

A Hybrid Position Feedback Controlled Bistable Metamaterial Concept

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Summary. The hybrid position feedback controller, proposed previously by the authors, is a control technique for bistable systems and it is based on the well-known positive-position-feedback controller. This controller is an unstable-then-stable position feedback controller, which is a second-order single-degree-of-freedom system in nature. The hybrid controller takes advantage of the resonant mode of a bistable system about one equilibrium position, destabilizes the system, and dynamically induces snap-through between the two equilibria. In this paper, a new multi degree of freedom metamaterial concept that utilizes the hybrid position feedback controller is introduced. An arbitrary number of bistable “segments” or “material elements” are attached to each other in a serial (or parallel) manner to generate a “distributed” bistable structure – also referred to as a metamaterial. Due to the simplicity of the hybrid controller, the physical implementation of proposed approach can be realized using simple circuit elements distributed in the material domain. This new metamaterial inherits the multiple bistable positions that its building blocks have; hence, the metamaterial becomes multi-stable. It can hold multiple positions without consuming power and has the capability of achieving many shapes. The proposed metamaterial concept can be used in various applications: locomotion in bioinspired systems, undulatory motion, morphing aerodynamic surfaces, wave guiding, and vibration attenuation. The concept can also be used in energy harvesting to enable maximum power extraction for a given vibratory input.

Introduction

Bistable structures are useful in many applications such as morphing aerodynamic surfaces, vibration energy harvesters, robotic actuators and mechanisms, and for locomotion of bioinspired systems where energy may be severely limited. A bistable system has two stable configurations and one unstable equilibrium.

The dynamics of bistable structures have been studied by various researchers [1-9]. The control of bistable structures using piezoelectric actuators has received significant attention in the last two decades. Shultz et al. [10] demonstrated one directional snap-through of a bistable plate using a piezoelectric actuator with static excitation. Arrieta et al. [11-13] studied the dynamic properties of bistable structures and achieved only one-directional snap-through. Later, Arrieta et al. [14] and Bilgen et al. [15] introduced resonant control with a surface bonded piezoelectric device. Zarepoor et al. [16] demonstrated the energy characteristic of a Duffing-Holmes (D-H) type bistable structure under dynamic forcing. Simsek et al. [17, 18] demonstrated an automated method for bidirectional state transfer on a wing-like cross-ply bistable plate using the hybrid control strategy and a piezocomposite actuator. Simsek et al. [19] demonstrated the prevention of chaotic behavior for Duffing-Holmes oscillator by applying the Hybrid Position Feedback (HPF) controller, which consists of the Negative Position Feedback (NPF) and Positive Position Feedback (PPF) controllers. Simsek et al. [20] analyzed the stability and response types of Duffing-Holmes oscillator with the HPF controller. Simsek et al. [21] demonstrated a piezoelectric-material induced monotonic snap-through without possibility of triggering cross-well oscillations or chaotic response. Crosswell oscillations are undesirable, as only one state is desirable (and nominal) for various applications. To this end, a hybrid position feedback (HPF) controller shows superior performance in terms of control, stability, and performance both theoretically and experimentally. In that work, the application of the HPF controller to a bistable unsymmetric cross-ply composite plate with surface-bonded piezoelectric actuators was presented.

Metamaterials or metastructures have engineered functionality that conventional materials or structures do not exhibit naturally. They have a broad range of applications from electromagnetics, acoustics, energy harvesting to vibration control. The assembly of HPF controlled bistable elements can be used to create a metamaterial or a metastructure. A system consisting of individual bistable units can form a desired configuration. Each individual element can be either at one state or the other which eventually forms a multi-stable metastructure. The proposed design can be implemented either using mechanical components such as mass-, damper- and spring-like elements, or it can be realized using circuit elements such as inductor-, resistor-, and capacitor-like elements.

The paper is organized as follows. First, the HPF control scheme is discussed. Next, the metamaterial concept is introduced. Then, the preliminary numerical simulation results are presented.

Hybrid Position Feedback Control

The hybrid position feedback controller was previously proposed by the authors to enable stable and monotonic cross-well transition of bistable (or multi-stable) structures [17, 18]. The controller is designed based on the dynamics of the stable equilibrium positions of a bistable structure around which it exhibits linear behavior for small perturbations. The control method first achieves system destabilization to make the structure move away from its current stable equilibrium position, and subsequently stabilize the system to the other (target) stable equilibrium position. The proposed hybrid control scheme employs the well-known PPF controller, and its modified version, the NPF controller. The PPF controller is a second order damped system (filter) that creates approximately 180° phase difference between input and output of the system in a certain gain range [22], which enables the system to dissipate energy through the controller. The NPF controller is a modified version of a PPF controller, here used to create a destabilizer by exploiting the phase

characteristics. As a destabilizer, the NPF controller, is expected to provide kinetic energy to the system until it reaches the desired threshold position (i.e. the unstable equilibrium.) Ref. [23] proposed and compared different switching methods for the HPF controller, and presented an analysis of the control effort and settling time.

The control diagram of the HPF controller is represented in Figure 1. In this diagram, the parameters of the NPF and PPF controllers are the same given by the block G. The reference signal $r(s)$ is considered as zero since system is in stabilizer and destabilizer mode. The authors have also investigated the system where the NPF and PPF controller parameters are individually selected and tuned.

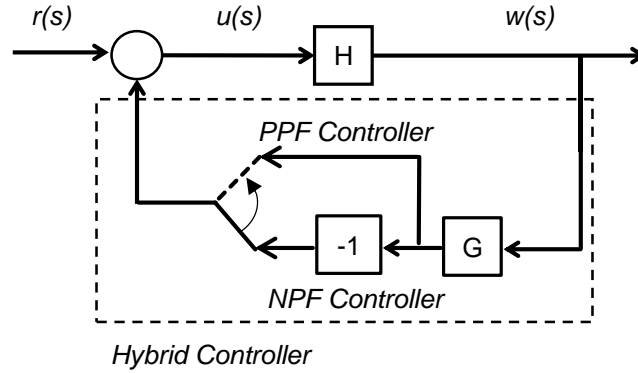


Figure 1: The block diagram for the hybrid position feedback controller.

