Frequency Stabilization of MEMS Oscillators Using Internal Resonance

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Summary. We study the frequency stabilization of a MEMS self-sustained oscillator operating in internal resonance. We show, both experimentally and theoretically, that coupling two vibrational modes with a frequency ratio of 1:3 through a nonlinear resonance makes it possible to stabilize the oscillation frequency of MEMS oscillators beyond any currently known frequency stabilization technique. Our analysis shows that this novel frequency stabilization stems from operation in a region of (nearly) zero frequency dispersion at large amplitudes. The large amplitude improves signal quality and the internal resonance provides both the zero dispersion as well as phase-locking between the modes, all of which are beneficial for noise reduction. Our findings provide a new strategy for engineering low-frequency noise oscillators capitalizing on the intrinsic nonlinear phenomena of micro- and nano-mechanical resonators.

Introduction

MEMS oscillators, which offer the potential for reduced power consumption, on-chip integration with CMOS, and a small footprint, are widely used for precision time keeping and as sensitive detectors. Therefore, their frequency stability is a key figure of merit. However, due to the small size of the mechanical vibrating structures in these MEMS oscillators, their vibrations are highly sensitive to noise and become nonlinear even for small amplitudes. Unlike linear vibrations, in nonlinear vibrations, the resonant frequency has a strong dependence on the oscillation amplitude (frequency dispersion), and therefore, amplitude fluctuations translate into frequency fluctuations. This amplitude-to-frequency (A-f) noise conversion considerably degrades the oscillator performance.

In this study, we show that a nonlinear resonance between a pair of modes that have a frequency ratio of 1:3 provides a zero-dispersion domain over which, local to the operating point, A-f effects are eliminated and the system regains some specific characteristics of linear vibrations, even though the operating point is well into the nonlinear regime. Moreover, the inter-modal phase locking of the nonlinear resonance produces a frequency stabilizing effect similar to that observed for pairs of synchronized oscillators. These effects both suppress frequency fluctuations in the primary mode, making the considered oscillator cleaner than its linear counterpart (which is ideally the cleanest oscillator).

Experimental method and observations

The resonator (i.e., the mechanical vibrating structure) of our MEMS oscillator is fabricated from single-crystal silicon and is composed of 3 beams connected at their centers to each other and to a pair of comb drives (see Refs. [1] for details). The resonantly interacting modes are the fundamental flexural mode (eigenfrequency ω_1) and the fundamental torsional mode (eigenfrequency $\omega_2 \approx 3\omega_1$); see the left panel of Fig. 1. The self-oscillation is achieved by a placing the resonator in a feedback loop with an amplifier and a phase shifter whose tuning both destabilizes the thermal vibrations and controls the amplitude and phase of the flexural mode. [4]

