Optimal design and tuning of an SMA-assisted PTMD system

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Summary. This work presents a first step to using a phenomenological model of an SMA spring as an auxiliary damping mechanism in a Pendulum Tuned Mass Damper (PTMD) setup under stochastic excitation. An SMA superelasticity model was considered to approach the hysteretic behaviour of an SMA material. In provision for susbsequent simulations, this model's optimal damping parameters were derived with particular care for the implicit relationship between the SMA model parameters and system mean square response.

Introduction

In the field of Pendulum Tuned Mass Dampers (PTMD), many damping modalities have been investigated. This paper presents a novel way of using hysteretic aspects of Shape Memory Alloy (SMA) spring as a damping mechanism in a PTMD system. This investigation looks into damping for a single SMA spring, allowed to move in a one-dimensional space along the horizontal displacement of the pendulum mass. It presents the results of the optimal parameters corresponding to maximum structure displacement variance reduction. The methods address the derivation of moment equations through symbolic computing and solving. These are then used to optimise the system parameters through numerically solving implicit relationship between the SMA parameters and the system's mean square response through an iterative process proposed in [2].

Methods

The system under consideration is presented in Fig. 1. The main structure was assumed to move only along the x-axis. The pendulum is attached to the structure at point A and has length L and angle of inclination θ. The SMA spring is attached at the bottom of the ball mass and at the side floor of the structure. The top floor and wall were assumed to move as a rigid body and the excitation, applied along the x-direction, is modeled as a white noise excitation.

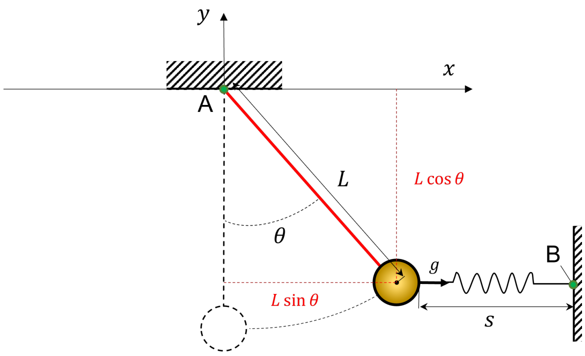

Figure 1: Schematic of the system under consideration

The elongation of the spring, denoted by s, has a minimum length of s_0. The restoring force g of the SMA spring is here assumed to behave according to the Yan-Nie equivalent linearization model [3],

$$g(s,\dot{s}) = \alpha k s + (1-\alpha) k z \tag{1}$$

where,

$$z = c_e\dot{s} + k_e s \qquad c_e = \frac{b-a}{\sqrt{2\pi}\sigma_{\dot{s}}}\left[1 - \mathrm{erf}\left(\frac{a}{\sqrt{2}\sigma_s}\right)\right] \qquad k_e = \frac{a+b}{\sqrt{2\pi}\sigma_s}e^{-\frac{a^2}{2\sigma_s^2}} \tag{2}$$

Here a and b are constants defining the phase transition lengths of the SMA material. Note that in order to use this linearized model for the restoring force, the standard deviations σ_s and $\sigma_{\dot{s}}$ of the elongation displacement and velocity must be known. In this case, according to the law of cosines, the elongation displacement s and velocity $\dot{s}$ could be expressed as,

$$s = s_0 - L\sin\theta \tag{3}$$

$$\dot{s} = L\cos\theta\dot{\theta} \tag{4}$$

This system's dynamics under white noise excitation can be described by,

$$\begin{pmatrix} M+m & mL\cos\theta \\ mL\cos\theta & mL^2 \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C & -mL\sin\theta\dot{\theta} \\ 0 & (1-\alpha)kc_eL^2\cos\theta^2 \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix}\begin{pmatrix} K & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ \theta \end{pmatrix} = \begin{pmatrix} 0 \\ -mgL\sin\theta - (\alpha k + (1-\alpha)kk_e)(L^2\sin\theta\cos\theta + Ls_0\cos\theta) \end{pmatrix} \quad (5)$$

where M, C and K are the primary structure mass, damping and stiffness, respectively. The constant m represents the pendulum mass. Here, $k = \frac{F_0 mg}{b}$ where g is the acceleration due to gravity. F_0, the normalized transformation strength, therefore drives the SMA stiffness. It was chosen to investigate the system with a mass ratio $\frac{m}{M} = 0.2$, a frequency of oscillation of 1.5 Hz and a damping ratio of 0.017. The SMA parameters were chosen to be $a = 0.005\text{m}$, $b = 0.08\text{m}$ and $\alpha = 0.06$.

Throughout the study, all the state variables (the structure displacement x and pendulum angle θ) were assumed to be zero-mean stationary Gaussian random variables. The following quantities were therefore determined using a small-angle approximation,

$$\sigma_s^2 = L^2E[\theta^2] \qquad \sigma_{\dot{s}}^2 = L^2E[\dot{\theta}^2] \quad (6)$$

Since the relationship between Eqs. (2) and (6) is implicit, it was proposed to use the iterative process described in [2] to determine adequate values for c_e and k_e upon convergence. This was allowed by first deriving the moment derivatives of this system, derived using principles devised in [1]. Additionally, this system's optimal operating point in a domain $L \in [0, 1]$, $F_0 \in [0.1, 0.95]$ was determine through the optimization of a performance index defined by the ratio of the optimized variance (σ_{optim}^2) and the determined uncontrolled primary structure's variance (σ_r^2). For the purpose of optimization, the MATLAB® *fmincon* function was used. Namely,

$$\begin{aligned} \min_{L,F_0} \quad & \frac{\sigma_{optim}^2}{\sigma_r^2} \\ s.t. \quad & 0 \le L \le 5 \quad (m) \\ & 0.1 \le F_0 \le 0.95 \end{aligned} \quad (7)$$

Results

Figure 2 demonstrates the results of a parameter sweep performed for the described range of values of L and F_0. It also presents the optimal operating point for this system with $L = 0.4932\ m$, $F_0 = 0.2066$ with an optimal reduction of the variance to 13.88% of the uncontrolled structure's variance.

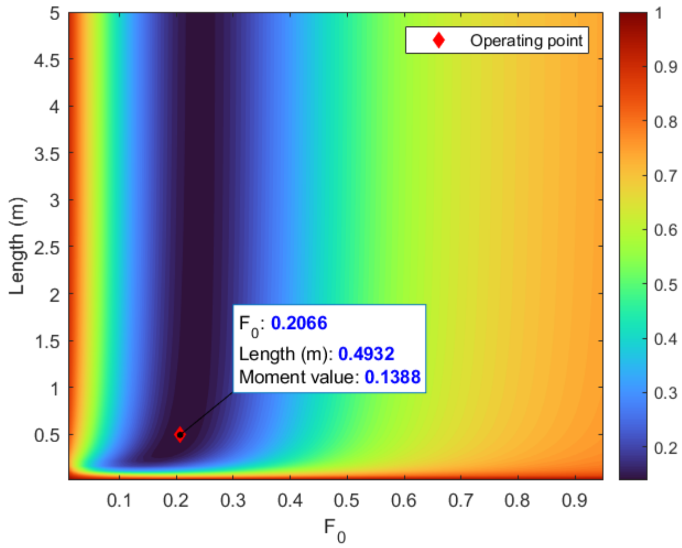

Figure 2: Contour plot of the ratio of derived variance to uncontrolled variance as a function of length and normalized transformation strength

References

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