Using Spectral Submanifolds for Forced Response Prediction in Nonlinear Finite Element Models: Direct and Nonintrusive methods

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Summary. Finite element models of realistic nonlinear structures are characterized by very high dimensionality that renders simulations of the full system infeasible. Exact model reduction aims to achieve a drastic reduction of the full system's variables in a mathematically justifiable fashion. Specifically, Spectral Submanifolds have recently been shown to result in exact reduced-order models for periodically and quasiperiodically forced, nonlinear mechanical systems. In this work, we demonstrate recent advances towards the computation of SSMs that enable the treatment of realistic finite-element models using direct as well as non-intrusive/data-driven methods.

Background

The prediction of a steady-state response to an externally applied dynamic load is of special significance in engineering applications. Mechanical structures are usually characterized by light damping which results in exceedingly long integration times before a steady state is reached. Despite the broad availability of dedicated software packages [2, 3], the computation and continuation of the steady-state in response to periodic forcing remains a serious computational challenge for full-scale nonlinear finite element models.

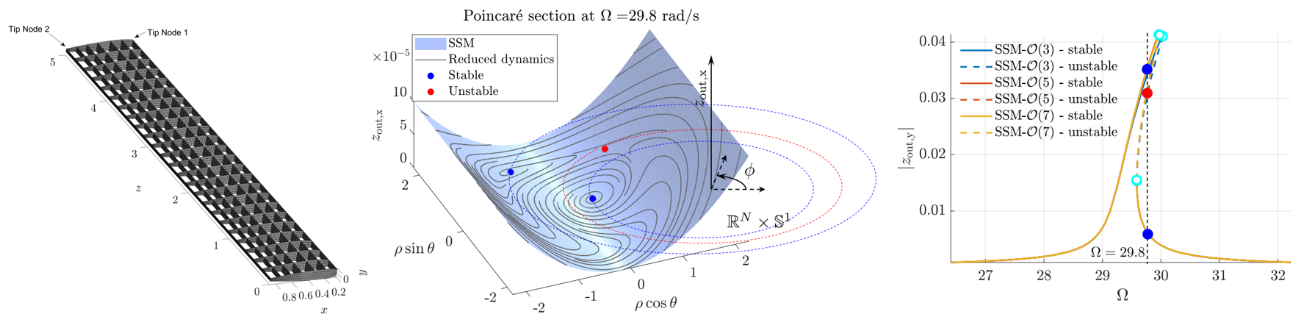
Direct computation of SSMs

We consider finite-element discretized system of second-order ordinary differential equations for the generalized displacement $\mathbf{x}(t) \in \mathbb{R}^n$ given as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) = \epsilon \mathbf{f}^{\text{ext}}(\Omega t), \tag{1}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K} \in \mathbb{R}^{n \times n}$ are the mass, stiffness and damping matrices; $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) \in \mathbb{R}^n$ is the purely nonlinear internal force; $\mathbf{f}^{ext}(\Omega t) \in \mathbb{R}^n$ denotes the external forcing, which is periodic in t with frequency Ω; and $0 < \epsilon \ll 1$ is a scalar forcing amplitude parameter.

The recent theory of Spectral Submanifolds (SSM) [1] has laid the foundation for a rigorous model reduction of such nonlinear systems, leading to reliable steady-state response predictions within feasible computation times. Further developments [4] have enabled the computation of SSMs and their reduced dynamics by solving the associated invariance equations directly in physical coordinates using only the eigenvectors associated to the master modal subspace. The software implementation of the method has been available in an open-source package, `SSMTool` [5], making direct SSM computations scalable to realistic, nonlinear finite-element models.



(a) (b) (c)

Figure 1: (a) The finite-element mesh of a geometrically nonlinear aircraft wing (illustrated after removing the skin panels) with n =133,290 degrees of freedom. The wing is cantilevered at the $z = 0$ plane (see ref. [6] for model details.) (b) Poincaré section of the non-autonomous SSM computed around the first mode for the near-resonant forcing frequency $\Omega = 29.8$ rad/s. (c) Forced response curves obtained using local SSM computations at $\mathcal{O}(3)$, $\mathcal{O}(5)$ and $\mathcal{O}(7)$ that converge towards a hardening response. (see ref. [4] for computational details.)