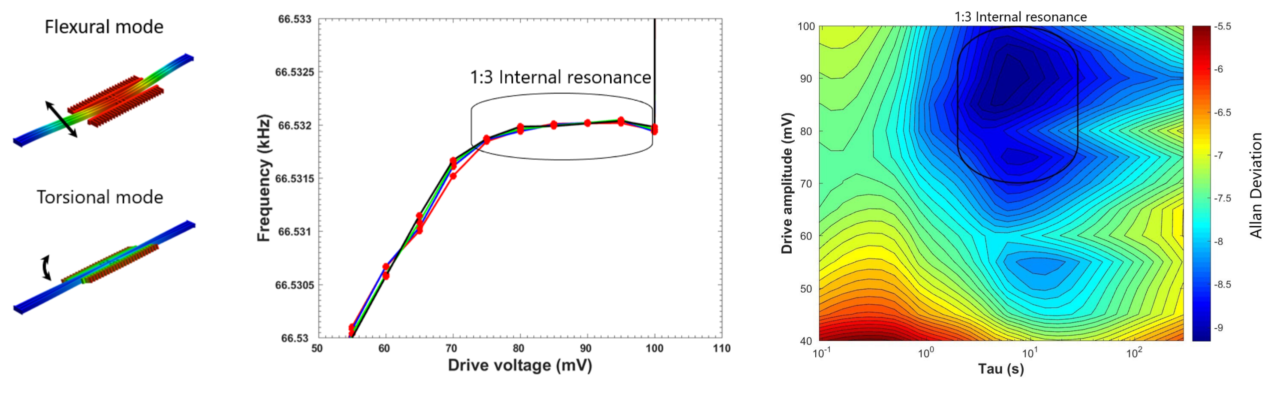

Figure 1: Device and experimental measurements. Left panel—the flexural (ω_1) and torsional (ω_2) modes from finite element models, which interact resonantly when $\omega_2/\omega_1 \approx 3$. Center panel—the operating frequency of the oscillator increases with the drive voltage, due to the Duffing nonlinearity, until it saturates with zero-dispersion in the vicinity of the 1:3 internal resonance. Right panel—at the internal resonance, the Allan deviation reduces dramatically, revealing the stabilizing effect of the nonlinear mode coupling.

The motion of the flexural (in-plane) mode is detected capacitively by one set of combs. The capacitance variation of the voltage-biased comb-drive electrode generates a current that is introduced into a current amplifier to produce a voltage output proportional to the oscillation amplitude of the flexural mode. This voltage is phase shifted and used to excite the beam through the other set of combs. The motion of the torsional (out-of-plane) mode is detected by an optical interferometry method, with the laser spot focused on the outer-most position of the movable structure. The interference signal is then amplified to produce a voltage output proportional to the oscillation amplitude of the torsional mode. The frequency stability of the system was analyzed in terms of the Allan deviation from the signal of the flexural mode [2] $\sigma_y(\tau) = \sqrt{\frac{1}{2(N-1)}\sum_{i=1}^{N-1}(\langle y_{i+1}^{\tau}\rangle - \langle y_i^{\tau}\rangle)}$, where $\langle y_i^{\tau}\rangle$ are the relative frequency fluctuations averaged over the i^{th} discrete time interval of τ. The center panel of Fig. 1 shows the zero dispersion by the flattening of the operating frequency as a function of amplitude, and the right panel clearly shows the attendant frequency stabilization obtained in the internal resonance by the reduction of the Allan variance at these amplitudes.

Analytical model and results

We consider the following closed loop system with a 1:3 internal resonance

$$\ddot{x}_1+2\Gamma_1\dot{x}_1+\omega_1^2(1+\eta_1)x_1+\gamma x_1^3+3\alpha x_1^2x_2 = S\cos(\omega_1 t+\phi_1+\Delta)+\xi_1,\ \ \ddot{x}_2+2\Gamma_2\dot{x}_2+\omega_2^2(1+\eta_2)x_2+\alpha x_1^3 = \xi_2,$$