First, the so-called NPF mode is used to start the cross-well transfer by making the PPF controller intentionally unstable. When the feedback gain is -1 , the system is destabilized, and the apparent dynamic stiffness of the system becomes negative. The controller induces an oscillatory response with increasing amplitude. When the system position reaches a threshold value at the unstable equilibrium, the feedback gain is changed to $+1$. This leads to a stable controlled (i.e. decaying) system response with decreasing amplitude. Figure 3 illustrates the cross-well motion of an example bistable system with the hybrid controller. In the figure, the stable states of the bistable structure are labeled as S1 and S2. The current position and the unstable equilibrium position are represented by w and w_{u0} respectively.

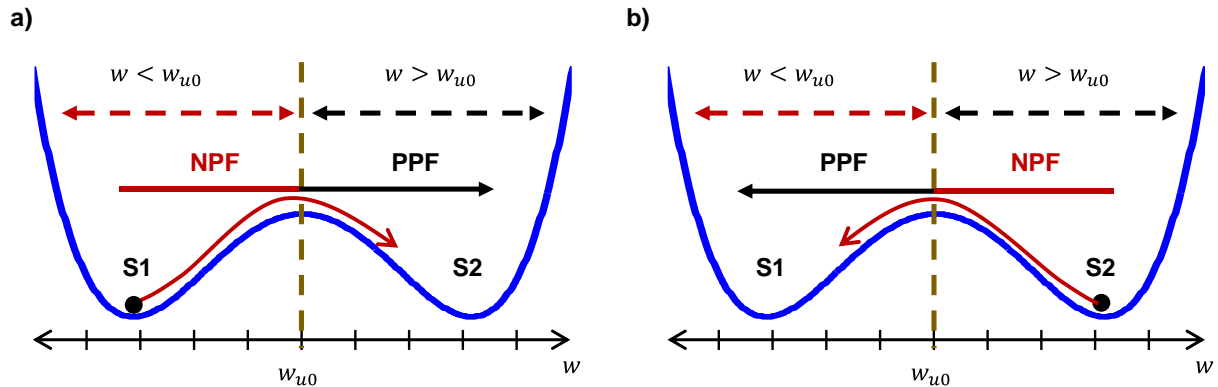


Figure 2: Illustration of state transfer a) from state 1 to state 2, and b) from state 2 to state 1.

For the state transfer from state 1 to 2, the structure is destabilized around equilibrium position S1 by using the NPF mode of the HPF controller. Due to the destabilization, the structure starts an increasing amplitude oscillation around the first equilibrium position. Once the amplitude reaches the unstable equilibrium, the structure snaps towards the second stable equilibrium position S2. The crossover is detected by an internal logic and the controller mode is switched to the PPF mode to attenuate the response around the target state of 2.

Figure 3 depicts the mechanical representation of the D-H system and HPF controller. In the system k_1 , c and k_{nl1} are linear stiffness, damping constant and cubic stiffness of system, respectively. k_c and c_c are the stiffness and damping parameter of HPF controller.

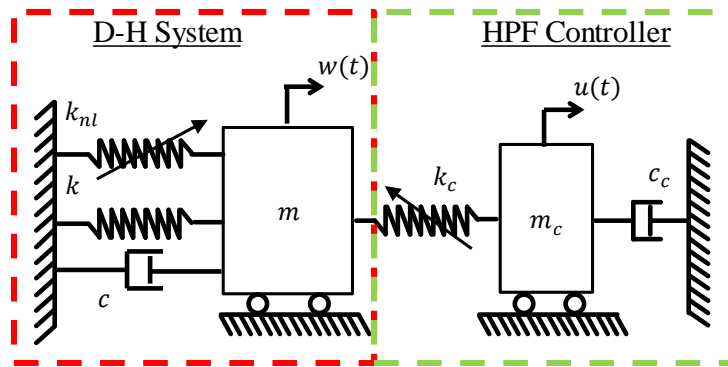


Figure 3: Mechanical model of the bistable Duffing-Holmes system with the hybrid position feedback controller.

The coupled system equations become:

$$\ddot{w} + 2\zeta\omega_n \dot{w} - \omega_n^2 w + \frac{k_{nl}}{m} w^3 = g_n \omega_n^2 u \quad (1)$$

$$\ddot{u} + 2\zeta_f \omega_f \dot{u} + \omega_f^2 u = g_f \omega_f^2 w \quad (2)$$

where ω_n and ζ represent natural frequency and damping ratio of the underlying linear system respectively, and g_n is the structure input gain. Here, it is noted that the actual value of input gain $g_n \omega_n^2$ is not constant in a physical implementation as the voltage/strain level changes [24, 25]. In this paper an effective g_n value previously determined from experiments is utilized [26]. The controller parameters ζ_f and ω_f correspond to damping ratio and natural frequency of the controller, respectively. The control gain is denoted as g_f and it is simply a proportional amplification of the feedback signal.

The complete system response depends on controller frequency, damping and gain, and depends on the system damping constant and other system parameters. The coupled system can yield three response types, namely, “intra-well,” “single cross-well,” and “multiple cross-well” types of responses. The “intra-well” response corresponds a response that will not reach to the threshold limit, while “single cross-well” response crosses the threshold limit once, and “multiple cross-well” response crosses twice or more.

The Metamaterial Concept and Mathematical Representations

In this paper, two different mechanical representations are considered. They are series and parallel configurations. The series configuration represents a cantilevered beam, and the parallel configuration represents an elastic foundation. Both of these configurations have different utilities in actual implementation.

