fixed parameters have been already articulated in [3, 4, 5] for membrane and Helmholtz resonators and has been programed for a loudspeaker in [6].

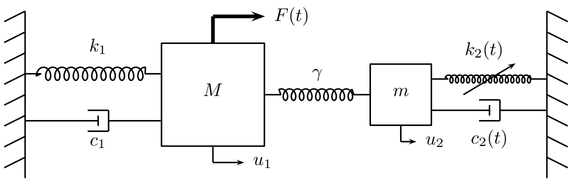


Figure 1: System with time-dependent cubic rigidity and damping

To study the behaviour of this system around a $1:1$ resonance, we consider the nondimensioned governing equations. Then, to have the envelope of the response of the system we introduce the complex variables of Manevitch [7]. To detect the different dynamics of the system, the multiple scales method [8] is used. Hence, we introduce fast and slow time scales. We keep only the first harmonics of the system via truncating other harmonics.

From the study of the fast dynamics of the system we can define the equation of the slow invariant manifold (SIM) which corresponds to the energy of the first linear oscillator (N_1) as functions of that of the absorber (N_2) and also the phase (δ_2). Moreover, we can detect the local extrema of the SIM. Introducing the perturbation in the complex variables of Manevitch, the unstable zone of the SIM can be clarified.

From the study of the slow dynamics of the system, we are able to define the equilibrium, singular points and the back-bone curve of the system. These developments permit to predict different possible periodic or non-periodic regimes as functions of excitation amplitude and frequency.

For the system with time dependent rigidity and damping, we represent in Fig. 2 the SIM with its unstable zone accompanied by results obtained from direct numerical integration of the governing equations of the system without external excitation but under initial conditions. We notice that the SIM depends on three parameters: the energy of the linear oscillator (N_1), the energy of the nonlinear oscillator (N_2) and its phase (δ_2) contrary to the case with constant parameters where the SIM only depends on N_1 and N_2. Moreover we can observe in the Fig. 2b that the numerical integration follows the SIM and therefore the analytical predictions. The Fig. 3 represents different views of the equilibrium points of this system and the unstable zone of the SIM. We observe that, there are two branches: a main branch (i) and an isola (ii).

Depending of progress of the work, some experimental results will be probably presented.

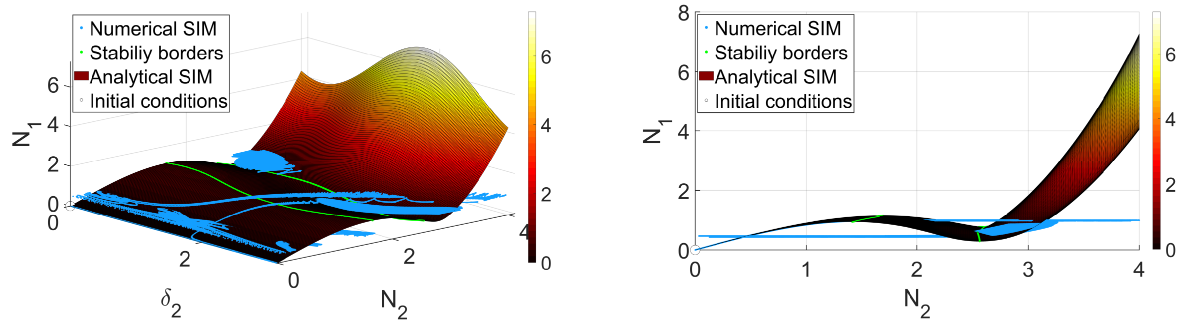
References

[1] Vakakis A. F., Gendelman O. V., Bergman L. A., McFarland D.M., Kerschen G. and Lee Y.S. (2008). Nonlinear targeted energy transfer in mechanical and structural systems. *Springer, Dordrecht*.

[2] Labetoulle A., Ture Savadkoohi A. and Gourdon E. (2021). Detection of different dynamics of two coupled oscillators including a time-dependent cubic nonlinearity. *Acta Mechanica*, doi: 10.1007/s00707-021-03119-w.

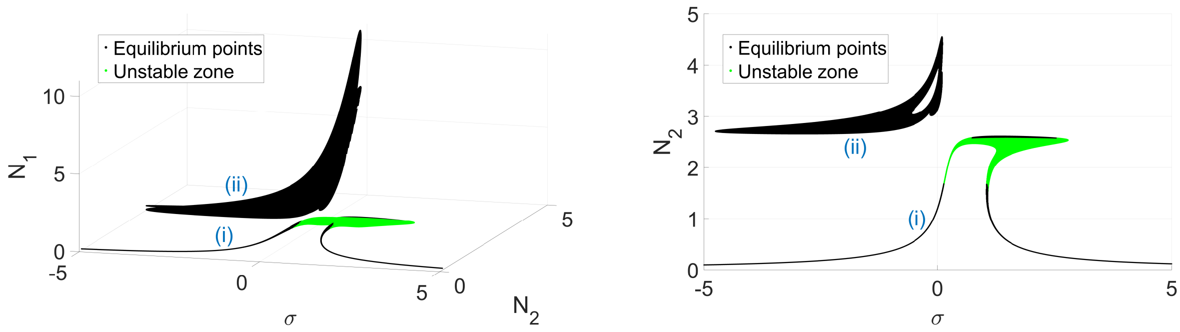
[3] Bellet R., Cochelin B., Herzog P., Mattei P.-O. (2010). Experimental study of targeted energy transfer from an acoustic system to a nonlinear membrane absorber. *Journal of Sound and Vibration* **329(14)**:2768-2791.

[4] Alamo Vargas V., Gourdon E. and Ture Savadkoohi A. (2018). Nonlinear softening and hardening behavior in Helmholtz resonators for nonlinear regimes. *Nonlinear Dynamics* **91(1)**:217-231.



(a) (b)

Figure 2: The SIM of the system with its unstable zone (green line) and numerical results (blue line). a) Three-dimensional view (δ_2, N_2, N_1); b) Two-dimensional view $(N_2$,$N_1)$.



(a) (b)

Figure 3: Different views of collected equilibrium points. σ is the de-tuning of the frequency of excitation for analyzing system behaviours around a $1:1$ resonance. a) (σ, N_2, N_1); b) (σ, N_2). The equilibrium points located in unstable zone of the SIM are represented in green.

[5] Gourdon E., Ture Savadkoohi A. and Alamo Vargas V. (2018). Targeted energy transfer from one acoustical mode to an Helmholtz resonator with nonlinear behavior. *Journal of Vibration and Acoustics* **140(6)**:061005 (8).

[6] Morell M., De Bono E., Collet M., Gourdon E., Ture Savadkoohi A. Innovative electroacoustic resonator control enforcing non-linear dynamics at low excitation levels. *SPIE, Smart Structures + Nondestructive Evaluation*, 6-10 March 2022, Long Beach, California, US.

[7] Manevitch L. I. (2001) The Description of Localized Normal Modes in a Chain of Nonlinear Coupled Oscillators Using Complex Variables. *Nonlinear Dynamics* **25**:95-109.

[8] A. H. Nayfeh and D.T. Mook (1995) Nonlinear oscillations. *Wiley classics library*.