Effect of Topology upon Relay Synchronization in Multilayer Neuronal Networks

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Summary. Relay synchronization is a dynamical phenomenon occurring in complex networks when remote parts of the network synchronize due to their interaction via a relay. This phenomenon can be observed in lasers, electronic circuits, biological systems. We investigate relay synchronization scenarios in a three-layer network and elaborate the effect of inhomogeneous network topology of the individual layers.

Complex networks are currently of great interest, since they allow to model various types of real-world systems, such as social networks, economical, biological, financial, transportation systems, as well as neural activity in the brain [1]. In many cases the network can be divided into several similar parts, called layers, where interaction inside the layers and between the layers can be qualitatively different [2]. Multiplex networks are a special class of such multilayer structures, where each layer consists of the same number of nodes and only one-to-one interaction between the corresponding nodes of neighbouring layers are allowed [3].

Relay (or remote) synchronization between layers which are not directly connected is an intriguing phenomenon, which has some similarities with relay synchronization of chaotic lasers [4]. In neuroscience various scenarios have been uncovered where specific brain areas act as a functional relay between other brain regions, having a strong influence on signal propagation, brain functionality, and dysfunctions [5, 6], as well as visual processing [7].

We examine relay synchronization in a three-layer neuronal network, where the dynamics of individual nodes is governed by the FitzHugh-Nagumo system, widely used to describe spiking dynamics of neurons. In the simplest configuration, the individual layers have a nonlocal ring topology, where each node interacts with its neighbours within some fixed coupling range. In such a network, we observe relay synchronization when spatio-temporal patterns of the two outer layers synchronize, but the middle layer which transfers the signal, performs different dynamics. Later on, we replace regular links with random shortcuts, which results in the formation of a small-world topology inside the layers. With the irregular topology in the layers we uncover dynamical scenarios of relay synchronization. In the focus of our study are chimera states- patterns of coexisting coherent and incoherent domains [8, 9, 10, 11], which can be observed in isolated layers of the network [12].

Our model is described by the following system of differential equations:

$$\dot{\mathbf{x}}_i^m = \mathbf{F}(\mathbf{x}_i^m(t)) + \frac{\sigma_m}{L_i^m}\sum_{j=1}^{N} \mathbf{G}_{ij}^m \mathbf{H}(\mathbf{x}_j^m(t) - \mathbf{x}_i^m(t)) + \sigma_{ml}\sum_{l=1}^{3} \mathbf{H}(\mathbf{x}_i^l(t) - \mathbf{x}_i^m(t)), \tag{1}$$

where $i = 1, ..., N$ numbers the nodes inside each layer, $m = 1, 2, 3$ labels the layer, $\mathbf{x} = (x, y)^T$ is a state variable, and the local dynamics is described by the FitzHugh-Nagumo system $\mathbf{F}(x, y) = \left(\frac{1}{\varepsilon}(x - \frac{x^3}{3} - y), x + a\right)^T$. The adjacency matrices $\mathbf{G}^m$ define the topology of each layer, and L_i^m is the number of links belonging to the ith node of the mth layer. σ_m is the strength of the couplings inside the layers, and σ_{ml} the strength of the couplings between the layers. The matrix $\mathbf{H}$ defines the interaction scheme between the two-dimensional individual systems. Usually, we allow not only direct, but also cross coupling between the variables x and y. Fig. 1 shows schematically the structure of the three-layer network, where each layer has a nonlocal coupling topology, the outer layers are shown in grey, and the middle relay layer is shown in red. As a measure for synchronization between the layers we employ the global synchronization error:

$$E^{ij} = \lim_{t\to\infty} \frac{1}{NT}\int_0^T \sum_{k=1}^{N} ||\mathbf{x}_k^j(t) - \mathbf{x}_k^i(t)||dt, \tag{2}$$

which takes values close to 0 when the patterns in the layers $m = i, j$ are synchronized. When coherent and incoherent domains coexist spatially in each layer, we observe nontrivial synchronization scenarios, where the coherent domains of

