Nonlinear Wave Disintegration in Phononic Material with Weakly Compressed Rough Contacts

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Summary. Phononic media with contact nonlinearity enable unique wave responses, which brings new capabilities in controlling the propagation of mechanical energy both passively and actively. Our previous studies of phononic media with periodic "rough" contacts have demonstrated different wave responses under *zero* or *strong* precompression, however, wave dynamics of these system under *weak* precompression is yet to be understood. Such understanding can help improve the dynamic response of these materials for wave propagation control. Here, we numerically study nonlinear wave propagation through phononic material with rough contacts such that the contacts are weakly compressed and exhibit a strong nonlinear response at high amplitude excitations. Different from uncompressed and strongly compressed media, this system disintegrates the excited waves into constant amplitude compression pulses followed by an oscillatory tail of decaying amplitudes. These two wave profiles are linked through a transition zone in the form of a rarefaction front. Such wave response is attributed to the mechanics of weakly-compressed contact that transmits tension-compression forces at low amplitudes but only compression forces at high amplitudes. We also demonstrate the tunability of the amplitude, speed, and energy of compression pulses via external precompression. Further, owing to the band gap characteristics of the underlying linear phononic media, these materials display spectral filtering of the harmonic waves. Overall, the capability of these materials to transfer information or energy via compression pulses, amplitude-dependent material response, energy transfer from the excitation frequency to other frequency-wavenumber regimes, and tunability through precompression could pave the way for the development of mechanical devices for advanced wave control.

Introduction

Phononic materials are periodic media that allow spatial and temporal control over mechanical waves. Specifically, Bragg scattering [1] and local resonance [2] phenomenon in these materials have enabled band gaps, which are frequencies of strong wave attenuation. These materials also exhibit unprecedented characteristics such as mode conversion [3], Anomalous polarization [3, 4] and wave directionality [5] by virtue of their unit cell geometry. While these studies are within the linear regime, incorporating nonlinearity further enhances dynamic properties in the form of amplitude-dependent response, irreversibility, and frequency conversion (see the review of [6]). All of these exotic properties make nonlinear phononic materials potential candidates for designing engineering materials and devices with advanced functionalities.

A route to achieve nonlinearity within phononic media is via contact interfaces. Phononic media with contact nonlinearity demonstrate excellent flexibility as they can be tuned from linear to a strongly nonlinear regime, through externally applied precompression. When uncompressed, these phononic materials does not support propagation of linear waves, essentially behaving as "sonic vacuum" [7]. On the other hand, high amplitude wave excitations can enable strongly nonlinear wave signatures. This has been demonstrated in granular crystals, which are ordered ensembles of particles (or grains) exhibiting *Hertzian* contact nonlinearity [8] and in continuum phononic materials with *rough* contacts [9]. While the type of contact nonlinearity and modeling of contacting bodies as discrete or continuum reveals interesting differences, the contact-based phononic media, in general, have shown excellent promise in enriching the wave dynamics.

Importantly, the combined effects of periodicity and nonlinearity in contact-based phononic media support different forms of nonlinear waves. For example, uncompressed (strongly nonlinear) granular crystals support the propagation of solitary waves, which are localized traveling waves with a hump-type wave profile propagating with constant speed and amplitude [7, 10]. Depending upon the mass of the striker, the system can support trains of solitary waves as well [11]. On the other hand, strongly compressed (weakly nonlinear) granular crystals support the propagation of harmonic (of oscillatory nature) waves. As a result, the system exhibits amplitude-dependent band diagrams with propagating and attenuation frequency zones [12]. Within a weakly nonlinear regime, granular crystals support energy transfer between frequencies due to harmonic generation [12]. The system can even support discrete breathers [13], which are spatially localized and temporally periodic modes, at band gap frequencies. Damped granular crystals support shock waves that gradually decay in time [14]. Beyond the elastic regime, asymmetric nonlinearity of elastic-plastic Hertzian contacts support the propagation of "signoton", which is a shock wave causing an instantaneous change in the deformation direction [15]. Our recent studies on continuum phononic media with rough contacts reported the emergence of stegotons (a different form of solitary waves with stepwise spatial profile) and acoustic resonances, when uncompressed (i.e. strongly nonlinear interfaces) [9], and frequency conversion from wave self-interactions when strongly compressed (i.e. weakly nonlinear interfaces) [16]. These recent studies further suggest the role of contact nonlinearity in phononic media in enabling unique wave responses with no analog in linear theory.

