Model updating for digital twins using Gaussian process inverse mapping models

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Summary. In engineering dynamics, model updating is typically applied to minimize the mismatch between a physical system and its digital twin. This paper proposes to use inverse mapping models, based on Gaussian Processes (GPs). The latter are trained offline using simulated data, enabling fast online updating of physically interpretable parameter values in first-principles-based nonlinear dynamics models. The GPs infer parameter values based on time-domain features measured on the real system. Additionally, GPs enables uncertainty quantification of the inferred parameter values. A nonlinear multibody model is used to illustrate the capability of this method to update parameter values, with high computational efficiency, and extract corresponding uncertainty measures.

Introduction

Digital twins allow engineers to optimize the design and performance of a (controlled) physical system, and monitor systems in real-time. Since, a model (i.e., a digital twin) is, not an exact representation of the physical system, fruitful employment of the digital twin is hindered. Model updating is therefore used to minimize the mismatch between the model and the measured system. In this research, we focus on updating parameter values of first-principles models with fixed model structures. To make model updating generally applicable in an online, digital twin context, the updating method should be: 1) computationally fast , 2) applicable to nonlinear models, 3) physically interpretable, and 4) able to quantify the uncertainty in the parameter estimates. The method introduced here uses inverse mapping models, based on Gaussian Processes (GPs): a set of measured features is mapped to a set of corresponding parameter values. Additionally, the GPs yield a quantification of the uncertainty in the estimated parameter values. Although this method has previously been applied to linear systems [3], here, we extend its application to nonlinear systems by using time-domain features.

Methodology

Inverse models are used to, online, rapidly map a set of measured time-domain output features to a set of physically interpretable parameter values of a first-principles (or forward) model, see Figure 1. Parameterizing the forward model with the inferred parameter values, results in an updated model that, when used in a simulation, yields a set of output features close in similarity to the original, measured features. In this research, the output features are defines as samples of the output signals at equidistant moments in time. In contrast to earlier work of the authors [2], in which a neural network is used to define the inverse mapping model, here, GPs are used. Due to the use of GPs, in addition to inferred parameter values, a quantification of the uncertainty in each inferred parameter value is obtained in the form of a standard deviation. For each updating parameter, a separate GP is trained offline using training data. These data are obtained by simulating the forward model and extracting features from the simulated output signals for a number of distinct combinations of updating parameter values, distributed in some admissible updating parameter space $\mathbb{P}$. Note that the excitation signals and initial conditions used to obtain the training data should be identical to those used for the real measurements as these are implicitly learned by the GPs. Furthermore, the forward model structure is assumed sufficiently accurate.

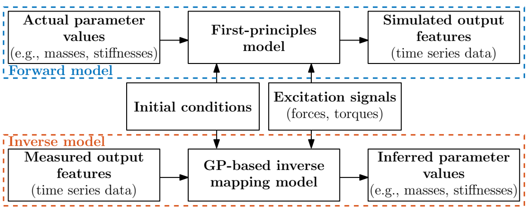

Figure 1: Schematic representation of in- and outputs to first-principles models and inverse models for model updating.

Case study: a nonlinear multibody system

The GP-based updating method is applied to a two-degrees-of-freedom nonlinear multibody system consisting of two connected rigid beams of mass m_1 and m_2, respectively, see Figure 2. The $n_p = 4$ updating parameters are: damping constants d_y, d_θ, and spring constants k_y, and k_θ, where we assume that their values lie between the bounds of the parameter set $\mathbb{P} \subset \mathbb{R}^{n_p \times 1}$, specified in Table 1. The system is simulated for 5 seconds with the static equilibrium position of the system, parameterized with parameter values in the center of $\mathbb{P}$, as initial condition. For the output features, 100 equidistant time samples, of both the $y(t)$ and $\theta(t)$ output signals, are used per sample. To mimic real measurements, these output signals are contaminated by output noise (zero mean, $\sigma_y = 5 \times 10^{-5}$ m, $\sigma_\theta = 0.015$ rad). The system is excited by an impulse-like excitation force $F(t)$ and moment $M(t)$:

$$F(t) = \begin{cases} 5 \quad \mathrm{N} & \text{if } 0.2 \leq t \leq 0.25 \\ 0 \quad \mathrm{N} & \text{else} \end{cases}, \tag{1}$$

$$M(t) = \begin{cases} 0.075 \quad \mathrm{N \cdot m} & \text{if } 0.2 \leq t \leq 0.25 \\ 0 \quad \mathrm{N \cdot m} & \text{else} \end{cases}. \tag{2}$$

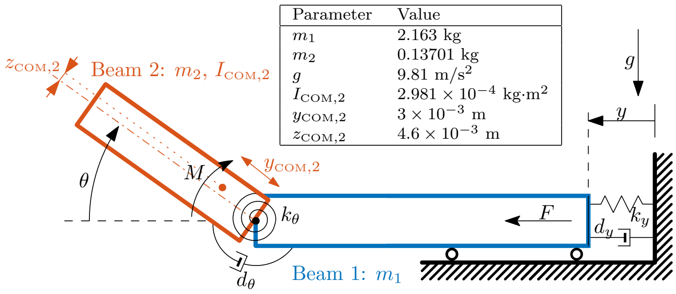

Figure 2: Demonstrator model with non-updating parameter values. The location of the Center Of Mass (COM) is indicated by $y_{\mathrm{COM},2}$ and $z_{\mathrm{COM},2}$. Furthermore, g represents the gravitational acceleration and $I_{\mathrm{COM},2}$ the mass moment of inertia about the COM.