Series Configuration

The so-called series configuration is presented in Figure 4. In this approach, the metamaterial concept is realized by connecting one bistable unit to next one with linear and cubic stiffnesses, and a viscous damper. A dedicated HPF controller is attached to each DOF. The series configuration represents a beam-like structure; however, with a multi-stable characteristic as opposed a “natural” monostable characteristic.

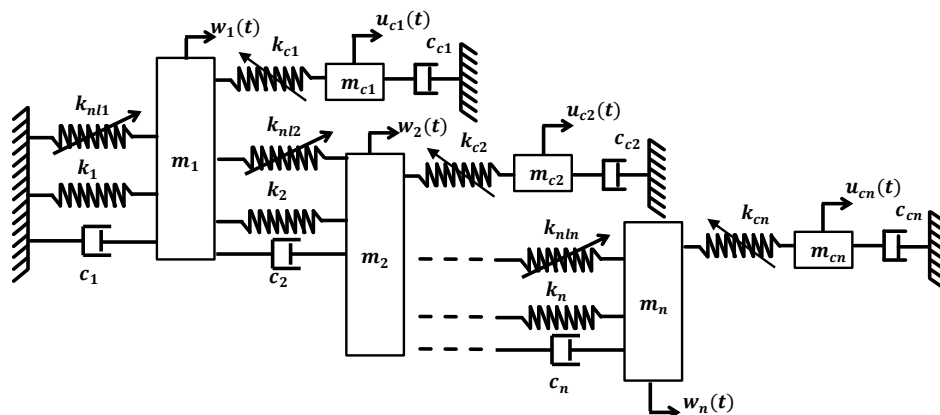


Figure 4: Mechanical model of the MDOF cascade bistable Duffing-Holmes system with the hybrid controller in series configuration.

The nonlinear coupling between the masses produces complex dynamics and a capability for the system to adopt various static or dynamic shapes. The proposed metamaterial concept is realized by adding multiple bistable oscillators in series. Figure 5 shows a representative model of cantilever beam with bistable elements connected in series.

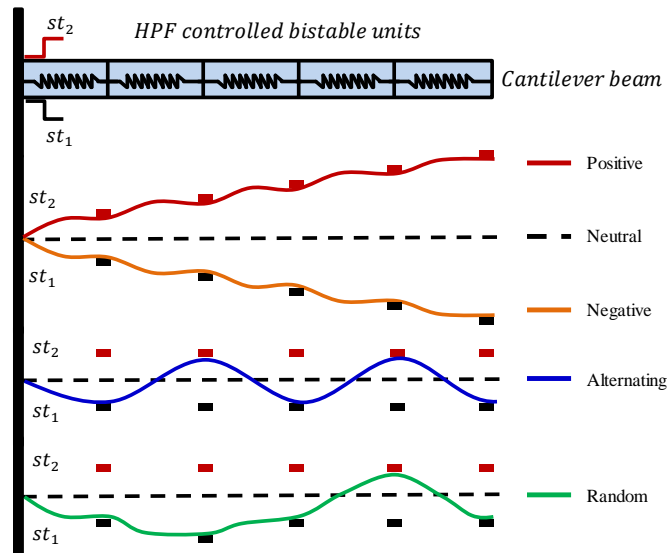


Figure 5: An illustration of a beam like metastructure with multiple bistable elements connected in series.

Parallel Configuration

Figure 5 shows an alternative, so-called parallel, configuration in which each DOFs is connected to the ground with a linear and cubic stiffness, and a viscous damper. In addition, a linear stiffness is used to connect the DOFs to each other. A dedicated HPF controller is attached to each DOF.

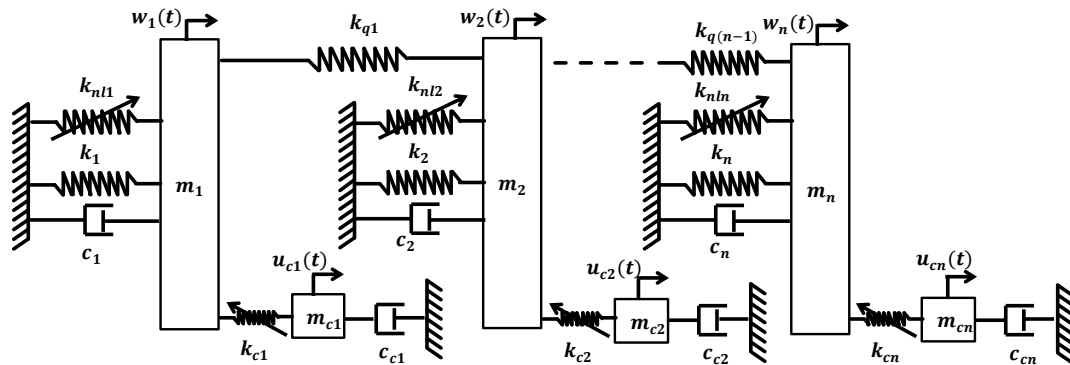


Figure 6: Mechanical model of the MDOF cascade bistable Duffing-Holmes system with the hybrid controller in parallel configuration.

This configuration reduces the coupling effect between the DOFs which minimizes the effort required to snap from one stable equilibrium while maintaining the connection between the DOFs. The proposed metamaterial concept is realized by adding multiple bistable oscillators in parallel. Figure 7 shows a representative model of a plate with bistable elastic foundation.

