Modal interactions in a non-linear mass-in-mass periodic chain

Jean Flosi†, Alireza Ture Savadkoohi† and Claude Henri Lamarque†
†*Univ Lyon, ENTPE, CNRS UMR5513, LTDS, France*

Summary. The multiple-scale dynamics of a forced periodic chain composed of 2 degree of freedom mass-in-mass cells with cubic non-linearity is studied. A continuous approach leads to dispersion equations and allows to define the modal decomposition of continuous physical coordinates of the system. Because of the system nonlinearity, a $1:3$ internal resonance is considered. The continuous model is projected on these two modes in order to study associated energy exchanges. Fast and slow dynamics leads to detection of slows invariant manifolds (SIM) and frequency responses of the chain.

Considered model of the chain

Meta-materials are developed to present unusual non natural responses against external or internal actions. In the vibro-acoustics domain, mass-in-mass systems are an example of meta-materials [1, 2]. We are considering the system presented in Fig.1: a L-periodic chain composed of mass-in-mass cells linearly linked two by two. Each cell is composed of a principal mass m_1 linked non-linearly to its inner mass m_2.

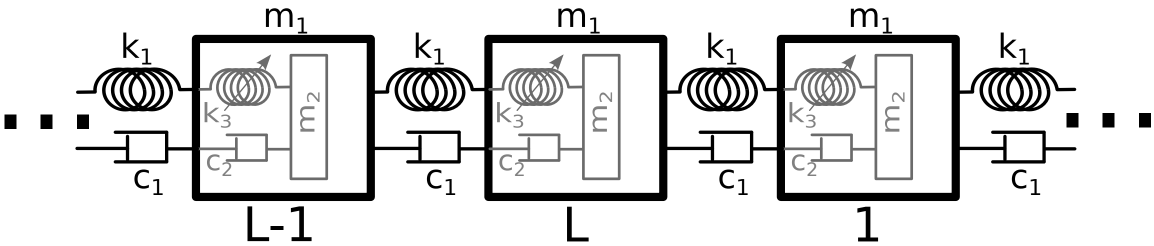

Figure 1: Scheme of the L-periodic chain composed by cubic nonlinear mass-in-mass cells

Governing equations are:

$$\left\{\begin{array}{l} m_1\dfrac{d^2u_1}{dt^2}+k_1(2u_1-u_L-u_2)+k_3(u_1-v_1)^3+c_1(2\dfrac{du_1}{dt}-\dfrac{du_L}{dt}-\dfrac{du_2}{dt}) \\ +c_2(\dfrac{du_1}{dt}-\dfrac{dv_1}{dt})=F_1\sin(\Omega t) \\ \\ m_1\dfrac{d^2u_j}{dt^2}+k_1(2u_j-u_{j-1}-u_{j+1})+k_3(u_j-v_j)^3+c_1(2\dfrac{du_j}{dt}-\dfrac{du_{j-1}}{dt}-\dfrac{du_{j+1}}{dt}) \\ +c_2(\dfrac{du_j}{dt}-\dfrac{dv_j}{dt})=F_j\sin(\Omega t) \text{ for } j\in[2,...,L-1] \\ \\ m_1\dfrac{d^2u_L}{dt^2}+k_1(2u_L-u_{L-1}-u_1)+k_3(u_L-v_L)^3+c_1(2\dfrac{du_L}{dt}-\dfrac{du_{L-1}}{dt}-\dfrac{du_1}{dt}) \\ +c_2(\dfrac{du_L}{dt}-\dfrac{dv_L}{dt})=F_L\sin(\Omega t) \\ \\ m_2\dfrac{d^2v_j}{dt^2}+k_3(v_j-u_j)^3+c_2(\dfrac{dv_j}{dt}-\dfrac{du_j}{dt})=0 \text{ for } j\in[1,...,L] \end{array}\right. \tag{1}$$

The periodicity leads to following boundary conditions:

$$\left\{\begin{array}{l} u_j(t)=u_{L+j}(t) \\ \dfrac{du_j(t)}{dt}=\dfrac{du_{L+j}(t)}{dt} \end{array}\right. \text{ pour } j\in[1,...,L] \tag{2}$$

Continuous approach

X is the initial spatial variable and the distance of two adjacent cells at rest is Δx. We introduce the continuous normalized space variable $x=\dfrac{X}{\Delta x}$ and new continuous functions:

$$\left\{\begin{array}{l} u(x,\tau)=u(x=j-1,\tau), x\in[0,L], j=\{1,...,L+1\} \\ v(x,\tau)=v(x=j-1,\tau), x\in[0,L], j=\{1,...,L+1\} \end{array}\right. \tag{3}$$

