Stability Analysis of Rotary Drilling Systems Associated with Multiple State-Dependent Delays

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Summary. This paper presents an algorithm to conduct numerical linear stability analyses of rotary drilling systems equipped with a realistic Polycrystalline Diamond Compact (PDC) bit. The interaction between a PDC bit and the rock introduces up to 100 state-dependent delays into the differential equations governing the rotary drilling system, here represented by a discrete multiple degrees of freedom model. A bit trajectory function is proposed to convert the time delays into their corresponding angular delays that are embedded in the PDC bit design. The proposed algorithm has the potential to be used for optimizing PDC bit design with the goal of postponing the occurrence of torsional stick-slips.

Introduction

Torsional stick-slip vibrations are destructive dynamic phenomena encountered during rotary drilling of oil and gas wells. The torsional stick-slips are characterized by alternating stick phases with the bit at rest for a period of time and slip phases, during which the angular velocity of the tool increases up to two times the nominal angular velocity. The root cause of the torsional stick-slip oscillations lies in the nature of bit-rock interaction as both laboratory and field measurement data have shown that PDC bit design has a significant influence on the torsional stick-slips. However, it is a challenging problem to model the interaction between a realistic PDC bit and the rock, which is associated with two major sources of nonlinearities: (i) the regenerative rock cutting process introduces up to 100 state-dependent delays due to the complex layout of PDC cutters; (ii) the unilateral nature of the frictional contact process introduces a discontinuity in the boundary conditions [1]. The search for the numerous state-dependent delays in numerical simulations is, however, CPU intensive, which renders the computational efficiency of conventional solution strategies unacceptable.

This difficulty has recently been overcome by Tian et al [1], who employed a bit trajectory function [2, 3], to reformulate the time delay-related variables. In this way, the state-dependent time delays are converted into the corresponding angular delays that are known from the PDC bit designs. The evolution of the bit trajectory function is governed by a partial differential equation (PDE), which is coupled by the system of ordinary differential equations (ODEs) that governs the dynamics of the discretized drillstring. By the application of the Galerkin method, the coupled system of PDE-ODEs are transformed into a system of ODEs, which can be efficiently integrated. The drillstring in [1] is, however, simplified as a low-dimensional (LD) discrete model that solely consists of two degrees of freedom (DOFs). This LD drillstring model considers only one torsional DOF, which is thus unable to capture the higher modes of torsional resonance that are usually responsible for the occurrence of torsional stick-slips [3].

This paper extends the LD drillstring model in [1] to a high-dimensional model (HD) with multiple DOFs. Instead of pursuing the time simulation of the HD model with strong nonlinearities, this paper focuses on its linear stability analysis, which provides useful information regarding the growth rate of torsional vibrations into stick-slip oscillations by the examination of unstable poles. The linear stability analysis thus has the potential application in the optimization of PDC bit designs in terms of postponing the occurrence of torsional stick-slips due to its high computational efficiency.

Numerical Stability Analysis

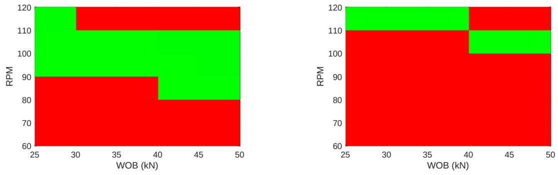
The PDC bit-rock interaction model is adopted from [1], while the HD drillstring model is the same as that in [3]. A combination of both leads to a system of nonlinear coupled PDE-ODEs governing the dynamics of the HD model. The reader is referred to the references [1, 3] for a full description of the derivation. Linearization of this nonlinear system is carried out around the equilibrium point by imposing small perturbations to the steady-state variables. The system of linear coupled PDE-ODEs that governs the perturbation of bit trajectory $\tilde{h}\left(\theta,\, \tau\right)$ function and state variables reads

$$\frac{\partial \tilde{h}}{\partial \tau} + \omega_0 \frac{\partial \tilde{h}}{\partial \theta} + \dot{\varphi}_b \frac{v_0}{\omega_0} = 0, \tag{1}$$

$$\hat{\boldsymbol{I}} \ddot{\boldsymbol{u}} + \hat{\boldsymbol{C}}_{\boldsymbol{a}} \dot{\boldsymbol{u}} + \hat{\boldsymbol{K}}_{\boldsymbol{a}} \boldsymbol{u} = \hat{\boldsymbol{W}}, \tag{2}$$

$$\hat{\boldsymbol{I}} \ddot{\boldsymbol{\varphi}} + \hat{\boldsymbol{C}}_{\boldsymbol{t}} \dot{\boldsymbol{\varphi}} + \hat{\boldsymbol{K}}_{\boldsymbol{t}} \boldsymbol{\varphi} = \hat{\boldsymbol{T}}, \tag{3}$$