Despite these studies, nonlinear wave propagation through phononic media with *weakly-compressed rough contacts* is yet to be understood. There are open questions such as (1) how the reported nonlinear responses of these phononic media (acoustic resonances, stegotons and harmonic generation) change when non-zero precompression and high amplitude excitation is involved and (2) whether both strongly and weakly nonlinear wave responses co-exist together. Further, weak precompression may in fact support additional nonlinear responses not possible with uncompressed or strongly

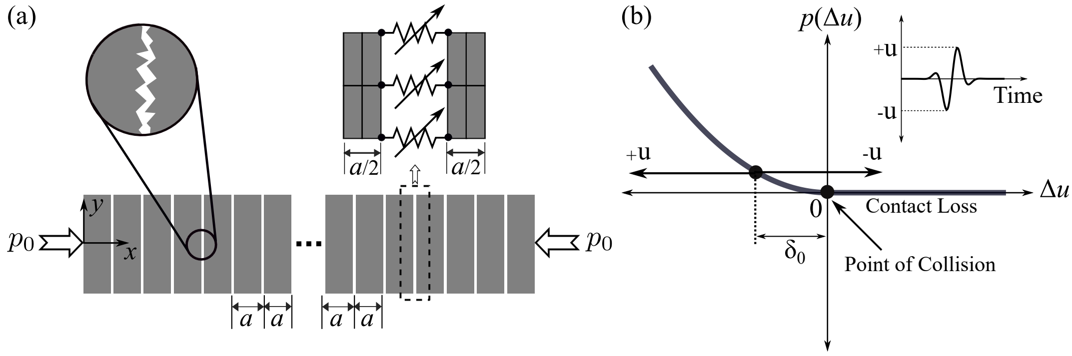

Figure 1: (a) Compressed phononic material with periodic contacts having rough features at the microscale. Schematic of dashed rectangle shows contact modeling as nonlinear springs between elastic layers. (b) Nonlinear contact pressure-gap relationship at rough contacts. The contacts exhibit strongly nonlinear responses as wave displacements, u, are larger than static predeformation, δ_0. Inset is a temporal schematic of the excited wave pulse.

compressed media.

In this paper, we numerically study nonlinear wave propagation through continuum phononic material with weakly compressed rough contacts. We excite wave pulse with amplitude larger than static predeformation to enable strong nonlinearity. We report the evolution of excited waves with propagation distance demonstrating the generation of localized traveling waves and subsequently their separation from low-amplitude harmonic waves. We study the wave response at different center frequency pulses and discuss the frequency-filtering abilities of the phononic material due to its dispersion characteristics. Finally, we also compare the speed and amplitude of generated pulses with the one formed in the uncompressed system, and show the tunability of propagation properties through precompression. The focus of the study is to illustrate nonlinear wave responses achievable in the weakly compressed system, different from uncompressed and strongly compressed configurations.

The paper is organized as follows: First, we describe the phononic material system and corresponding numerical setup. Then, we illustrate the transformation of input waves into waves of different forms due to nonlinearity and dispersion. The tunability of wave propagation is discussed later. Finally, we conclude the paper and provide potential future directions.

Phononic material with weakly compressed rough contacts

The studied phononic material is a one-dimensional system consisting of linear elastic layers of aluminum ($E = 69$ GPa, $\nu = 0.33$, $\rho = 2700$ kg/m^3) of identical length, a, with roughness on either side at the microscale [Fig. 1(a)]. These layers are compressed externally under a precompression, p_0, thus forming a continuum with periodic rough contacts. The layers and contact are assumed to be infinite in the y-direction. The study assumes the wavelength of propagating waves to be three orders of magnitude larger than the rough features of the contacts, allowing contacts to be modeled as nonlinear springs [17]. However, we study wave propagation at wavelengths comparable to layer thickness, thus capturing the effects of continuum layers as well. Due to the inherent feature of contacting bodies, this material does not support any tensile forces during a loss of contact. We assume the roughness on the contacting surface such that the asperities (uneven features on nominal surfaces) are uniformly distributed and that hysteresis associated with their plastic deformation is already removed through multiple loading-unloadings. Under these assumptions, the contacts follow a purely quadratic nonlinear relation between contact pressure, $p(\Delta u)$, and gap, Δu [Fig. 1(b)] [18]. Thus, the contact nonlinearity within our system is as follows:

$$p(\Delta u) = \begin{cases} 0, & \Delta u > 0, \\ \frac{C^2 \Delta u^2}{4}, & \Delta u \leq 0. \end{cases} \tag{1}$$

where $C = 6 \times 10^{10} \sqrt{\text{Pa}}/\text{m}$ [18]. Since $u > \delta_0$ in our analysis (i.e. wave displacement is larger than the predeformation caused by p_0), the dynamics of rough contacts is non-smooth (strongly nonlinear) as contact surfaces can collide and separate [Fig. 1(b)].