We use the new continuous coordinates:

$$\left\{\begin{array}{l} U(x,t)=u(x,t) \\ V(x,t)=u(x,t)-v(x,t) \end{array}\right. \tag{4}$$

and applying change of variables $\tau = \omega t = \sqrt{\frac{k_1}{m_1}} t$, $\varepsilon\Lambda = \frac{k_3}{k_1}$, $\varepsilon\chi_1 = \frac{c_1}{\sqrt{k_1 m_1}}$, $\varepsilon\chi_2 = \frac{c_2}{\sqrt{k_1 m_1}}$, $\varepsilon f_j = \frac{F_j}{k_1}$ and $\mu = \frac{\Omega}{\omega}$, continuous expression of system 1 is:

$$\begin{cases} \frac{\partial^2 U}{\partial \tau^2}(x,\tau) - \frac{\partial^2 U}{\partial x^2}(x,\tau) + \varepsilon\Lambda V^3(x,\tau) - \varepsilon\chi_1 \frac{\partial}{\partial \tau}\frac{\partial^2}{\partial x^2} U(x,\tau) + \varepsilon\chi_2 \frac{\partial}{\partial \tau} V(x,\tau) \\ = \varepsilon f(x)\sin(\mu\tau) \\ \\ \frac{d^2(U-V)}{d\tau^2}(x,\tau) - \Lambda V^3(x,\tau) - \chi_2 \frac{\partial V}{\partial \tau}(x,\tau) = 0 \end{cases} \tag{5}$$

The study of the linearized system leads to the dispersion equation of the system. We consider a $1:3$ internal resonance and we define:

$$\begin{cases} U(x,\tau) = p_1(\tau)\cos(\omega_1 x + \Theta_1) + p_3(\tau)\cos(\omega_3 x + \Theta_3) \\ V(x,\tau) = q_1(\tau)\cos(\omega_1 x + \Theta_1) + q_3(\tau)\cos(\omega_3 x + \Theta_3) \end{cases} \qquad \omega_k = \frac{2k\pi}{L}, k \in [1,\dots,L] \tag{6}$$

Fast and slow dynamics study

In order to detect the different dynamics of the system, multiple scale method [3] is applied. Modal projection allows to obtain different set of governing equations from the continuous Eq.5 and asymptotic responses of the system are considered [4]. Detection of fast dynamics [5] leads to the determination of the two SIMs associated to projected system on the first and third mode. Example of SIM obtain from projection on first mode is plotted in Fig.2 from two different angles, where N_1 stands for associated amplitude of the outer mass for first mode and M_1 and M_3 stand for associated amplitudes of the inner mass for first and third modes respectively. Analytical SIM is plotted in blue while numerical integration using Runge Kutta is plotted in red. Slow dynamics study at the order ε^1 allows to determine singular and

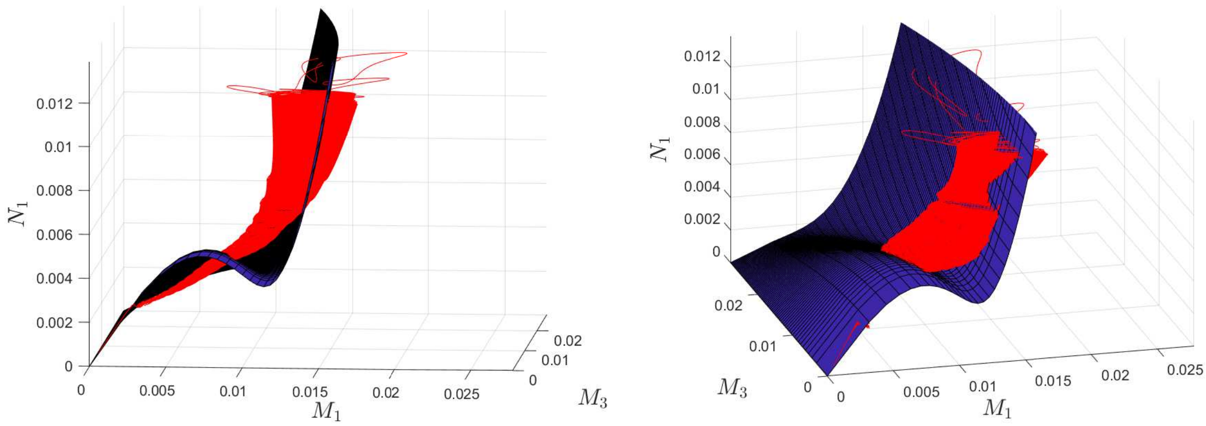

Figure 2: Multiangle representation of the SIM obtained after projection on first mode for free system with initial deformation.

equilibrium points. These points lead to the prediction of periodic and quasi-periodic behaviors. Study can be generalized for the $1:k$ resonance.

References

[1] Cveticanin, L. (2018) Influence of nonlinear subunits on the resonance frequency band gaps of acoustic metamaterial,Nonlinear Dynamics 93.

[2] Wang, X. (2014) Dynamic behaviour of a metamaterial system with negative mass and modulus, International Journal of Solids and Structures 51.

[3] Nayfeh, A.H. and Mook, D.T. (1979) Nonlinear Oscillations,

[4] Manevitch, L. (2001) The Description of Localized Normal Modes in a Chain of Nonlinear Coupled Oscillators Using Complex Variables, Nonlinear Dynamics 25.

[5] Charlemagne, S.,Ture Savadkoohi, A. and Lamarque, C. H. (2018) Dynamics of a linear system coupled to a chain of light nonlinear oscillators analyzed through a continuous approximation, Physica D: Nonlinear Phenomena.