# Dynamical Analysis of a Multibody Wave Energy Converter excited by Random Waves

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*Summary*. The dynamics of a novel multibody wave energy converter based on inclined single modules connected to a frame are investigated, on which generators convert the corresponding relative motion into electrical power. Thereby, it is studied under which conditions the inclined individual modules perform the largest relative motions in regular and irregular waves. For this, different setups are analyzed in the presence of wave excitations, which is generated by a random non-white Gaussian stochastic process.

## Introduction

Renewable energies play an increasingly important role in modern energy systems. As a consequence, hydropower, solar and wind energy are becoming more and more important. In addition to these well-known examples of renewable energy generation, there is also the possibility to obtain energy from ocean waves. Since wave energy has a high power density compared to wind and solar energy, it is also promising for energy generation [1]. Therefore, several new concepts of wave energy converters (WEC) have been studied in the last years. For example a pendulum energy converter was investigated, whereby its pivot is excited by water waves in such a way that a rotational motion of the pendulum is generated, which can be converted to electrical energy, cf. [2, 3, 4, 5].
This paper deals with the analysis of the dynamical behavior of a multibody WEC, where generators mounted on a frame are each excited by a randomly moving cylindrical floating body (CFB). The CFBs are floating in ocean waves and the energy generation of the generators results in additional damping. Our results consider the case of excitation by a non-white Gaussian random process, which can for example be encountered in real sea states.

## Description of the mechanical system

The mechanical system mainly consists of a frame and $N$ CFBs. Figure 1 shows the side view of the structure for the case of $N = 2$ in still water and in the presence of harmonic water waves. In this work, only the motion of the system in the $xy$-plane with horizontal coordinate $x$ and vertical coordinate $y$ is considered. Each CFB moves along guided rods with corresponding displacement $\xi_i$ in a plane, which is inclined with respect to the frame by the corresponding adjustable inclination angle $\varepsilon_i$, $i \in \{1, \dots, N\}$. The frame can freely move in the $xy$-plane, whereby the angle of rotation is denoted by $\beta$ and the horizontal and vertical displacements of the frame are denoted by $x_{\mathrm{F}}$ and $y_{\mathrm{F}}$, respectively. It is assumed that the frame is in contact with the sea surface and that only the CFBs are excited by water waves. Thereby, the CFBs and the frame are connected by springs and mechanical friction is accounted for by a velocity dependent damping force. Generators convert the relative motion between the CFBs and the frame to electrical energy, leading to an additional electrical damping force.
Let

$$\mathbf{z} = [x_{\mathrm{F}}, y_{\mathrm{F}}, \beta, \xi_1, \dots, \xi_N]^{\mathrm{T}} \tag{1}$$

be the vector of all degrees of freedom of the multi body WEC. Then, the general equation of motion of the mechanical system can be written as

$$\mathbf{M}(\mathbf{z}, t)\ddot{\mathbf{z}} + \mathbf{k}(\mathbf{z}, \dot{\mathbf{z}}, t) = \mathbf{q}(\mathbf{z}, \dot{\mathbf{z}}, t), \tag{2}$$

whereby $t$ is the time, $\mathbf{M}$ is the generalized mass matrix, $\mathbf{k}$ is the vector of Coriolis, centrifugal and gyroscopic forces and $\mathbf{q}$ is the vector of the applied forces.
In order to compute the hydrodynamic forces, the water pressure $p$ must be integrated over the wetted surface of each CFB. Using potential flow theory and Bernoulli's equation of fluid dynamics, the radiation problem as well as the diffraction problem of a moving cylinder has to be solved in order to calculate the hydrodynamic forces [6]. For incoming harmonic water waves with wave amplitude $A$ and wave frequency $\omega$, the corresponding sea surface is given by

$$\eta(x, t) = \mathrm{Re}\{A \exp(\mathrm{i}(\kappa x - \omega t))\} \quad \text{with} \quad \omega^2 = \kappa g \tanh(\kappa H), \tag{3}$$

whereby $\kappa$ denotes the wave number, $g$ the gravity constant and $H$ the water depth. For this type of water waves, Yeung [7] and Garrett [8] have computed the velocity potential of radiation and diffraction by expressing them as a series of eigenfunctions for the case of a single truncated cylinder. Using this theory, it is assumed that the motion of CFBs does not lead to hydrodynamic forces, which affect other CFBs.