To study nonlinear wave propagation through these phononic materials, we construct a finite element (FE) model using COMSOL Multiphysics 5.5a with solid mechanics module based on our previous work [16, 9]. The layers are considered as continua while the contacts are modeled through spring elements with nonlinear characteristics as defined in Eq. 1. The numerical simulation is conducted in two stages: first, the model solves the static problem of external precompression to determine the deformed state of the phononic material and predeformation in the springs. Then, a wave propagation study is conducted through transient analysis. The output of the static analysis is considered as the initial conditions for the wave propagation problem. We model a larger number of unit cells than what we analyze to avoid the effect of end reflections, essentially simulating a phononic material infinite in the positive x-direction. A longitudinal wave pulse [wave schematic in Fig. 1(b)] of amplitude, U, and center frequency, f, with a Gaussian modulation (parameters, $\zeta = 2/f$, $\sigma = 0.5/f$ for Eq. 4 of [9]) was excited from the entire left edge of the FE geometry. Note that the maximum displacement of the

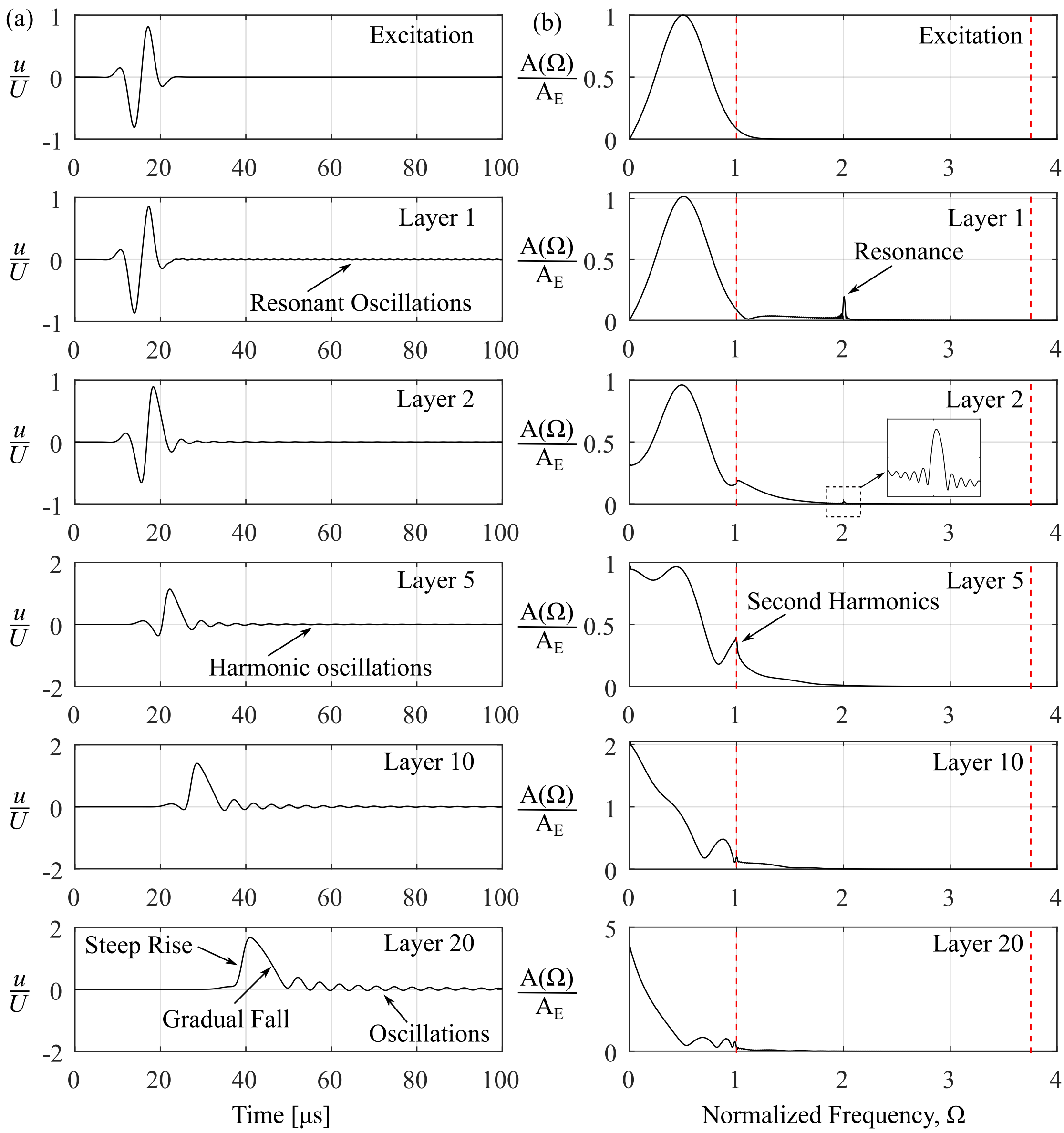

Figure 2: Evolution of input pulse wave with center frequency at $\Omega = 0.5$. (a) Displacement-time profile and (b) frequency content of the propagating wave at multiple locations inside the phononic material. Wave displacements, u, are normalized by pulse excitation amplitude, U, and spectral amplitudes are normalized by spectral excitation amplitude, A_E. The signals are recorded at the center point in the corresponding layers and the layer indices are numbered from the excitation boundary. Red dashed lines are linearized band gap edges.

input pulse [max($\mid +u \mid, \mid -u \mid$)] is smaller than excitation amplitude, U, due to Gaussian modulation. This specific wave profile corresponds to the typical response of a broadband ultrasonic contact transducer.

