Asymptotic analysis of transient behavior of two coupled exciters

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Summary. A model of two coupled exciters is considered to investigate transient behavior of self-synchronizing systems with non-negligible damping. An averaging method for partially strongly damped systems is used for asymptotic analysis. Stationary solutions of the system are derived. Attraction domains of different types of solutions are identified and depicted in phase space.

Introduction

Application of multiple exciters and utilization of self-synchronization in mechanical systems led to development of a new generation of vibratory machines including self-synchronous vibrating feeders, conveyors, screens, grinders and so on. Such systems replace kinematic connections like gears or chains with self-synchronization to generate required excitation forces. They also have the advantage of distributing and decreasing the load on bearings, if instead of just one big exciter, multiple smaller exciters are used.
The synchronization theory of mechanical exciters was first proposed by Blekhman [1]. Since then, many different synchronous systems are investigated. Previous works mostly just analyze synchronous solutions, their stability and existence with the assumption of negligible damping and that synchronization occurs far away from resonance. Goal of this work is to analyze transient behavior of synchronizing systems, examine different types of solutions in phase space and determine attraction domains of stable solutions.

Investigated model

Investigated model of two coupled exciters is shown in Fig. 1. It consists of a carrier of mass M, which is elastically suspended with a spring-damper element of stiffness c and damping d in horizontal direction. Two unbalanced rotors of mass m_i, moment of inertia J_i and eccentricity e_i, where $i = 1, 2$ is the index describing the number of rotors, are mounted on the carrier. They are driven in the same direction by induction or DC engines of limited power with a linearized torque characteristic given as $T_i = U_i(\omega_i^* - \dot{\varphi}_i)$. The parameter U_i describes the slope of motor characteristic and ω_i^* the nominal rotation speed. The equations of motion read

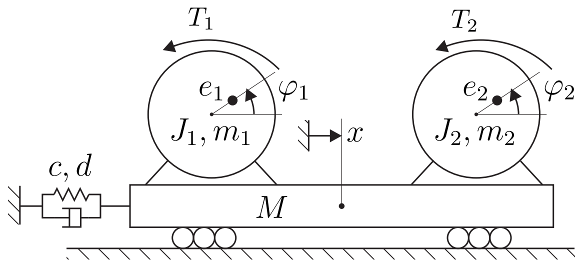

Figure 1: Investigated model

$$\xi'' + 2\sigma\xi' + \xi = \sum_{i=1}^{2} \mu_i \nu_i (\varphi_i'' \sin\varphi_i + \varphi_i'^2 \cos\varphi_i), \quad \varphi_i'' = \varepsilon \left(\frac{s_i}{\nu_i} \xi'' \sin\varphi_i + u_i(\lambda_i - \varphi_i') \right) = \varepsilon f_{\varphi_i}, \qquad i = 1, 2,$$

with the non-dimensional parameters and variables

$$\mu_i = \frac{m_i}{M^*}, \quad \nu_i = \frac{e_i}{e^*}, \quad \varepsilon s_i = \frac{1}{1 + J_i/m_i e_i^2}, \quad e^* = (e_1 + e_2)/2, \quad u_i = \frac{U_i s_i}{k m_i e_i^2}, \quad \lambda_i = \frac{\omega_i^*}{k},$$
$$\xi = \frac{x}{e^*}, \quad k^2 = \frac{c}{M^*}, \quad 2\sigma = \frac{d}{kM^*}, \quad M^* = M + m_1 + m_2, \quad \tau = kt.$$

The parameter ε is assumed to be small and the damping parameter σ is not small. The investigation is performed for a system with two identical rotors with different nominal speeds, which means that parameters of both rotors are identical except $\lambda_1 \neq \lambda_2$.

Asymptotic analysis

The averaging method for partially strongly damped systems is applied, see [2, 3]. Motion of the carrier can be replaced by its forced solution and the differential equation for ξ can be neglected in further analysis. Equations in standard form for a second order approximation with $\sqrt{\varepsilon}$ as the small parameter can then be acquired as

$$\frac{\mathrm{d}\delta}{\mathrm{d}\psi} = \frac{2\sqrt{\varepsilon} v}{p}, \qquad \frac{\mathrm{d}v}{\mathrm{d}\psi} = \frac{2\sqrt{\varepsilon}(f_{\varphi_2} - f_{\varphi_1})}{p}, \qquad \frac{\mathrm{d}p}{\mathrm{d}\psi} = \frac{2\varepsilon(f_{\varphi_2} + f_{\varphi_1})}{p}, \tag{1}$$

with the new variables $\omega_i = \varphi_i'$, $\psi = (\varphi_1 + \varphi_2)/2$, $\delta = \varphi_2 - \varphi_1$, $p = \omega_1 + \omega_2$, $v = (\omega_2 - \omega_1)/\sqrt{\varepsilon}$. By averaging these equations, a third-order system can be derived for the averaged variables $\bar{\delta}$, $\bar{v}$ and $\bar{p}$.

