Broadband Stabilization with Combined Anti-Resonances

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Summary. The study of anti-resonances in parametrically excited systems in the recent years was focused mainly on bimodal systems. Assuming it to be strictly a bimodal coupling phenomenon, anti-resonances in systems with multiple degrees of freedom (DoF) were seen as a generalization without added effects. Recent findings, however, hint at an interesting behavior when multiple anti-resonances arise in close vicinity of each other. In the present contribution, these effects will be discussed and assessed with perturbational methods, aiming at understanding the underlying phenomenon, which is expected to be advantageous especially in enhancing the robustness and intensity of vibration mitigation.

Introduction

Ever since the discovery of anti-resonance by Tondl [8], further research by Schmieg [7], until recent investigations conducted by Dohnal [1, 2] and finally comprehensive semi-analytical describtion by Karev [3, 4, 5], anti resonance was interpreted as coupling between two modes. However, recent findings suggest that multiple anti-resonances can not be viewed in isolation in all cases. Following [6], Lyapunov characteristic exponents (LCEs) are used to gain insight into the system's behavior. Floquet theory is applied to the parametrically excited system with excitation period time T, resulting in Floquet multipliers ρ_i, from which the LCEs λ_i can be derived by

$$\lambda_i = \frac{1}{T} \ln |\rho_i|. \tag{1}$$

Numerical Observations

In [6], it was shown that in systems with multiple degrees of freedom (MDoF), under the right conditions anti-resonances may appear at multiple excitation frequencies Ω. This is illustrated in Fig. 1, using the system introduced in [2, 9] and studied further in [6] as an example. The anti-resonances are approximately at the combination frequencies $\Omega = \Sigma_{21} = \omega_1 + \omega_2$ and $\Sigma_{32} = \omega_3 + \omega_2$.

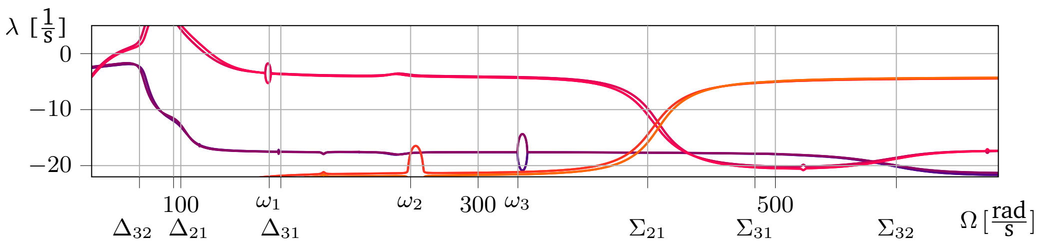

Figure 1: LCEs λ_i of example system [6] over excitation frequency Ω for $\kappa = 1$, $\varepsilon_p = 0.15$. Two sepearte anti-resonances are visible.

The numerical results obtained by applying Floquet theory indicate that for a certain amplitude ε_p of parametric excitation, while having asymmetrically skewed excitation terms by a factor κ in a MDoF (see (2)), a minimum of the largest of the LCEs involved in anti-resonances $\max(\lambda_i)$ can be achieved. This leads to the anti-resonances laying on top of each other, as visible in Fig. 2.

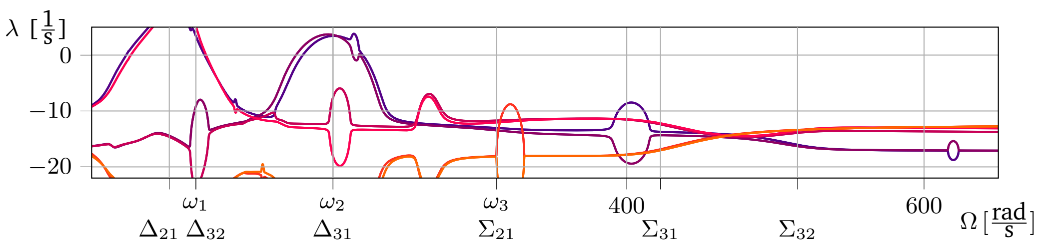

Figure 2: LCEs λ_i of example system [6] over excitation frequency Ω for $\kappa = 0.729$, $\varepsilon_p = 0.158$. Two anti-resonances occur at the same frequency.