Nonlinear wave disintegration

In this section, we study the evolution of the input wave profile in the phononic material with weak external precompression and high amplitude excitation. Specifically, we discuss an example case of $\delta_0/u \approx 0.83$, which allows us to enable strong nonlinearity as well as study the effect of external compression. To inform the selection of frequencies for analysis, we obtained the linearized dispersion of the phononic material [16]. A non-dimensional frequency, Ω, is introduced such that the frequencies are normalized by the lower-edge frequency of the first band gap of the linearized phononic media for the applied p_0 (i.e. band gap starts at $\Omega = 1$). We study two representative cases of input pulse: (1) with center frequency in pass band ($\Omega = 0.5$) and (2) with center frequency at band edge ($\Omega = 1$). The first pulse, consisting of wide band of frequencies within $0 < \Delta\Omega < 1$, was excited to analyze nonlinear wave propagation in a highly dispersive frequency regime of the linearized phononic media. On the other hand, the second pulse, consisting of wide band of frequencies within $0 < \Delta\Omega < 2.5$, was excited to analyze the effect of band gaps on nonlinear wave propagation.

The wave profile changes as the wave propagates through the phononic material [Fig. 2(a)]. Particularly, the input pulse that originally consists of both compression and extension displacement, gradually transforms to a profile which is predominantly in compression only [compare wave profile in layer 1 vs in layer 20 in Fig. 2(a)]. As the wave propagates, the tensile portion of the pulse vanishes with each layer [note gradual reduction in negative displacements from layer 2

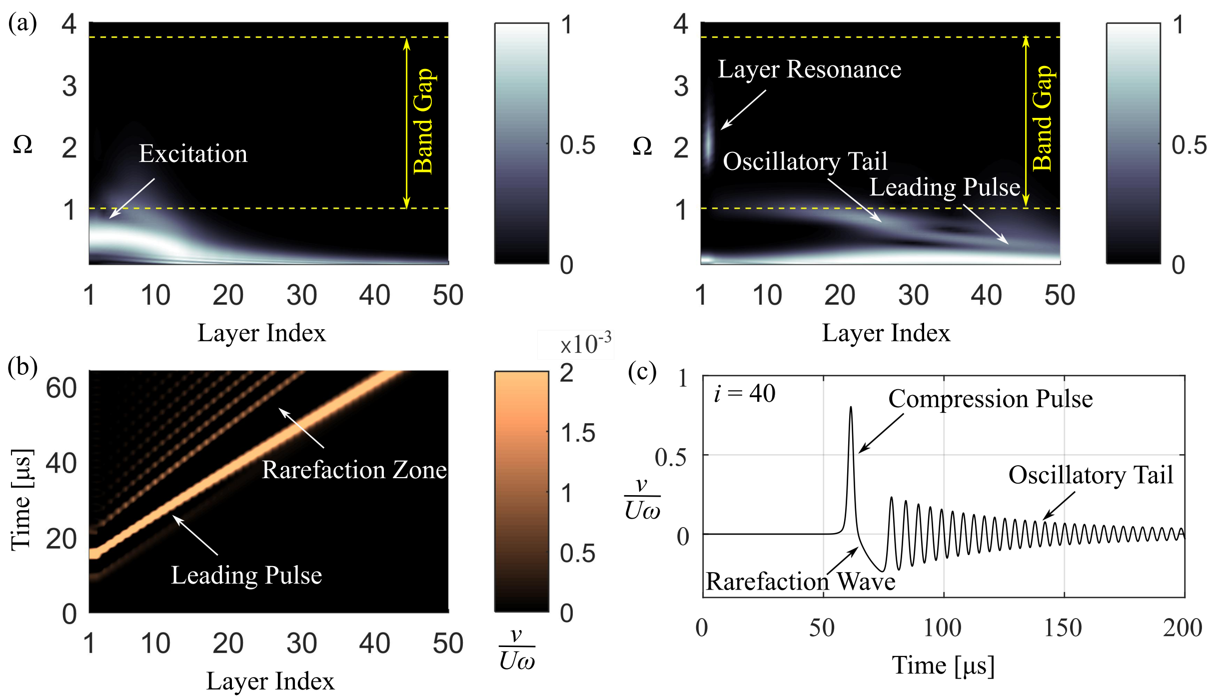

Figure 3: (a) Spatio-spectral and (b) spatio-temporal plots of nonlinear wave propagation for excitation at $\Omega = 0.5$. Spatio-spectral plot (normalized by its maximum value) is shown at an instant when the leading pulse is between the 10th and 20th layer (left) and between the 40th and 50th layer (right). Dashed yellow lines are linearized band gap edges. Layer indices are marked from the excitation boundary. (c) Temporal profile of the disintegrated wave inside the 40th layer. Wave amplitude is denoted in terms of particle velocity, v, normalized by excitation amplitude, U, and angular frequency, ω.