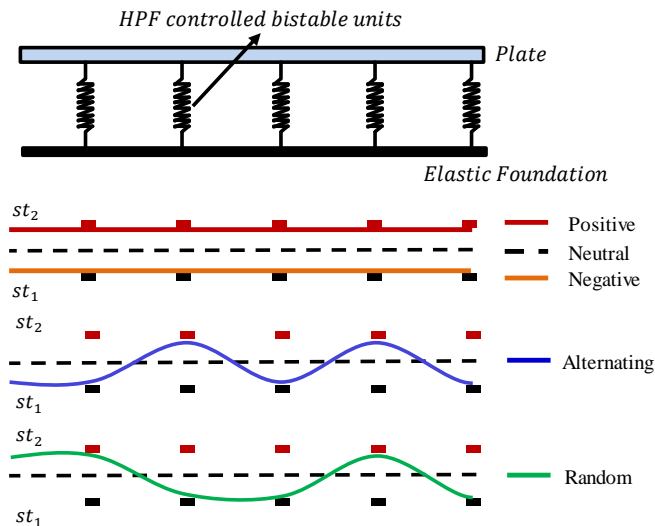


Figure 7: An illustration of a plate like metastructure with multiple bistable elements connected in parallel.

Simulation Results and Feasibility Analysis

Both the series and parallel configurations are examined through numerical integration. The dynamic response of the system is obtained using Dormand-Prince numerical integration method.

Series Configuration

The dynamic response of the series configuration is studied. Figure 8 presents the mechanical model of the 2DOF cascade bistable Duffing-Holmes system with the hybrid controller.

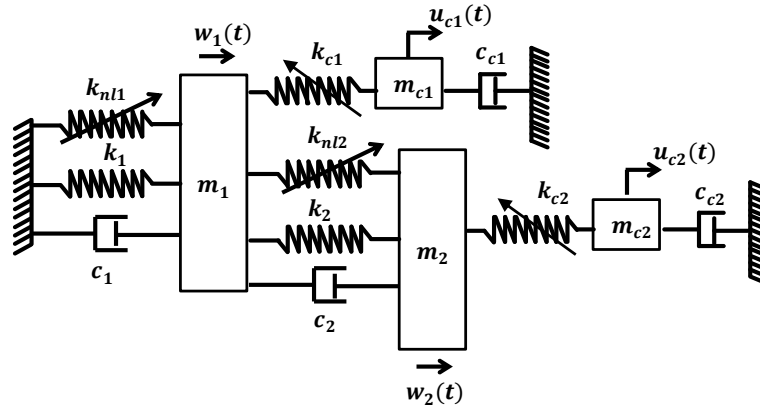


Figure 8: Mechanical model of the 2DOF cascade bistable Duffing-Holmes system with the hybrid controller in series configuration.

The governing equation for the series configuration is written as follows assuming unit mass for each DOF,

$$\ddot{w}_1 + 2\zeta_{s1}\omega_{s1}\dot{w}_1 - \omega_{s1}^2 w_1 + k_{nl1}w_1^3 - k_{nl2}(w_2 - w_1)^3 - k_2 w_2 - c_2 \dot{w}_2 = \omega_{s1}^2 u_{c1} \quad (3)$$

$$\ddot{u}_{c1} + 2\zeta_{c1}\omega_{c1}\dot{u}_{c1} + \omega_{c1}^2 u_{c1} = g_{f1}\omega_{c1}^2 w_1 \quad (4)$$

$$\ddot{w}_2 + 2\zeta_{s2}\omega_{s2}\dot{w}_2 - \omega_{s2}^2 w_2 + k_{nl2}(w_2 - w_1)^3 - k_2 w_1 - c_2 \dot{w}_1 = \omega_{s2}^2 u_{c2} \quad (5)$$

$$\ddot{u}_{c2} + 2\zeta_{c2}\omega_{c2}\dot{u}_{c2} + \omega_{c2}^2 u_{c2} = g_{f2}\omega_{c2}^2 w_2 \quad (6)$$

where $\omega_{s1}, \omega_{c1}, \omega_{s2}, \omega_{c2}$ and $\zeta_{s1}, \zeta_{c1}, \zeta_{s2}, \zeta_{c2}$ represent frequencies and damping ratios of the system and controllers, g_{f1} and g_{f2} is the controller gains for each DOF. k_{nl1} and k_{nl2} are cubic stiffness terms for the first and second DOFs, respectively. k_2 and c_2 are stiffness and damping terms representing linear coupling between the masses.

The dynamic response of the series configuration is presented in Figure 9. The HPF authority for two different scenarios is tested for weakly coupled 2DOF system. In the first scenario, HPF switches the first and second DOFs forward. In the second scenario, the HPF moves both DOFs backward from one state to another.

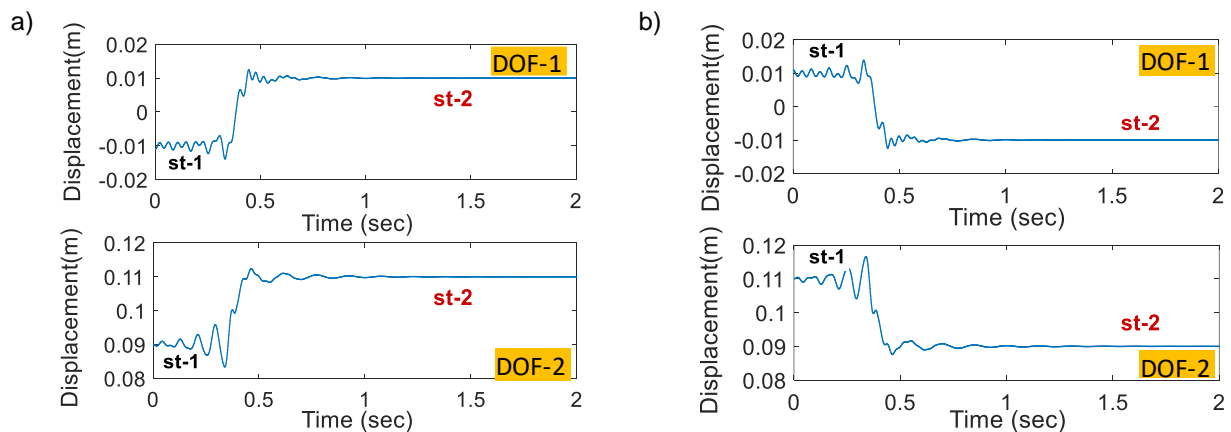


Figure 9: Example time response for the series 2DOF cascade bistable Duffing-Holmes system with the hybrid controller: a) DOF-1 st1->st2 and DOF-2 st1->st2, b) DOF-1 st2->st1 and DOF-2 st2->st1.