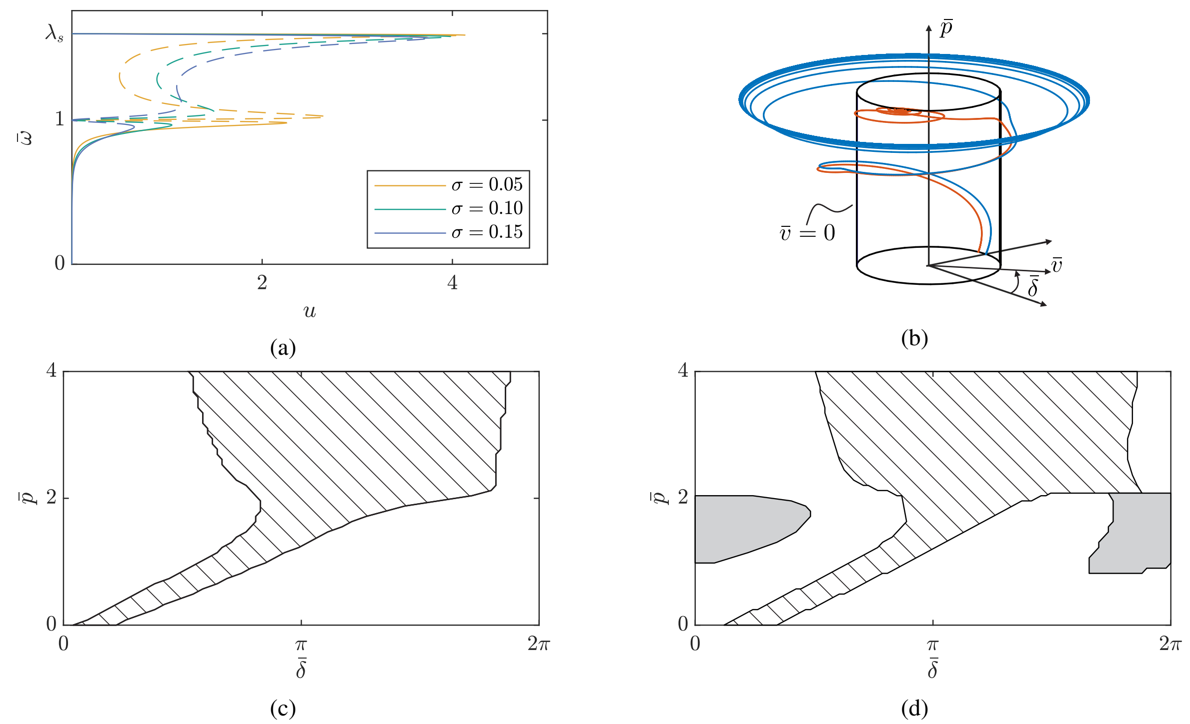

Figure 2: Results: (a) Stable (full) and unstable (dashed) stationary solutions of the averaged system for different values of damping, $\lambda_1 = 1.5$, $\lambda_2 = 1.7$, synchronization frequency $\lambda_S = 1.6$ derived according to [1]. (b) Two trajectories in cylindrical phase space. (c) A section of the phase space showing attraction domains of overcritical (white) and synchronous (hatched) solutions. (d) Same section of the phase space for smaller damping, where overcritical, synchronous and capture into resonance (grey) solutions all coexist.

Results

From the equation for $\bar{\delta}$ in Eq. (1) follows, that all stationary solutions of the system, which can be obtained under the assumptions made, are synchronous solutions ($\bar{v} = 0$, i.e. $\bar{\omega}_1 = \bar{\omega}_2 = \bar{\omega}$), including capture into resonance of both rotors. The stationary solutions as a function of parameter u ($u_1 = u_2 = u$) corresponding to the slope of motor characteristic and for different values of damping parameter σ are shown in Fig. 2a. First stable solution branch depicts capture into resonance, where both rotors cannot cross the resonance frequency of carrier ($\bar{\omega} = 1$). Second stable solution branch depicts synchronous solution at synchronization frequency λ_S. So Figure 2a can be interpreted as existence conditions of capture into resonance and self-synchronization as functions of engine power and damping. The overcritical solution, where both rotors reach their nominal speeds is not shown in Fig. 2a, because it is not a stationary solution.
To depict the phase space of the averaged system, cylinder coordinates are chosen, where $\bar{p}$ is the vertical, $\bar{v}$ is the radial and periodic variable $\bar{\delta}$ is the tangential coordinate. Since the variable $\bar{v}$ can become negative an arbitrary radius is chosen for $\bar{v} = 0$, see Fig. 2b. In the same Figure, two trajectories are shown, which start at resting position with different initial phase differences. One trajectory ends with self-synchronization, which is a point at $\bar{v} = 0$ and the other one converges to the overcritical periodic solution, which is a limit cycle in phase space. Lastly, sections of attraction domains of different solutions at the surface $\bar{v} = 0$ are shown in Fig. 2c,d. The trajectories in Fig. 2b can be explained with the attraction domains in Fig 2c. There is a narrow region, where the system can directly run-up to the synchronous solution form resting position. If the damping is chosen smaller, the three solution types (capture, synchronous and overcritical) can also coexist, see Fig 2d.

Conclusion

A model of two coupled exciters is investigated using an averaging method for partially strongly damped systems. Averaged equations of second order approximation in $\sqrt{\varepsilon}$ are analyzed. Stationary solutions and their existence conditions are discussed. Phase space of averaged system is described. Attraction domains of different types of solutions are exemplary depicted.

References

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