to 20 in Fig. 2(a)]. Moreover, the transmitted compression part is followed by small-amplitude oscillatory waves. The Gaussian input pulse eventually evolves into a waveform that contains three distinct features: (1) a steep increase in profile at the wave front, followed by (2) a gradual amplitude reduction, and (3) an oscillatory tail. This transformation can be understood as follows: The profile of the pulse transmitted across a contact is a combined effect of contact nonlinearity and weak compression. The contacts transmit all compressive forces, whereas only tensile forces that are just enough to counteract the external precompression are transmitted. As a result, once tensile forces exceed external precompression, a contact loss takes place. Therefore, the only harmonic waves supported inside the phononic material are those that contain small amplitude displacements.

This wave evolution also causes frequency conversions, meaning that energy is transferred within the spectral domain. In fact, both strongly and weakly nonlinear effects occur within the phononic material. Specifically, layer 1 resonates at a frequency, $\Omega = 2$ [note resonant oscillations in Fig. 2(a) and a spectral peak in Fig. 2(b) for layer 1]. This resonance corresponds to the mode of the layer under the fixed-forced boundary condition. Note that layer 1 is held fixed at the left end by virtue of excitation profile whereas the right end is a rough contact. This mode is at a frequency slightly higher compared to the first fixed-free mode of the layer, i.e. acoustic resonance given by $\Omega_r = 1.876$ ($\Omega_r = c_0/4a$, where c_0 is the wave speed of the bulk material). This is because, unlike the uncompressed system, the weakly compressed system experiences a non-zero spring force at the contact end, which pushes the mode of the layer to a higher frequency. Even, layer 2 shows a spectral peak at the same frequency [refer inset in Fig. 2(b) for layer 2]. This is surprising since, in an uncompressed system, the second layer from the excitation boundary resonates at a frequency twice the resonance of layer 1 [9]. This is because layer 2 loses its contact with adjacent layers after wave interaction and the contact loss remains in effect due to the lack of external precompression. As a result, layer 2 resonates under free-free boundary conditions. In the current system, however, both layers (1 and 2) are in contact due to external precompression - a state not possible in an uncompressed system. Due to their persistent contact even after the wave-contact interactions, the resonant energy of layer 1 is partly transmitted to layer 2 causing a spectral peak at $\Omega = 2$. In subsequent layers, second harmonics are generated, i.e. $\Omega = 1$ [refer to the spectral peak at Fig. 2(b) for layer 5] - a feature typical of weakly nonlinear systems. These harmonics, however, lie on the band gap edge and beyond. Thus, they do not propagate and therefore their spectral amplitude vanishes at later layers, for example, layers 10 and 20. Frequencies within the pass band propagate while those close to the band gap edge propagate with extremely slow speed. Eventually, the propagating frequency components consist primarily of low frequencies close to 0 Hz, i.e. a localized traveling wave.

One can think of the transformation of the input wave profile and its frequency content as a “disintegration" of the input pulse. Here, we refer to disintegration as a transformation of input pulse into (1) leading pulses of compressive nature, (2) transitioning zone of rarefaction nature, and (3) tails of decaying amplitudes of harmonic nature. This can be seen in terms of the spatial distribution of frequency content [Fig. 3(a)] and temporal dependence of particle velocity [Figs. 3(b) and (c)]. Despite exciting energy in the system around $\Omega = 0.5$, the energy content within phononic material spreads spatially

