The effect of additional masses on the dynamic buckling of a like-beam structure

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Summary. A mechanical model of a beam subjected to impact velocity is analyzed. It consists of a beam with additional mass-spring systems. We investigate the distribution of the axial stress along the beam, and the effect of different parameters (masses and their distribution, spring's stiffness) on the stability of the beam.

Introduction

In industrial safety, dynamic buckling is one of the most important considerations to design structures subjected to sudden loadings. For instance, spacer grids in nuclear fuel assembly should have sufficient buckling strength in case of major accidents as earthquakes. Several studies using finite elements models with experimental validation for static buckling and post-buckling of spacer grids were conducted [1]. In literature, most studies focus only on inner characteristics of the spacer grid components without a fully dynamic analysis of the fuel rods movements [2]. Dynamic buckling of structural elements (columns, plates, shells...) under impulsive axial loading has been studied using different approaches. It has been widely investigated for imperfection-sensitive structures with neglected axial inertia forces [3]. For nearly perfect structures, other studies have shown that the axial inertia forces must be considered, particularly in the case of high impact velocities [4].

Mechanical model

We propose a simplified beam model to reproduce the effect of fuel rods, as lumped masses, on the dynamic buckling of the spacer grid. In the present study, we conduct a stability analysis based on eigenvalues evolution for the systems shown in figure 1 and figure 2. The occurrence of buckling and its characteristics (time to buckling, evolution of eigenvalues) are affected by additional masses due to axial stress waves. This effect is illustrated through the impact response of the system with an initial velocity for different configurations. Each configuration is defined by a specific distribution of mass-spring systems and by their frequencies.

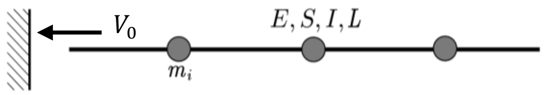

Figure 1: Model of a beam with additional masses

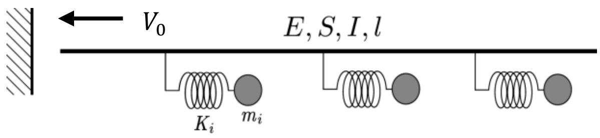

Figure 2: Model of a beam with additional mass-spring systems

To predict a potential buckling of the beam, we solve the wave equation by considering the additional masses

$$E\,\frac{\partial^2 u}{\partial x^2} = \rho\,\frac{\partial^2 u}{\partial t^2} \tag{1}$$

To obtain the geometric stiffness matrix at each time step, we consider the first nonlinear term in transverse direction in the axial strain of the beam

$$E\,I\,\frac{\partial^4 v}{\partial x^4} + \frac{\partial}{\partial x}\left(E\,A\,\frac{\partial u}{\partial x}(x,t).\frac{\partial v}{\partial x}\right) + \rho\,A\,\frac{\partial^2 v}{\partial t^2} = 0 \tag{2}$$

Then an eigenvalue problem is obtained at each time step

$$[K_{elas} + K_{geom}(t)].\,X + M.\ddot{X} = 0 \tag{3}$$

In fact, the system is instable if one eigenvalue of the system is positive.

Results

The study is carried out for several configurations with multiple distributions of the masses or the mass-spring systems. Herein we present the results obtained for two configurations. The first configuration consists of a beam with three masses characterized by the mass ratio: $r_m = \frac{m_i}{M_{beam}}$, with M_{beam} is the beam's mass. The second one consists of a beam with three mass-spring systems characterized by the frequency ratio: $r_f = \frac{f_i}{f_{beam}}$ where f_{beam} is the first natural longitudinal frequency of the beam and f_i is the frequency of the mass-spring system. The distribution of the axial stress along the beam at each time step is shown in figures 3 and 4 for the first and the second configurations, respectively. As shown in figure 5 the beam tends to buckle for heavy additional masses. Furthermore, for a given additional mass, the frequency of the mass-spring system needs to be smaller than the natural frequencies of the beam to avoid buckling (figure 6).