A parametric analysis is carried out to determine the controller parameters that satisfy the desired switch properties for different cases. The proposed schematic for series configuration can be realized when two DOFs achieve forward, backward, converse state transfer. In the analysis, two controller parameters ζ_{npf} and g_{npf} are chosen since they have the major effect on switching behavior. In the analyses, the ζ_{npf} and g_{npf} are equal for the two DOFs – they are swept

through a range to determine the optimum values that satisfy the desired switching behaviors. Figure 10 presents a parametric analysis for the series configuration.

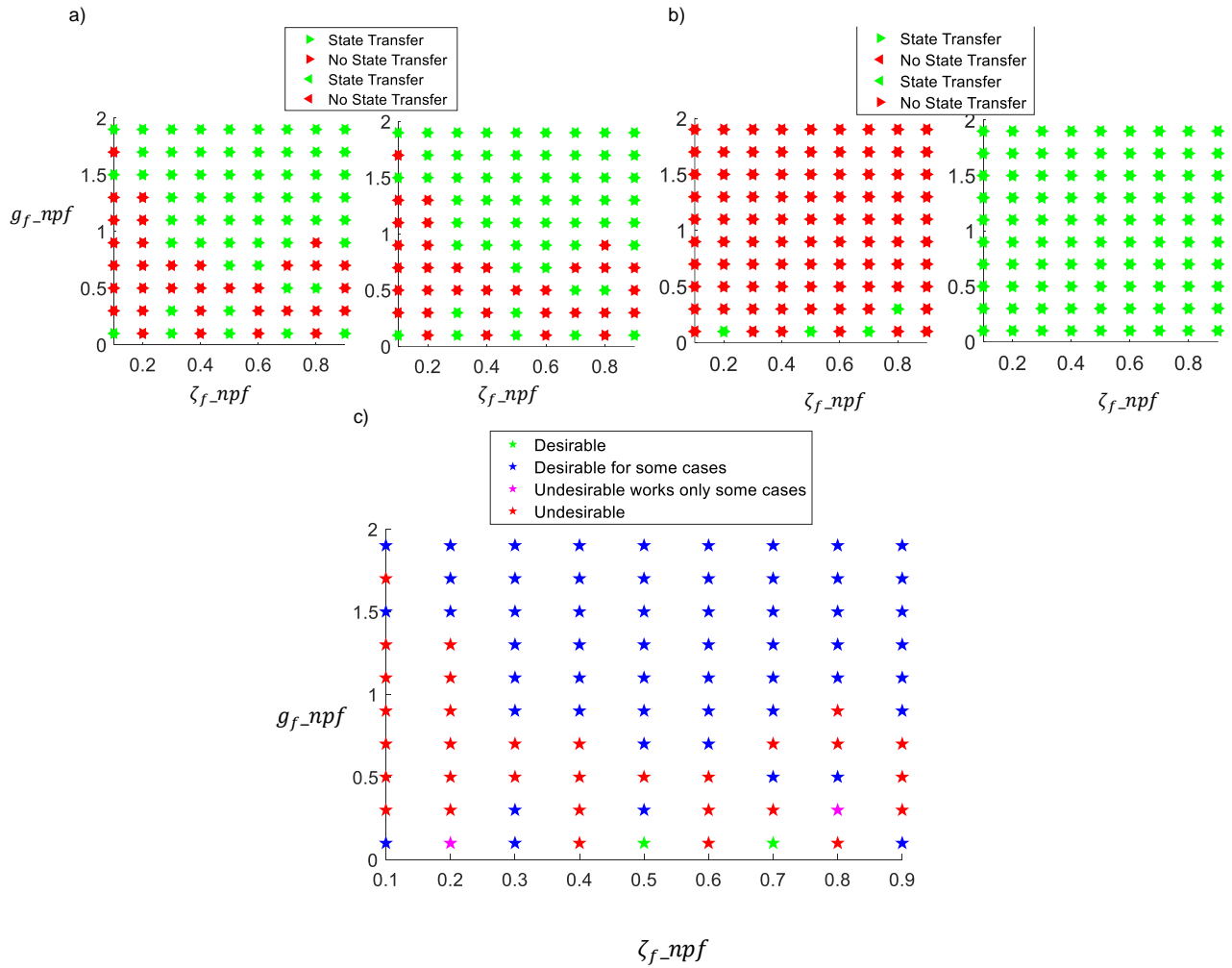


Figure 10: Parametric analysis for the series 2DOF cascade bistable Duffing-Holmes system with the hybrid controller a) Forward (DOF-1 and DOF-2: st1->st2) and Backward (DOF-1 and DOF-2: st2->st1) state transfer case, b) Converse State transfer case (DOF-1: st1->st2 and DOF-2: st2->st1, vice versa), c) all cases together.

Figure 10a presents a forward and backward state transfer behavior with respect to the controller parameters. The triangle pointing left or right shows the configuration of controller. The green and red colors indicate that initial configuration is desirable (i.e., intended state transfer) and undesirable (i.e., no state transfer), respectively. Figure 10b demonstrates the analysis carried out for converse state transfer where DOF-1 is set to achieve state transfer from state 1 to state 2 while DOF-2 is set to achieve state transfer from state 2 to state 1. Also, vice-versa case simulation is presented in this figure. Figure 10c demonstrates the behavior of controller for all cases mentioned above. The green color indicates the controller parameters that achieve successful state transfer for all scenarios. Figure 11 shows the time responses for desired configuration.