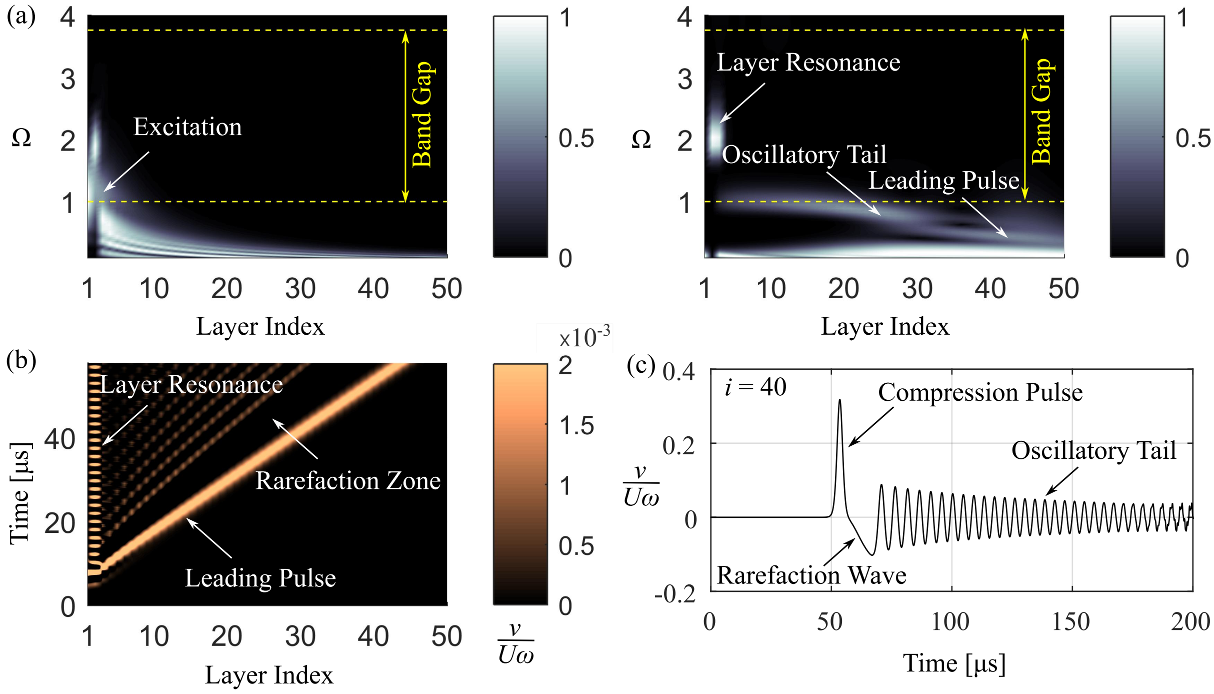

Figure 4: (a) Spatio-spectral and (b) spatio-temporal plots of nonlinear wave propagation for excitation at $\Omega = 1$. Spatio-spectral plot (normalized by its maximum value) is shown at an instant when the leading pulse is between the 1st and 10th layer (left) and between the 40th and 50th layer (right). Dashed yellow lines are linearized band gap edges. Layer indices are marked from the excitation boundary. (c) Temporal profile of the disintegrated wave inside the 40th layer.

due to dispersion while some remains trapped in the form of layer resonances [Fig. 3(a)]. The oscillations correspond to frequencies near the band edge that propagate at a much slower speed. The leading pulse propagates faster than the oscillatory tails and thus overtakes them, causing disintegration, which is clearly visible away from the excitation boundary. The pulse and oscillatory wave profiles are linked through a rarefaction front [Fig. 3(c)]. As a result, the duration over which a rarefaction wave exists in a layer increases with propagation distance [Fig. 3(b)]. This shows that phononic material with contact nonlinearity can support rarefaction waves given non-zero external precompression. Interestingly, similar wave disintegration has been reported in nonlinear metamaterials but with softening-type nonlinearity, i.e. where the exponent of power-law nonlinearity is < 1 [19, 20]. Specifically, these studies reported that an excited compression pulse transforms into a leading rarefaction pulse in tensegrity [19] and origami metamaterials [20].

A similar wave disintegration occurs even in the case when the center frequency of the excited pulse is at the linearized band gap edge, i.e. $\Omega = 1$ (Fig. 4). Importantly, the frequencies $\Omega > 1$ do not propagate through the phononic material, indicating that the band gap exists at these frequencies in this strongly nonlinear regime. In this case, oscillatory tails consist of frequencies within the pass band only. Since the excitation frequency is closer to the resonance frequency of layer 1, there are stronger resonant oscillations of layer 1 compared to when the excitation center frequency is $\Omega = 0.5$ [Compare oscillations at layer index = 1 in Fig. 4(b) and Fig. 3(b)]. The generated compression pulses in these phononic materials are in fact localized traveling waves that propagate with constant speed and amplitude. Due to the hybrid nature of the phononic material, which is continua connected through discrete nonlinearity, these compression pulses are in the form of stegotons showing a step-wise spatial profile [9].

Tunability of compression pulses

In this section, we characterize the dependence of propagation properties of nonlinear waves on external precompression. Specifically, we discuss how external precompression can be used to tune and control the leading compression pulses. The ability to control the speed and amplitude of these pulses can allow greater flexibility and feasibility in utilizing these phononic materials for engineering applications.