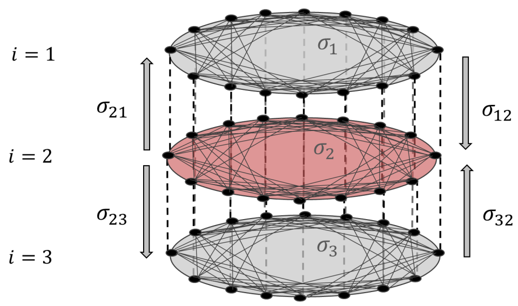

Figure 1: Schematic structure of a three-layer network

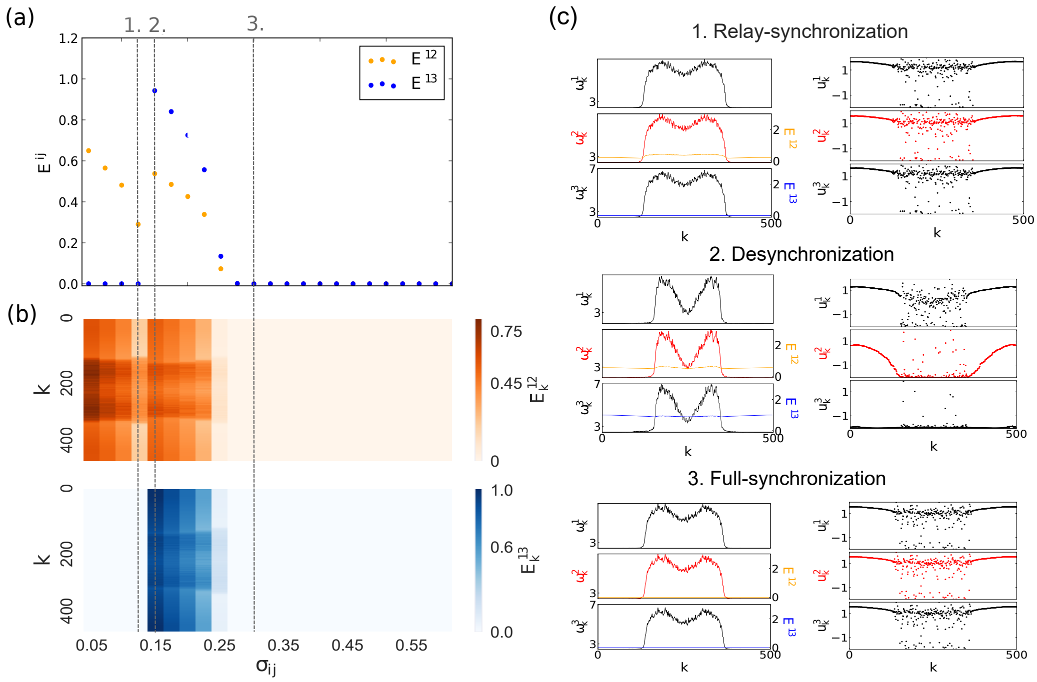

Figure 2: Scenario of synchronization transitions depending on the inter-layer coupling strength σ_{ij}. (a) Global synchronization error; (b) local synchronization error; (c) mean phase velocity profiles and inter-layer synchronization errors (left column) and snapshots (right column) for three values of σ_{ij} marked by vertical lines in (a), (b).

the patterns synchronize while the incoherent domains perform different dynamics. To distinguish this special kind of partial synchronization, we introduce the local synchronization error

$$E_k^{ij} = \lim_{t\to\infty} \frac{1}{T} \int_0^T ||\mathbf{x}_k^j(t) - \mathbf{x}_k^i(t)|| dt. \tag{3}$$

When the topology of the layers in the network is regular, we observe complete relay synchronization of chimera patterns in the outer layers, as well as partial relay synchronization, where only the coherent domains of patterns synchronize [13]. If we change the topology of the outer layers by replacing the regular links with random shortcuts, we find the scenarios shown in Fig. 2. Here the global and local inter-layer synchronization errors for increasing coupling strength between the layers, and examples of snapshots of chimera states and their mean phase velocities are depicted. We analyze the role of the system parameters and uncover parameter regions where full and partial relay synchronization occurs. Our results may be useful for understanding remote synchronization in brain networks.

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