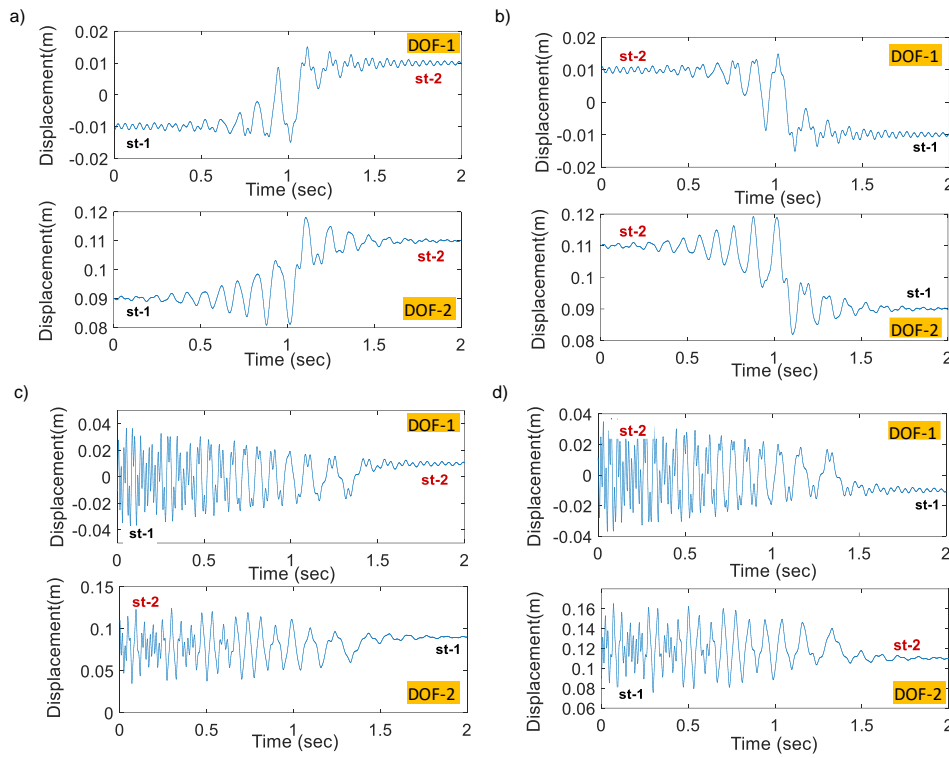


Figure 11: Time response for the parameters $\zeta_{fnpf} = 0.5$ and $g_{fnpf} = 0.1$ that achieves all scenarios of series configuration.

Parallel Configuration

The dynamic response of the parallel configuration is studied. Figure 12 presents the mechanical model of the 2DOF cascade bistable Duffing-Holmes system with the hybrid controller.

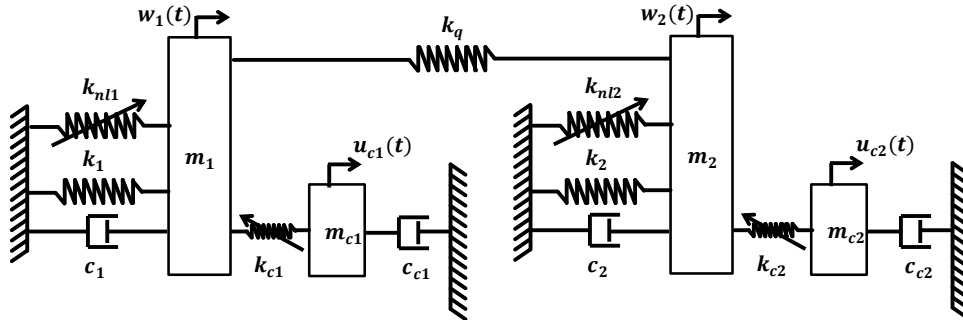


Figure 12: Mechanical model of the 2DOF cascade bistable Duffing-Holmes system with the hybrid controller in parallel configuration.

The governing equation for parallel configuration is written as follows assuming unit mass for each DOF,

$$\ddot{w}_1 + 2\zeta_{s1}\omega_{s1}\dot{w}_1 - \omega_{s1}^2 w_1 + k_{nl1}w_1^3 - k_q(w_2 - w_1) = \omega_{s1}^2 u_{c1} \quad (7)$$

$$\ddot{u}_{c1} + 2\zeta_{c1}\omega_{c1}\dot{u}_{c1} + \omega_{c1}^2 u_{c1} = g_{f1}\omega_{c1}^2 w_1 \quad (8)$$

$$\ddot{w}_2 + 2\zeta_{s2}\omega_{s2}\dot{w}_2 - \omega_{s2}^2 w_2 + k_{nl2}w_2^3 + k_q(w_2 - w_1) = \omega_{s2}^2 u_{c2} \quad (9)$$

$$\ddot{u}_{c2} + 2\zeta_{c2}\omega_{c2}\dot{u}_{c2} + \omega_{c2}^2 u_{c2} = g_{f2}\omega_{c2}^2 w_2 \quad (10)$$

where $\omega_{s1}, \omega_{c1}, \omega_{s2}, \omega_{c2}$ and $\zeta_{s1}, \zeta_{c1}, \zeta_{s2}, \zeta_{c2}$ represent frequencies and damping ratios of the system and controllers, g_{f1} and g_{f2} is the controller gains for each DOF. k_{nl1} and k_{nl2} are cubic stiffness terms for the first and second DOFs, respectively.

The dynamic response of the parallel configuration is presented in Figure 13. The control authority for four different scenarios is tested for a weakly coupled 2DOF system. In the first and second scenario, the HPF keeps the first DOF at the first state while switching the second DOF forward and backward from one state to another. In the third and fourth scenario, the first DOF is attracted to the second state while the second DOF switches forward and backward between the states.