By controlling external precompression while keeping the input amplitude the same, we can control the amplitude of the generated compression pulse. The absolute amplitude of these pulses increases with an increase in precompression [Fig. 5(a)]. This is surprising since as precompression increases, the ratio between predeformation and wave amplitude (δ_0/u) increases. As a result, the strength of nonlinearity decreases, which in turn is expected to cause relatively weaker momentum transfer across the contacts. In other words, increasing precompression is expected to generate compression pulses with decreasing amplitudes. However, keeping the same excitation displacement amplitude in the analysis requires larger external work to be done on the system for larger precompressions. This is because the dynamic excitation has to work against the stronger contact forces developed from increased precompressions. Overall, increasing precompression draws more input energy for the same excitation frequency and amplitude, which subsequently generates compression pulses

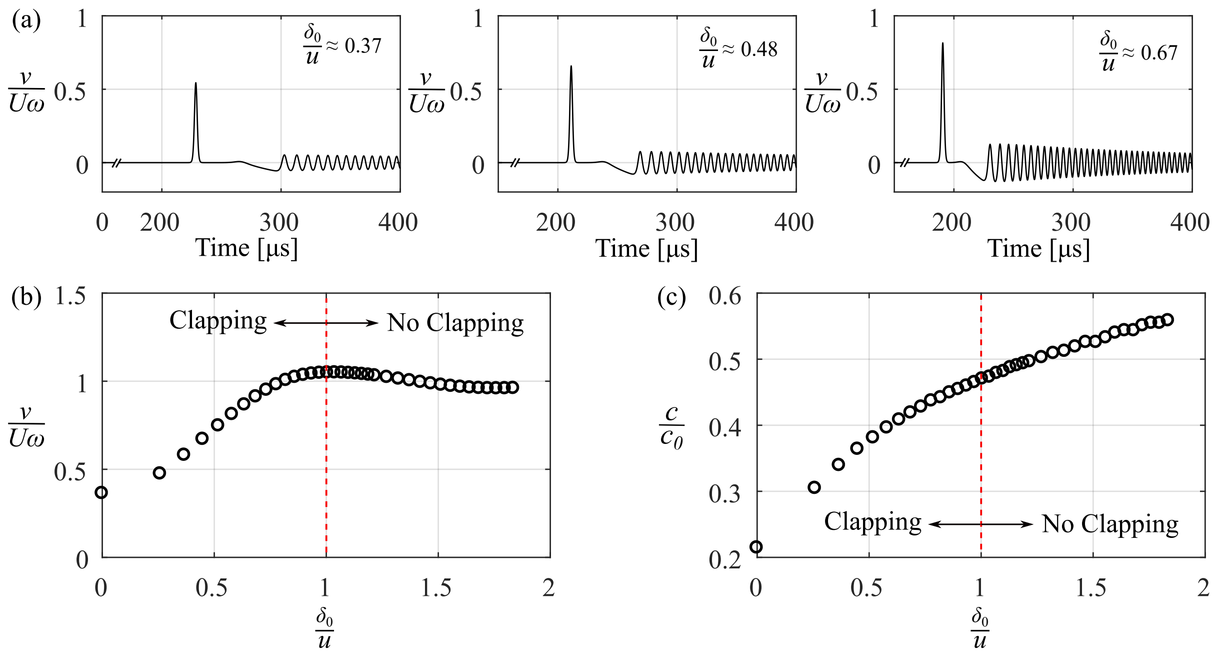

Figure 5: Tunability of nonlinear waves through precompression. (a) Disintegrated wave profile inside the 150th layer for three different precompressions. Dependence of pulse (b) amplitude and (c) wave speed on precompression. Red dashed line is the precompression threshold below which contact clapping occurs.

with larger amplitudes. The rarefaction and harmonic wave amplitudes also increase with compression pulse amplitudes. In other words, stronger precompression supports tensile waves of high amplitudes. Based on these observations, we can conclude that weakly compressed systems exhibit nonlinear wave responses that are a mix of uncompressed and strongly compressed systems, and further allow control over these responses. Specifically, the high amplitude portion of the excitation contributes to strongly nonlinear responses (i.e. compression pulses) as seen in the uncompressed system and low amplitudes contribute to weakly nonlinear responses (i.e. frequency filtering and harmonic waves) as seen in the strongly compressed systems. However, the transition zone from compression pulse to harmonic oscillations is seen only in weakly compressed systems. As waves propagate, compression pulses separate from the rarefaction waves. It is worth noting that the time delay between the compression pulse and rarefaction front can also be controlled through precompression [Fig. 5(a)].