where $\Gamma_{1,2}$ are the dissipation rates, $\omega_{1,2}$ are the modal eigenfrequencies ($\omega_2 \approx 3\omega_1$), S is the drive level (set by the amplifier), $\phi_1(t)$ represents the phase of the flexural mode, Δ is the imposed phase shift from the feedback loop, $\eta_{1,2}(t)$ are the frequency noises, and $\xi_{1,2}(t)$ are the additive thermal noises. We have assumed that the coupling stems from a single-term potential $U_{cpl} = \alpha x_1^3 x_2$, that the flexural mode nonlinearity is a simple Duffing type, γx_1^3, and the torsional mode operates in its linear range. Using the method of stochastic averaging [3], we derive a pair of Langevin equations for the phase sum $\phi = \phi_1+\phi_2$ and difference $\psi = 3\phi_1-\phi_2$ (ϕ_2 is the phase of the torsional mode). Near a stable operating point, it can be shown that diffusion of the phase difference remains constant and small. In contrast, the diffusion of the phase sum is always strong and associated with a variance that increases linearly in time ($\langle\varphi^2\rangle - \langle\varphi\rangle^2 = D_\varphi t$). However, in the zero-dispersion domain, D_φ reduces drastically due to elimination of the A-f noise conversion. Moreover, we can neglect the diffusion of ψ, which is considerably smaller than the strong diffusion of φ, and then approximate, using the relations $\phi_1 = (\varphi+\psi)/4$ and $\phi_2 = (3\varphi-\psi)/4$, the diffusion constants of the individual phases ($\langle\phi_{1,2}^2\rangle - \langle\phi_{1,2}\rangle^2 = D_{\mathrm{T}_{1,2}}t$) as $D_{\mathrm{T}_1} \approx (1/4)^2 D_\varphi,\ D_{\mathrm{T}_2} \approx (3/4)^2 D_\varphi$. Therefore, the phase-locking mechanism ($\dot{\psi} = 0$) of the internal resonance leads to a further reduction in the phase noise of the first mode (D_{T_1}). We note that the flexural mode is subjected to the noises of the feedback circuitry while the torsional mode is largely isolated from such noise sources. Thus, if the second mode (nearly purely mechanical) is significantly cleaner than the first mode, i.e., $D_{\xi_2} \ll D_{\xi_1} \ll$ and $D_{\eta_2} \ll D_{\eta_1}$, then the cleaner second mode, which has a threefold stronger influence, cleans the noisy first mode and its diffusion constant is reduced by a factor of 1/16, i.e., $D_{\mathrm{T}_1}|_{\text{Inside IR}} = (D_{\mathrm{T}_1}|_{\text{Outside IR}})/16$. This cleaning effect can be readily seen in Fig. 2, where in the zero-dispersion domain of the coupled-mode oscillator, the diffusion constant of the first mode is significantly lower than the diffusion constant of even the highly ideal single-mode linear oscillator. Note that without the phase cleaning effect, the diffusion constants would be (nearly) equal in the zero-dispersion domain.

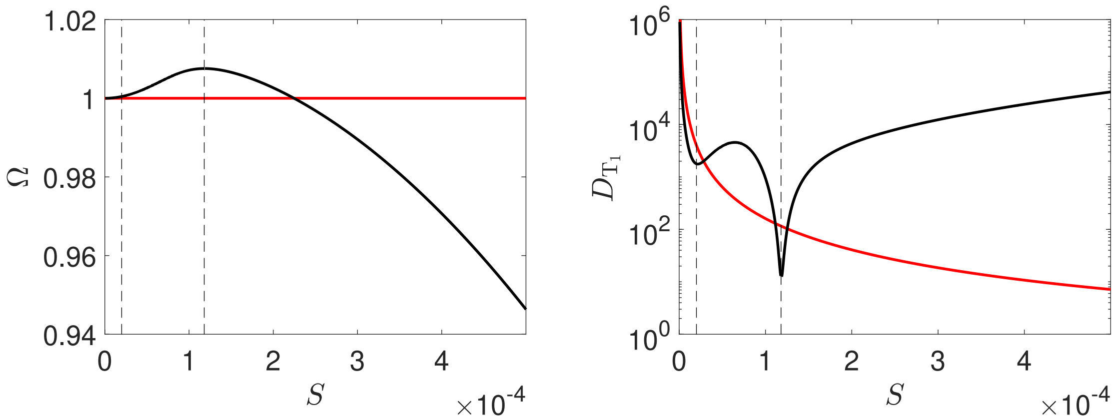

Figure 2: Operating frequency Ω (left panel) and total diffusion constant of the phase of the first mode (right panel) as functions of drive level for single-mode linear (red), and coupled-mode (black) closed-loop oscillators, with $\omega_1 = 1, \omega_2 = 3.06, \gamma = 3, \alpha = 10, \Gamma_1 = 10^{-3}, \Gamma_2 = 10^{-4}\Delta = \pi/2, D_{\eta_1} = 1, D_{\eta_2} = 0.1, D_{\xi_1} = 0.4$ and $D_{\xi_2} = 0.04$. Only the stable states in which $\Omega < \omega_2/3$ are shown. The diffusion constant of the linear oscillator decreases as S^{-2}, whereas the diffusion constant of the coupled-mode oscillator has a dual local minima (denoted by the dashed vertical lines), the lower of which is the usual operating point for a Duffing oscillator without zero dispersion. The cancellation of amplitude to frequency noise conversion at the zero-dispersion domain along with the phase constraint yield a diffusion constant for the coupled-mode oscillator that is even smaller than that of the linear oscillator.

References

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