Axisymmetric, nonlinear capillary waves: dimple and jet formation

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Summary. We study axisymmetric, nonlinear capillary waves in confined cylindrical geometry. Extending our recent analytically and computational studies[5, 2], we present a third order theory [6] which can describe dimple formation in nonlinear axisymmetric capillary waves for moderately large values of the wave steepness parameter ϵ. For $\epsilon > 1.8$, we show that the dimple produces a jet which evolves self-similarly in a window of time and space. Analytical predictions are compared to numerical solutions.

Jet formation from an axisymmetric capillary wave

Jet formation from bursting bubbles have attracted persistent attention from the scientific community for more than half a century [3]. In addition to scientific curiosity, the phenomena is also of significant meterological interest as the bursting of bubbles on the open ocean surface (due to wave breaking) causes ejection of liquid droplets which are a source of sea-salt aerosols and can serve as cloud condensation nuclei. In a series of recent studies [5, 2] it was observed that similar jet ejection can also occur in apparently simpler conditions where the free-surface of quiescent liquid in a cylindrical container of radius R_0 is deformed as a an eigenmode of the linearised system viz. a Fourier-Bessel mode of the form $a_0 J_0(k_0 r)$. It was shown that [5, 2] that the resulting surface oscillation due to the restoring force of gravity and capillarity produces a trough whose subsequent evolution in time, closely resembles a collapsing cavity [4]. When the steepness parameter $\epsilon \equiv a_0 k$ is sufficiently large ($\mathcal{O}(1)$), a nonlinearity induced dimple like structure is produced at the base of the trough leading to the formation of a jet at the axis of symmetry [2, 5], much analogous to what is observed in the bursting of a bubble at a free-surface [8, 4]. While the $\mathcal{O}(\epsilon^2)$ non-linear theory presented in [2] was able to capture the inception of the dimple, it was unable to describe the same at later times. An important conclusion from [2] was that the lengthscale of the dimple rendered it completely dominated by capillary effects with the gravitational acceleration being insignificant. In this study, we lend further insight into this dimple formation focusing our attention on pure capillary waves (no gravity) [6]. A novel $\mathcal{O}(\epsilon^3)$ solution to the intial-value problem has been calculated [6] which significantly improves upon the second order calculation presented earlier in [2]. Fig. 1a depicts the initial axisymmetric, surface perturbation of the form $\eta(r,0) = \epsilon J_0(k_0 r)$ and fig. 1b shows the formation of the dimple for a moderately large $\epsilon = 0.8$, and is described quite accurately by our $\mathcal{O}(\epsilon^3)$ calculation. This is the solution to the initial-value problem using ϵ as a perturbation parameter and the Lindstedt-Poincaré technique. It has the form

$$\eta(r,\tau) = \epsilon\cos(\tau)\mathrm{J}_0(r) + \epsilon^2\sum_{j=1}^{\infty} f^{(j)}(\tau)\mathrm{J}_0(\alpha_{j,p}r) + \epsilon^3\left[\sum_{m=1}^{\infty} g^{(m)}(\tau)\mathrm{J}_0(r) + \sum_{j=1,\, j\neq p}^{\infty} h^{(j)}(\tau)\mathrm{J}_0(\alpha_{j,p}r)\right]$$

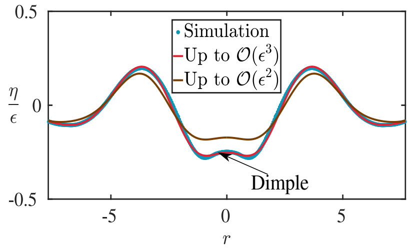
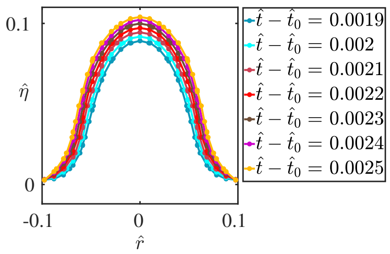
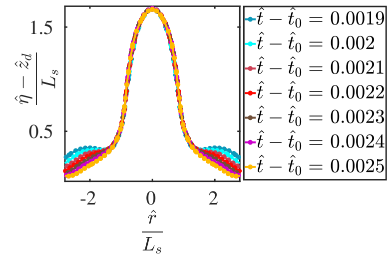


Figure 1: Comparison of the surface evolution of the interface at steepness $\epsilon = 0.8$. Numerical solution to the Euler's equation (•) using Basilisk [9], $\mathcal{O}(\epsilon^3)$ (——), $\mathcal{O}(\epsilon^2)$ (——). Third order theory captures the dimple very accurately. Here η is non dimensional interface and r is non-dimensional radial distance.