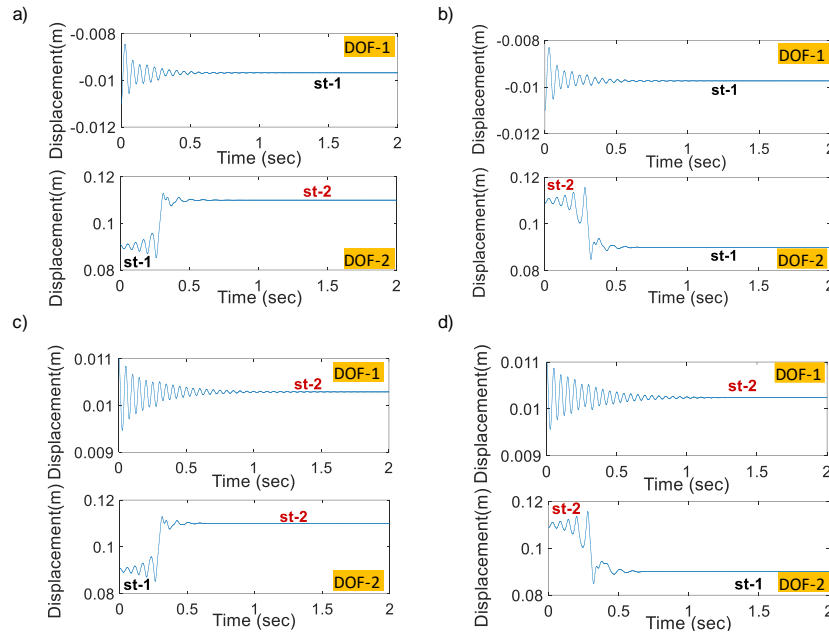


Figure 13: Example time response for the parallel 2DOF cascade bistable Duffing-Holmes system with the hybrid controller: a) DOF-1@st1 and DOF-2 st1->st2, b) DOF-1@st1 and DOF-2 st2->st1, c) DOF-1@st2 and DOF-2 st1->st2 and d) DOF-1@st2 and DOF-2 st2->st1.

A parametric analysis is carried out to determine the controller parameters that satisfy the desired switch properties for different cases. The proposed schematic for parallel configuration can be realized when two DOFs achieve forward, backward, converse state transfer. In the analysis, two controller parameters ζ_{npf} and g_{npf} are chosen since they have the major effect on switching behavior. In the analyses, the ζ_{npf} and g_{npf} are equal for both DOFs – they are swept through a range to determine the optimum values that satisfy the desired switching behaviors. Figure 14 presents a parametric analysis for the parallel configuration.

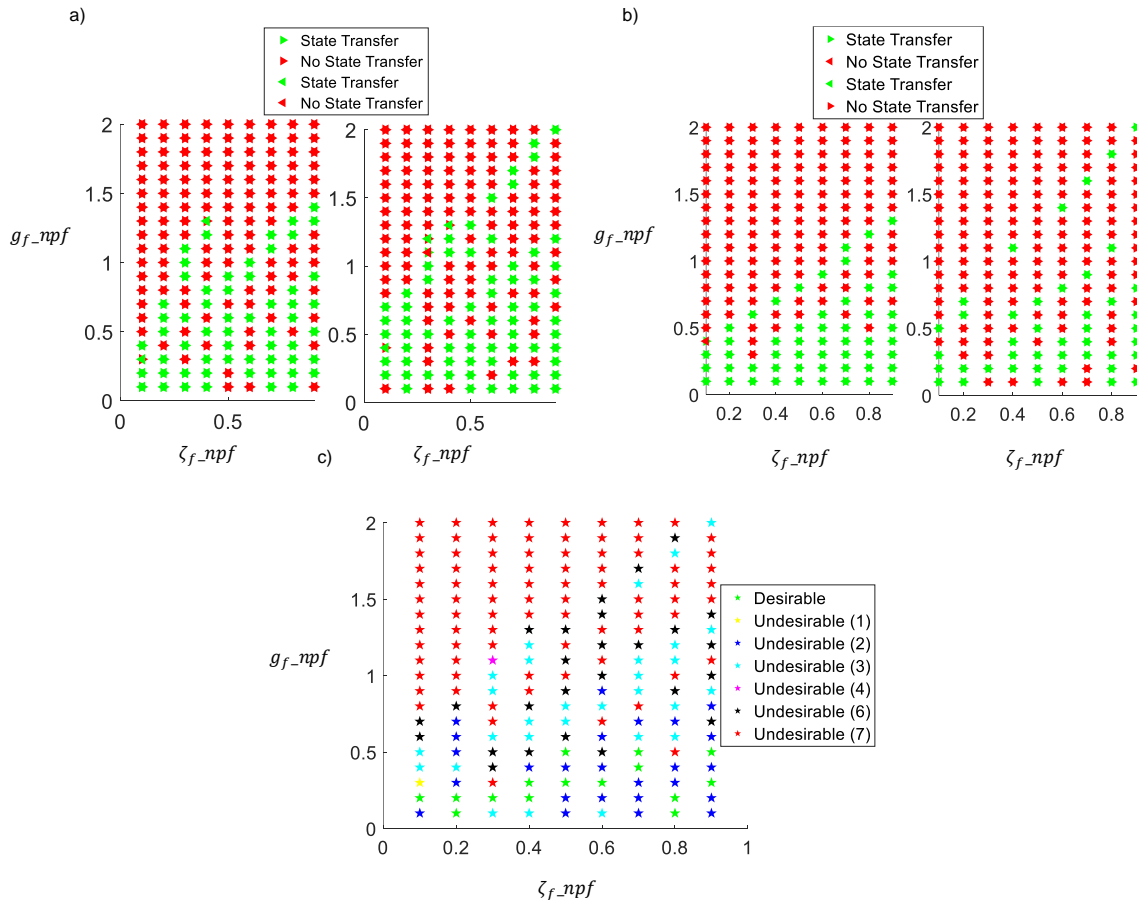


Figure 14: Parametric analysis for the series 2DOF cascade bistable Duffing-Holmes system with the hybrid controller a) Forward (DOF-1 and DOF-2: st1->st2) and Backward (DOF-1 and DOF-2: st2->st1) state transfer case, b) Converse State transfer case (DOF-1: st1->st2 and DOF-2: st2->st1, vice versa), c) all cases together.