As discussed, the compression pulses are in fact solitary waves as these pulses propagate with constant speed and amplitude. Next, we discuss how the amplitude and speed of these pulses depend on precompression and the threshold value of precompression that can still generate these strongly nonlinear responses. Interestingly, the compression pulses are generated even for the cases where contact clapping is restricted, i.e. when predeformation is larger than the excitation amplitude ($\delta_0/u > 1$) [Fig. 5(b)]. Despite the lack of clapping, this regime is not necessarily weakly nonlinear since excitation amplitudes are not small enough. Note that there exists quadratic nonlinearity between contact force and displacement due to the deformation of rough asperities. This is possibly causing the balance between nonlinearity and dispersion to form a compressive pulse. Further, the amplitude of generated pulses decreases beyond $\delta_0/u > 1$ - a threshold beyond which no clapping is possible. The change in amplitude of compression pulses is less sensitive to $\delta_0/u > 1$ compared to $\delta_0/u < 1$ [note the change in pulse amplitude dependence on precompression before and after the red dashed line in Fig. 5(b)]. The maximum amplitude of the compression pulse is attained at $\delta_0/u = 1$. This is the threshold where clapping is possible and energy added to the system is more than the energy added for $\delta_0/u < 1$.

Contrary, the wave speed of the compression pulses, c, (normalized by bulk wave dilatational wave speed, c_0) increases monotonically with precompression [Fig. 5(c)]. This is consistent with Fig. 5(b) for $\delta_0/u < 1$ since the amplitude and speed of these waves are proportional to each other. We also observe that wave speed changes dramatically for low precompression and depends linearly on precompression at larger values. For $\delta_0/u > 1$, wave speed increases despite a decrease in amplitude. This indicates that the pulses generated beyond $\delta_0/u > 1$ have a different speed-amplitude relationship than solitary waves generated from clapping.

As expected, the fraction of energy carried by the compression pulses as a function of total energy in the system decreases with an increase in precompression (Fig. 6). In fact, the energy carried by the pulses is almost negligible (two orders smaller) for precompressions $u/\delta_0 > 1.5$. This indicates the region of precompression where the phononic material behavior changes into a weakly compressed system.

Conclusions

In this paper, we studied high amplitude nonlinear wave propagation through phononic material with weakly compressed rough contacts. The source of nonlinearity within the material comes from two mechanisms: (1) contact separation and

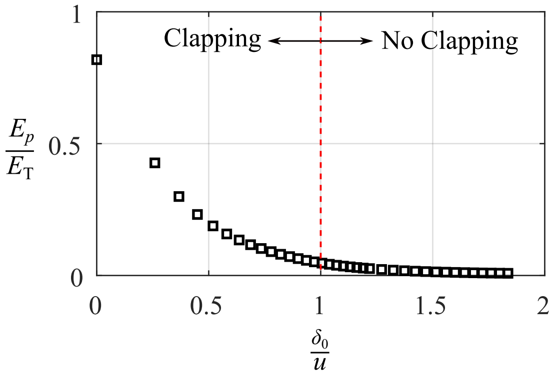

Figure 6: Dependence of the energy carried by the compression pulses (E_p) as a fraction of total energy (E_T) on precompression.

collision at high amplitudes and (2) quadratic contact force-displacement relationship during the contact. Application of non-zero precompression during analysis revealed that both strongly and weakly nonlinear wave responses can coexist. Specifically, the phononic material supports both solitary waves as well as frequency filtering. Importantly, an additional wave signature in the form of wave disintegration is observed in the weakly compressed system. The input pulse breaks down in three different wave forms: (1) a localized traveling wave in the form of compression pulse, (2) a rarefaction wave front, and (3) an oscillatory tail of decaying amplitude. A compression pulse initially leads the wave propagation before separating from other wave forms due to its greater wave speed. Oscillatory tails are associated with the periodic waves at frequencies in the vicinity of band gap and therefore spread spatially. Finally, the propagation properties of the compression pulse, i.e. their amplitude and speed, can be tuned through external precompression.

Contact-based phononic materials would practically require non-zero precompression to keep all surfaces in contact. The results presented in this paper give an idea of how a slight deviation in system parameters dramatically changes the behavior of the phononic material. The effect not only influences the values of propagation parameters but in fact introduces new types of waves. From an application perspective, these weakly compressed phononic materials can be used for developing mechanical sensors and delay switches. Particularly, the rate at which information is carried from one end to the other can be simply controlled in-situ through external precompression. The combined ability of information transfer and frequency filtering could help develop new imaging devices to isolate spurious scattering while transmitting captured data.

While we elucidated wave responses from weak compression, a detailed study of the effects of excitation frequency, pulse bandwidth, and different contact laws is necessary to fully understand the capability of the system. Higher frequency excitation may cause stronger momentum transfer across contacts generating strong compression pulses while different contact laws may dictate the speed-amplitude relationship. Wave excitation with narrow bandwidth may help study higher harmonic generation while wide band excitation could cause wave mixing within input frequencies. These complex behavior may help advance the understanding of how contacts can be exploited to manipulate the propagation of mechanical energy in an unprecedented way.

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