(a) A rising jet



(b) Self-similar behaviour of jet

Figure 2: Panel (a), (b): Self similar behaviour of the jet at $\epsilon = 2.0$ and cylinder radius $\mathrm{R}_0 = 2.936$ cm, where R_0 is the radius of the cylinder. Here $\hat{t}_0$ is the dimple formation time, $\hat{\eta}$ is dimensional interface and $\hat{r}$ is dimensional radial distance, $\hat{z}_d$ is the height of the jet base (all variables with hat are dimensional)

At larger values of $\epsilon = 2.0$, a clear jet emerges from the dimple as shown in fig. 2a. At large steepness($\epsilon > 1.8$), we find that the jet evolution in time becomes self-similar. This is shown in fig. 2b where the length scale $L_s = \frac{(\hat{t}-\hat{t}_0)^{2/3}}{\left(\frac{\rho}{\mathrm{T}}\right)^{1/3}}$ [7, 10] has been used to scale the vertical and radial coordinate. Note that ρ, T are density and surface tension and $\hat{t}_0$ is the dimple formation time.

Modal analysis

In fig. 3, we plot the surface energy of the system from modal analysis [2]. The figure shows the Fourier-Bessel modes which are excited at the chosen instant of time. For the case $\epsilon = 2.0$, the time instant has been chosen to lie within the self-similar window shown earlier in fig. 2b. Comparing $\epsilon = 2.0$ with $\epsilon = 1.0$, it is clearly seen that a large number of modes are excited in the case of $\epsilon = 2.0$ compared to $\epsilon = 1.0$. Note that we have plotted $\left|\hat{H}(l_j)\right|$ which comes from the coefficient of the Dini series and provides an instantaneous measure of the potential energy contained in various wavenumbers [1] (l_j is the j^{th} nontrivial root of the Bessel function $\mathrm{J}_1(\cdot)$).

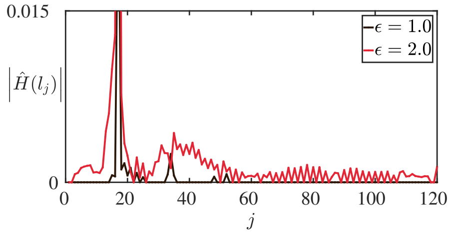
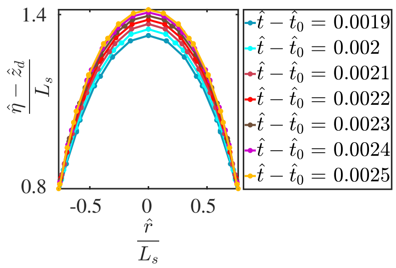


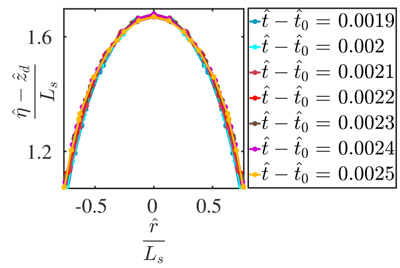
Figure 3: The potential energy between $\epsilon = 1.0$ (——) and $\epsilon = 2.0$ (——) are compared at the same time instant $\hat{t} = 0.0092$ s. At this instant the $\epsilon = 2.0$ simulation displays self similar collapse depicted earlier in fig. 2b. The simulation with $\epsilon = 1.0$ however does not show self similar behaviour at this instant or at later time.

Confinement and loss of self-similarity

We find that strong confinement (i.e. shrinking the radius of the container) leads to loss of self-similarity. This is shown in fig. 4(a) and (b) where both panels represent simulations with $\epsilon = 0.2$. In panel (a) the radius is small ($R_0 = 0.038$ cm) and the evolution of the jet is not self-similar in marked contrast to panel (b).



(a) R_0=0.038 cm



(b) R_0=2.936 cm

Figure 4: Panel (a), (b): Effect of container radius on the self similar evolution of jet

Conclusions

We investigate dimple and jet formation from nonlinear capillary waves in axisymmetric cylindrical confined geometry. The initial value problem has been solved to $\mathcal{O}(\epsilon^3)$ using the Lindstedt Poincarè perturbation technique and it is found that the weakly nonlinear solution can predict the formation of the dimple quite well for moderately large value of ϵ. Using simulations, we have shown that for $\epsilon > 2.0$, the evolution of the jet happens in a self similar manner in a narrow window of space and time. The physical reasons for this self-similarity as well as the eventual loss of the same will be discussed at the meeting. All simulations have been performed using the open source Basilisk [9].

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