Figure 14a presents a forward and backward state transfer behavior with respect controller parameters. The triangle pointing left or right shows the configuration of controller. The green and red colors indicate that initial configuration successful and unsuccessful, respectively, as described previously. Figure 14b demonstrates the analysis carried out for converse state transfer where DOF-1 is set to achieve state transfer from state 1 to state 2 while DOF-2 is set to achieve state transfer from state 2 to state 1. Also, vice-versa case simulation is presented in this figure. Figure 14c demonstrates the behavior of controller for all cases mentioned above. The green color indicates the controller parameters that achieve convergence to the desired state for all scenarios. The other colors in the figure are undesirable with the number of unsatisfied case indicated in the legend. Figure 15 shows the time responses for desired configuration.

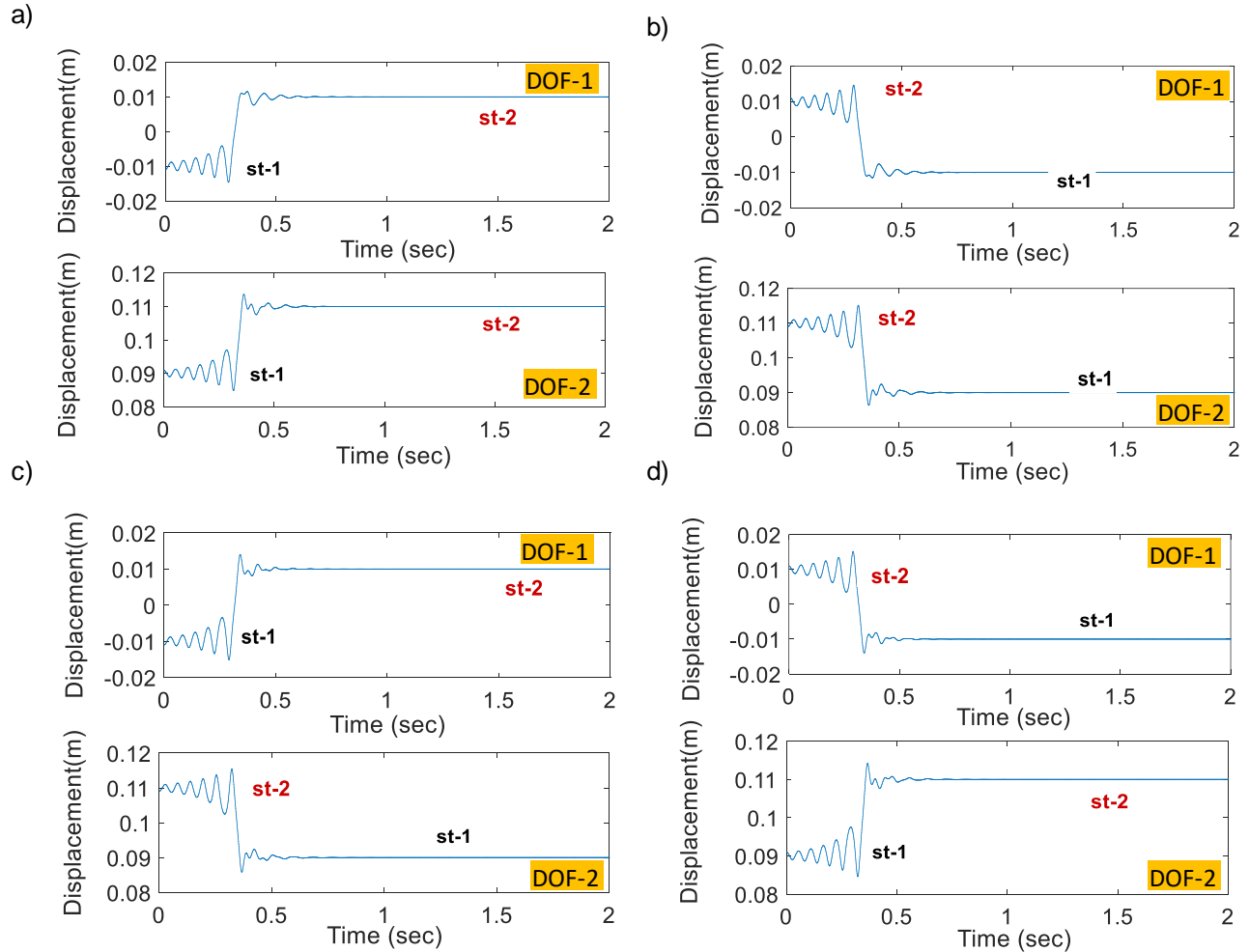


Figure 15: Time response for the parameters $\zeta_{f_{npf}} = 0.5$ and $g_{f_{npf}} = 0.3$ that achieves all scenarios of parallel configuration.

Conclusions

A multi degree of freedom metamaterial concept that utilizes the hybrid position feedback controller is introduced. An arbitrary number of bistable “segments” or “material elements” are attached to each other in a serial or parallel manner to generate a “distributed” bistable structure. The initial results, derived for 2DOF systems, show the merit of the proposed concept which can be applied to many applications. Due to the simplicity of the hybrid controller, the physical implementation of the proposed approach can be realized using simple circuit elements distributed in the material domain. The proposed metamaterial concept can be used in various applications: locomotion in bioinspired systems, undulatory motion, morphing aerodynamic surfaces, wave guiding, and vibration attenuation. The concept can also be used in energy harvesting to enable maximum power extraction for a given vibratory input.

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