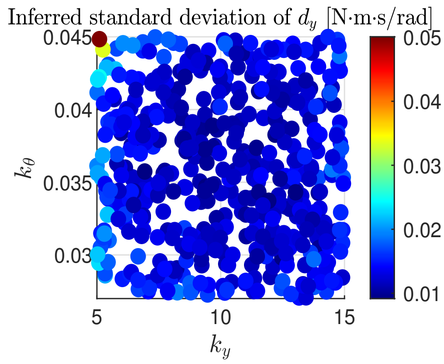

Figure 3: Inferred standard deviation of parameter value estimates for d_y in the subspace of $\mathbb{P}$ spanned by k_y and k_θ.

Table 1: Updating parameters and their lower and upper bound of $\mathbb{P}$, bias, standard deviation, and mean absolute relative error.

Parameter	Lower bound	Upper bound	$\boldsymbol{\mu}_\epsilon$ [%]	$\boldsymbol{\sigma}_\epsilon$ [%]	$\boldsymbol{\mu}_{\lvert\epsilon\rvert}$ [%]
d_y	0.9 N·s/m	1.1 N·s/m	0.1063	0.9374	0.7412
d_θ	1.75× 10^{-4} N·m·s/rad	2.25× 10^{-4} N·m·s/rad	-0.0991	1.3673	1.0880
k_y	5 N/m	15 N/m	-0.0004	0.3117	0.2128
k_θ	0.027 N·m/rad	0.045 N·m/rad	-0.0356	0.2581	0.2026

All n_p GPs are trained using the same 250 training samples, generated for as many combinations of updating parameter values, sampled using a Latin hypercube from $\mathbb{P}$. The squared exponential kernel is used in combination with a Gaussian likelihood and a constant valued mean function, of which the hyperparameters are optimized by minimizing the negative log marginal likelihood. For more information about these settings, the reader is referred to [1].
To demonstrate the proof of principle of the proposed method, output features of $n_t = 500$ test samples are simulated for distinct parameter values $\boldsymbol{p}(i) \in \mathbb{P}$ (where i indicates the sample), using equivalent settings for the simulation as for the training data generation. Here, all instances of $\boldsymbol{p}(i)$ are randomly distributed (uniformly) in $\mathbb{P}$. Then, the trained GPs are used to infer parameter values $\hat{\boldsymbol{p}}(i) \in \mathbb{P}$ from the simulated output features. To asses accuracy and precision of these inferred parameter values, the relative estimation error is calculated:

$$\boldsymbol{\epsilon}_{\mathrm{rel}}(i) = \left(\hat{\boldsymbol{p}}(i) - \boldsymbol{p}(i)\right) \oslash \boldsymbol{p}(i), \tag{3}$$

and used to determine the bias and standard deviation of the relative error, and the mean absolute relative error:

$$\boldsymbol{\mu}_\epsilon = \frac{1}{n_t}\sum_{i=1}^{n_t} \boldsymbol{\epsilon}_{\mathrm{rel}}(i), \qquad \boldsymbol{\sigma}_\epsilon = \sqrt{\frac{1}{n_t - 1}\sum_{i=1}^{n_t} \left(\boldsymbol{\epsilon}_{\mathrm{rel}}(i) - \boldsymbol{\mu}_\epsilon\right) \otimes \left(\boldsymbol{\epsilon}_{\mathrm{rel}}(i) - \boldsymbol{\mu}_\epsilon\right)}, \qquad \boldsymbol{\mu}_{|\epsilon|} = \frac{1}{n_t}\sum_{i=1}^{n_t} \left|\boldsymbol{\epsilon}_{\mathrm{rel}}(i)\right|. \tag{4}$$

In (3) and (4), $\oslash$ and $\otimes$ denote the entrywise division and multiplication operators, respectively. In Table 1, these error metrics are listed for all updating parameters. As displayed by the low error metrics, these parameters are inferred accurately (low bias) and precisely (low standard deviation). Furthermore, Figure 3 shows the inferred standard deviation in d_y, representing the quantification of the uncertainty in the estimated parameter values, as obtained by the GP, for all test samples in the subspace of $\mathbb{P}$ spanned by k_y and k_θ. Note that the largest inferred standard deviations are located at the edges of $\mathbb{P}$ (especially the edge where $k_y \approx 5$ N/m). The time required to infer all parameter values and their inferred standard deviations is only 13 ms, enabling fast, credible parameter value updating.

Conclusions and future work

In this work, Gaussian Processes are used as inverse mapping models to efficiently update physically interpretable parameter values of a nonlinear multibody model using time-domain features. Additionally, inferred standard deviations, provided by the GPs, provide a quantification of the uncertainty in the updated parameter values. However, costwise, GPs scale poorly with an increasing number of training samples [1]. Consequently, applications to updating problems with many updating parameters may become infeasible. Therefore, in future work, we will investigate Bayesian neural networks as an alternative for inverse mapping models. Furthermore, to improve the sensitivity of the inverse mapping model, optimal excitation design, feature extraction, and feature selection techniques should be explored further.

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References

[1] Bishop C.M. (2006) Pattern Recognition and Machine Learning. Springer, Cambridge.

[2] Kessels B.M., Fey R.H.B., Abbasi M.H., van de Wouw N. (2022) Model Updating for Nonlinear Dynamic Digital Twins Using Data-Based Inverse Mapping Models. *Submissions of IMAC-XL conference, 7-11 February, Orlando, USA.* (accepted for publication)

[3] Zhou K., Liang G., Tang J. (2014) Efficient Model Updating using Bayesian Probabilistic Framework based on Measured Vibratory Response. *Proceedings of SPIE-The International Society for Optical Engineering, Vol. 9063, 7-13 March 2014, San Diego, USA.*