The exact relation between the values of all LCEs λ_i and the equivalent damping of the system is yet to be fully understood. Thus, numerical simulation of the system's amplitudes after an initial displacement is used to clarify the effects of the two combined anti-resonances shown in Fig. 2. Fig. 3 shows the summed squares of amplitudes of all DoF of the example system. With the dashed line indicating the reference time at which the unexcited system's amplitudes become negligibly small, it is apparent that a stabilizing effect is not only caused around the frequency of anti-resonances but for all frequencies apart from resonances.

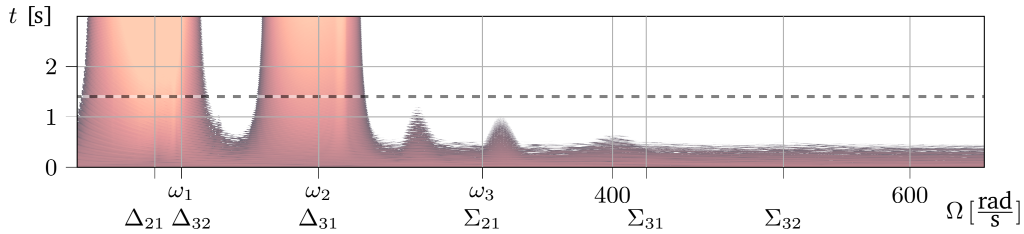

Figure 3: Summed squares of amplitudes in all DoF after initial pertubation recorded over time t and with respect to the parametric excitation frequency Ω. Reference time where unexcited system reaches negligible small amplitudes shown with dashed line (- - -). Lighter shade means larger amplitudes.

Perturbation analysis

In order to get a better insight into the observed phenomenon, the system is to be analyzed analytically. As a first step, the system model is reduced for the sake of generalizing the studied system, which leads to a three-dimensional equation of motion

$$\begin{aligned} &\ddot{\underline{u}} + \varepsilon \begin{pmatrix} \mu_1 & 0 & \mu_{13} \\ 0 & \mu_2 & 0 \\ \mu_{13} & 0 & \mu_3 \end{pmatrix} \dot{\underline{u}} + \left[\begin{pmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{pmatrix} + \dots \right. \\ &\left. \dots + (1-\kappa) \begin{pmatrix} 0 & k_{12} & k_{13} \\ k_{21} & 0 & k_{23} \\ k_{31} & k_{32} & 0 \end{pmatrix} + \left[\begin{pmatrix} 0 & f_{12} & f_{13} \\ f_{21} & 0 & f_{23} \\ f_{31} & f_{32} & 0 \end{pmatrix} - \kappa \begin{pmatrix} 0 & f_{21} & f_{31} \\ f_{12} & 0 & f_{32} \\ f_{13} & f_{23} & 0 \end{pmatrix} \right] \varepsilon_p \cos(\Omega t) \right] \underline{u} = 0, \end{aligned} \tag{2}$$

with the displacement $\underline{u}$, damping μ_i, eigenfrequencies of the undamped unexcited system ω_i^2, off-diagonal stiffness terms k_{ij}, parametric excitation coefficients f_{ij} and frequency Ω, small parameter ε, amplitude of parametric excitation $\varepsilon_p = \mathcal{O}(\varepsilon)$ and asymmetry parameter κ.

The findings are to be analyzed using multiple scales. For this purpose, the displacement $\underline{u}$ and asymmetry parameter κ are perturbated, described by a power series with respect to the small parameter ε:

$$\begin{aligned} \underline{u} &= \underline{u}_0 + \varepsilon \underline{u}_1 + \varepsilon^2 \underline{u}_2 + \mathcal{O}(\varepsilon^3), \\ \kappa &= 1 + \varepsilon \kappa_1 + \varepsilon^2 \kappa_2 + \mathcal{O}(\varepsilon^3). \end{aligned} \tag{3}$$

The ongoing study aims then at using the outcomes of the multiple scales analysis in understanding the reason behind the stabilizing effect of coinciding anti-resonances at approximately all excitation frequencies. In this way, a generalization of this phenomenon can be achieved for a 3 DoF system and, moreover, extended to generic MDoF systems. Such an effect could be a powerful tool in mitigating vibrations in industrial applications.

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