where $v_0\, (\omega_0)$ is the non-dimensional nominal axial (torsional) velocity; $\dot{\varphi}_b$ is the perturbed bit angular velocity; $\hat{\boldsymbol{I}}$ is the unit diagonal matrix; $\hat{\boldsymbol{C}}_{\boldsymbol{a}} \left(\hat{\boldsymbol{C}}_{\boldsymbol{t}}\right)$ and $\hat{\boldsymbol{K}}_{\boldsymbol{a}} \left(\hat{\boldsymbol{K}}_{\boldsymbol{t}}\right)$ are the dimensionless damping and stiffness matrices for axial (torsional) motion; $\boldsymbol{u}$, $\boldsymbol{\varphi}$, $\hat{\boldsymbol{W}}$, $\hat{\boldsymbol{T}}$ represent, respectively, the vectors of dimensionless axial and angular displacement, external axial force and torque. The perturbed bit trajectory function $\tilde{h}\left(\theta, \tau\right)$ can be approximated by



(a) (b)

Figure 1: Stability map in the space of the operating parameters (weight-on-bit and rotary speed) for two PDC bit designs: (a) bit A and (b) bit B. The red color denotes the region of operating parameters with large real parts, while the green color represents the region with smaller real parts.

$$\tilde{h}(\theta,\tau) = \tilde{a}_0\left(1-\frac{\theta}{2\pi}\right) + \tilde{a}_1\frac{\theta}{2\pi} + \sum_{k=1}^{N-1}\tilde{a}_{k+1}\sin\left(\frac{k\theta}{2}\right), \quad \theta\in[0,2\pi), \tag{4}$$

Substituting the approximation (4) into the linearized PDE (1) and invoking the Galerkin method to minimize the resulting residual $\tilde{\mathcal{R}}$ yields a system of first-order ODEs that governs the evolution of the perturbed coefficients

$$\int_0^{2\pi}\tilde{\mathcal{R}}\frac{\theta}{2\pi}d\theta = 0, \int_0^{2\pi}\tilde{\mathcal{R}}\sin\left(\frac{m\theta}{2}\right)d\theta = 0, \ m = 1,...,N-1. \tag{5}$$

This systems of ODEs together with the system of ODEs in (2)-(3) can be written in the matrix-vector form as

$$\boldsymbol{A}\dot{\tilde{\boldsymbol{X}}} + \boldsymbol{B}\tilde{\boldsymbol{X}} = \boldsymbol{0} \tag{6}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are coefficient matrices and $\tilde{\boldsymbol{X}}$comprises the perturbed state variables and coefficients $\tilde{a}_i, \ i = 1,2,...,N$. The linear stability analysis of the system of coupled PDE-ODEs (1)-(3) has thus been replaced by the equivalent stability analysis of the linear system of ODEs (6). If the system is stable, the perturbed vector $\tilde{\boldsymbol{X}}$ converges to the null vector; otherwise, $\tilde{\boldsymbol{X}}$ grows exponentially. The instability of the linear system of ODEs (6) is indicated by positive real part of the eigenvalues of the matrix $\boldsymbol{A}^{-1}\boldsymbol{B}$.

Numerical results

Surface rotary speed (RPM) and weight-on-bit (WOB) are two operating parameters that are commonly adjusted to alleviate drillstring vibrations. Preliminary linear stability analyses have shown that the drillstring system is unstable within the practical ranges of the operating parameters (weight-on-bit and rotary speed). This indicates that the torsional vibrations will always degenerate into stick-slips given enough time.

Extensive simulation results have shown that the most unstable pole with the maximum real part of the system (6) corresponds to the axial dynamics of the drillstring. The magnitude of the real part of the most unstable pole determines the rate at which the axial dynamics evolves eventually to the axial stick-slips. Therefore, the values of the operating parameters that lead to smaller real parts of the most unstable poles will delay the occurrence of axial stick-slips. Figure 1 presents the distributions of maximum real parts of the eigenvalues for different combinations of operating parameters for two PDC bit designs. It is observed that different PDC bit designs have different optimum pairs of the operating parameters.

Furthermore, the unstable axial dynamics influences the torsional dynamics via the coupling caused by the bit-rock interaction. It is observed from the time simulation results that the axial stick-slips always precede torsional stick-slips. Therefore, the occurrence of torsional stick-slips can be possibly delayed by selecting PDC bit designs for which the most unstable pole has the smallest real part for prescribed values of the operating parameters.

Conclusions

This paper presents an algorithm for the stability analysis of a high-dimensional drillstring model in conjunction with PDC bit-rock interaction. It is shown that PDC bit designs affect the stability of the drillstring. By recognizing that the most unstable pole corresponds to the axial dynamics and that the axial stick-slips always precede torsional stick-slips, it is possible to delay the occurrence of torsional stick-slips by optimizing PDC bit design that leads to the smallest real part of the most unstable pole. The validity of this approach is currently being verified by time domain simulations.

References

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