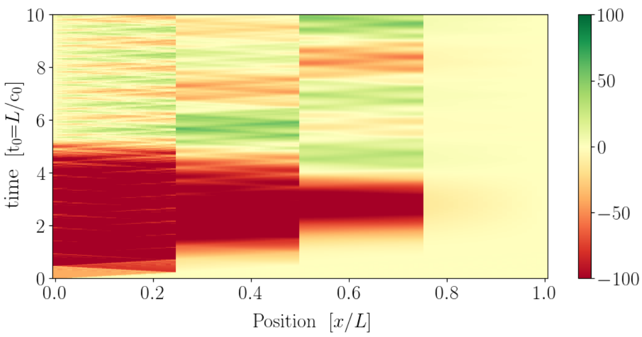

Figure 3: axial stress in the beam with masses for $r_m = 1$

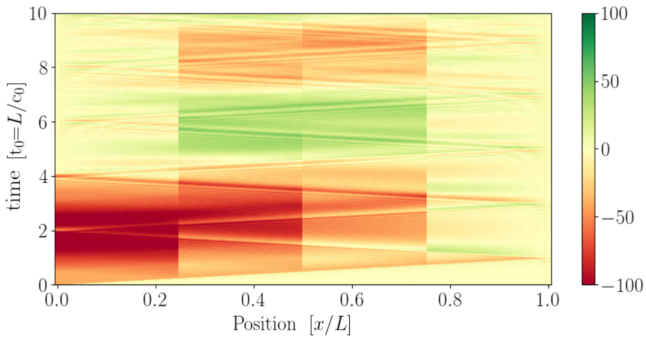

Figure 4: axial stress in the beam with mass-spring systems for $r_f = 1$

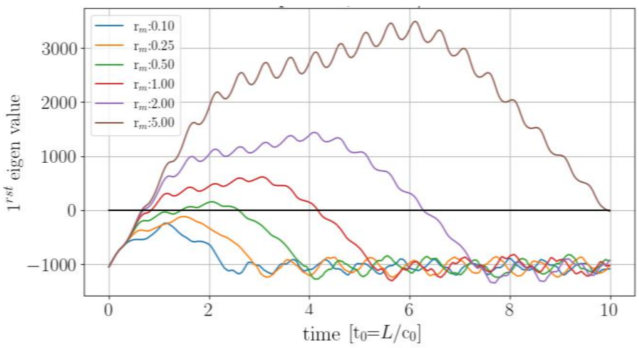

Figure 5: Evolution of the maximum real part of eigenvalues of the beam with masses for different mass ratios

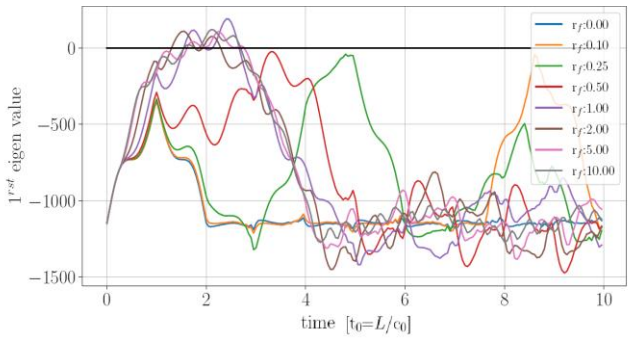

Figure 6: Evolution of the maximum real part of eigenvalues of the beam with mass-spring systems for different frequency ratios

Conclusion and comparison with experimental data

In this study we highlight the importance of considering the compression wave in the prediction of the buckling of a beam with an impact velocity. A further study considering non-linear springs is currently carried out to investigate their effect on the axial wave motion.

For the beam model with mass-springs, the validation of this analysis is considered by setting up a prototype of a structure with rigid point mass inclusions and an adapted experimental protocol under impact conditions.

References

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