## Description of random sea waves

A well-known model of random long-crested sea waves is given by the superposition of harmonic waves with wave frequencies $\omega$ and corresponding wave numbers $\kappa(\omega)$. With this, the wave amplitude of each harmonic wave component depends on the underlying sea state, which is given by the corresponding one-sided spectral density $S(\omega)$, cf. [9, 10]. Then, a one dimensional irregular long-crested wave surface can be written as

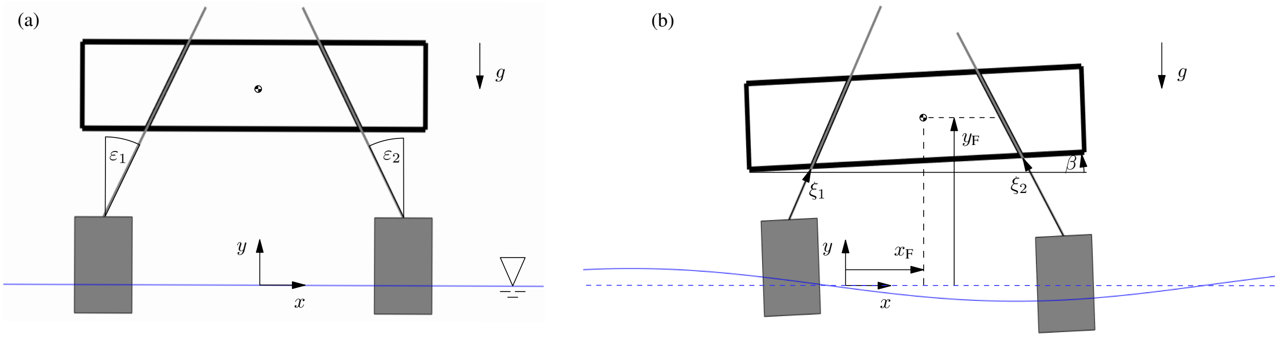


Figure 1: Sketch of the mechanical system in still water position (a) and in the presence of harmonic water waves (b).

$$Z(x,t) = \int_0^\infty \cos\left(\omega t - \kappa(\omega)x + \varepsilon(\omega)\right) \sqrt{2S(\omega)\mathrm{d}\omega}, \tag{4}$$

whereby the integral is not Riemann integral but a summation rule over the frequencies $\omega$. An example for such generated sea waves is shown in Figure 2, where either the space or time is fixed at $x = 0$ or $t = 0$, respectively.
With this developed mechanical model and the study of corresponding numerical results, a detailed analysis is performed showing the influence of design parameters, like the inclination angles $\varepsilon_i$ or the number $N$ of CFBs, in order to maximize the harvested energy.

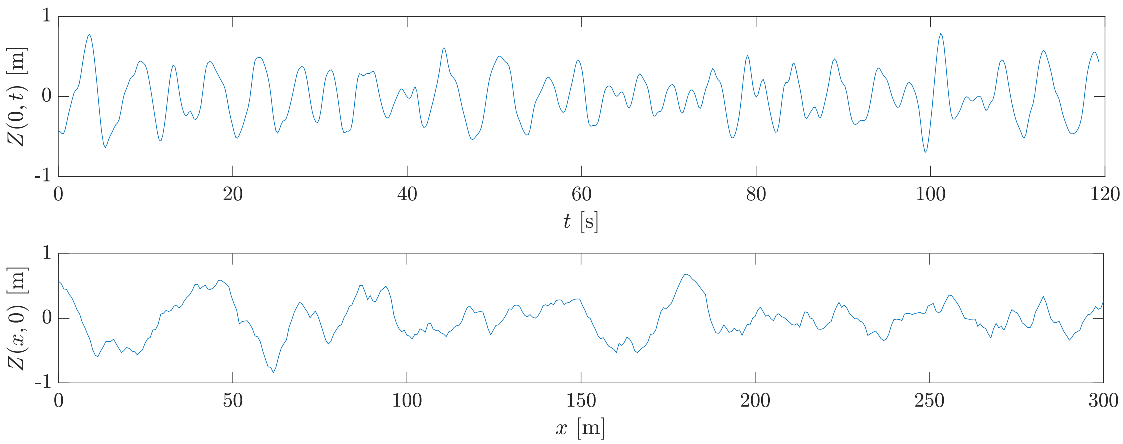


Figure 2: Evolution of a random sea surface in time and space.

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