In this contribution, we demonstrate applications of SSMs towards the extraction of *forced response curves* (FRC) directly from finite element models. For instance, Figure 1a shows the finite element mesh of a geometrically nonlinear aircraft wing structure containing 133,920 degrees of freedom. We extract the forced response of this model around its first natural frequency by analyzing the reduced dynamics on the two-dimensional SSM associated to the first mode, as shown in Figure 1b. The hyperbolic fixed points of the reduced dynamics on this SSM in polar-coordinates (ρ, θ) directly provide us the stable (blue) and unstable (red) periodic orbits on the FRC for different values of forcing frequency Ω (see Figure 1c.) We refer to ref. [4] for further details.

Nonintrusive computation of SSMs

As commercial finite element software seldom provide intrusive access to the nonlinearity $\mathbf{f}$ of system (1), the direct SSM computation procedure discussed above has limited applicability for the users of commercial codes. At the same time, however, recent advances [7, 8] towards data-driven computation of SSMs and their reduced dynamics have paved the way for nonintrusive applications of SSM theory.

To this end, we construct data-driven, nonlinear reduced-order models on SSMs using the open-source package `SSMLearn` [9]. We base this construction on unforced decaying trajectory data obtained from black-box finite-element simulations. We also take advantage of the knowledge of the master modes associated to the underlying SSM, which is readily available from commercial finite element software and aids computations. Thanks to the rigorous theory backing SSMs, our reduced-order models, which are trained on a minimal number of *unforced* simulation trajectories, are capable of accurately predicting the forced response of the full nonlinear mechanical systems.

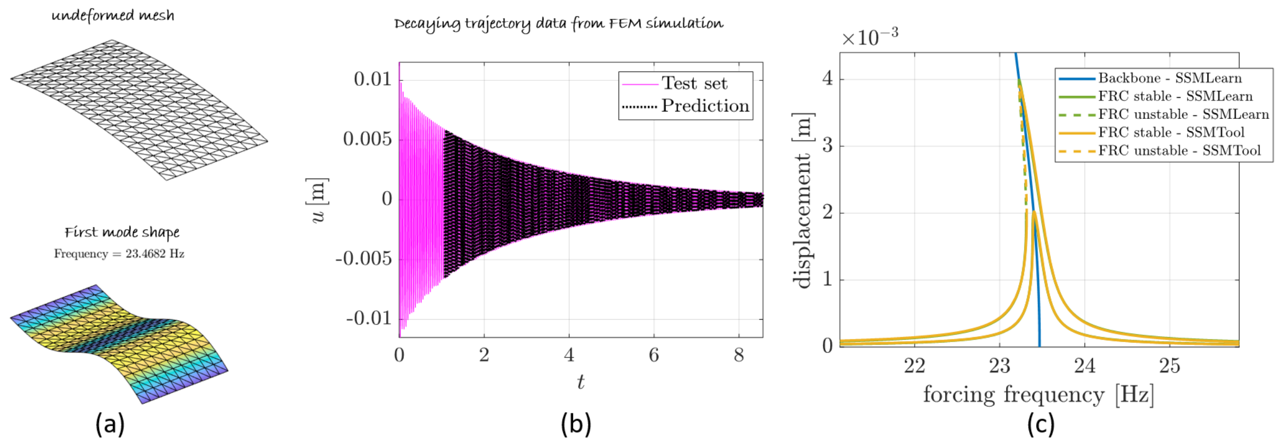


Figure 2: (a) The finite element mesh and the first mode shape of a shallow curved rectangular arch which is simply-supported on opposite ends. (b) Decaying trajectory data of the unforced system (c) Forced response predictions obtain from the data-driven reduced-order model trained on unforced trajectory data via `SSMLearn` agree with the analytic predictions based on direct SSM computations via `SSMTool`.

As an illustration, we consider a finite element model of a geometrically nonlinear shell structure [4] with 1,320 degrees of freedom, as shown in Figure 2b. Assuming linear viscous damping, we simulate two trajectories decaying from initial deflections given to the structure in the shape of the first vibration mode scaled by two different magnitudes. We use truncated simulation data from one of these trajectories to learn the slowest two-dimensional SSM and its nonlinear reduced dynamics up to an a priori determined accuracy. The second trajectory, which is unseen for the training procedure, is then used for testing the prediction of our data-driven model, as shown in Figure 2b.

Finally, we observe that our data-driven model is capable of predicting the forced response of the full system around the first natural frequency as shown in Figure 2c. Here, we obtain agreement with the analytic predictions of the reduced-order model on the same SSM when computed directly [4] via `SSMTool`.

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