Bifurcations of an Optically Excited Achiral Nano-Ellipsoid in a Stationary Fluid

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Summary. We derive and investigate a system of equations describing the three-dimensional dynamics of a prolate and oblate achiral ellipsoidal particle subjected to a linearly polarized, electromagnetic excitation. A half-wave plate rotates the polarization direction of the transmitted light by an angle Ω and the linearly polarized electric field induces a dipole moment in the particle that depends on the product of the polarizability tensor and the electric field. To minimize energy, the induced dipole moment tends to align with the field yielding an optical torque. The basic assumption in the development of the equations of motion was that the given ellipsoid rotates around its center of mass with negligible inertia due low Reynolds number. The nanoscale dimensions of the given particle enable problem formulation in the Rayleigh regime which yields a set of three nonlinear equations for the angular velocities in terms of Euler angles. Transformation of the equations of motion to an autonomous dynamical system enabled a linear stability analysis of multiple coexisting equilibria corresponding to stable and unstable periodic orbits in the lab frame. A set of Hopf bifurcations in the autonomous system revealed existence of nonstationary quasiperiodic and chaotic motions in the lab frame governed by three nondimensional parameters.

Introduction

The advent of the laser sparked numerous research areas and one of these has been and manipulation of matter by light. In 1986 Ashkin and co-workers showed that a single tightly focused beam could be used to hold, in three dimensions, a microscopic particle near beam focus, which is now known as optical tweezers. The potential uses of contact free control of microscopic and nanoparticles has maintained high scientific interest for more than three decades. Major advances in the field have been coupled with technological innovations such as dynamic control using holographic optical tweezers, engineering of beam shapes and ultraprecise tracking techniques. Optical trapping at the micro [1] and nano [2] scales is well documented in several reviews. We note that the dynamics of tri-axial ellipsoids in shear flow reveal existence of periodic [3], quasiperiodic [4] and chaotic [5] solutions. However, to date ellipsoids immersed in a fluid and excited by a modulated optical field, have been shown to be periodic or quasiperiodic [6]. Thus, our aim in this work, is to provide a sptio-temporal physical model of the rotational dynamics of a triaxial nano-ellipsoid (Fig.1 left), to validate the existence of periodic and quasiperiodic rotations documented for long cylindrical achiral nanorods [7] and to determine existence of nonstationary chaotic rotations. We consider an achiral ellipsoidal particle with principal axes $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ and permittivity $(\boldsymbol{\varepsilon_2})$ suspended in a liquid of permittivity $(\boldsymbol{\varepsilon_1})$. A rotating frame of reference is attached to the particles mass center where the Eulerian angles $(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\psi})$ correspond to pitch, yaw and roll (Fig.1 left). The ellipsoid is nonmagnetic so the relative magnetic permeability can be considered as a unity.

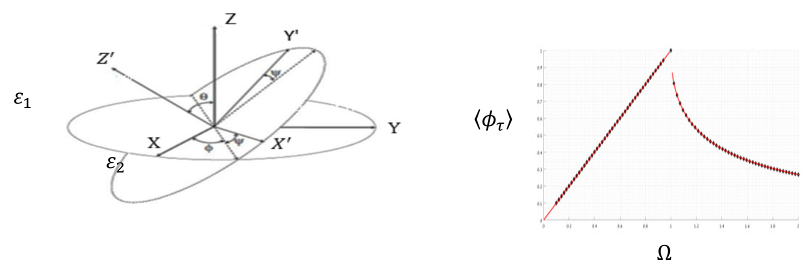

Fig.1: Definition sketch of a nanoscale ellipsoid (left) and the bifurcation diagram for a spheroidal particle (right).

A linearly polarized impinging electric field goes through a half-waveplate that rotates the polarization direction. This exciting electric field induces a dipole moment that depends on the product of the polarizability tensor and the electric field. To minimize energy, the induced dipole moment tends to align with the field yielding an optical torque. The basic assumption in the development of the equations was that the given particle is trapped, i.e., not subjected to gradient force and rotates around its center of mass. Since the dimensions of the ellipsoid are significantly small compared to the wavelength of the exciting field, the phase is considered constant over the particle. Moreover, since the half wave plate rotates at an angular frequency which is much slower than the angular optical frequency the total torque is averaged over one optical cycle. Under the assumption of low Reynolds number, inertial terms are neglected [8], which means that the angular drag is equal to the total optical torque on the particle. For example, it has been shown that for planar rotational dynamics of dielectric nanorod, close to a critical frequency, the optical torque is a factor of much higher than the corresponding rotation due to Brownian motion during a given time interval [6]. Furthermore, we note that the bifurcation structure of a nano-spheroid (a>b=c) includes periodic rotations (Fig.1 right-$\Omega < 1$) or quasiperiodic motions (Fig.1 right-$\Omega > 1$) which have been demonstrated experimentally for a long cylinder [6].

Results

The non-dimensional equations governing the motion were constructed in the theoretical framework of Newtonian rigid body dynamics yielding a third-order autonomous system controlled by nondimensional parameters that depend on the particle material properties, and the angular drag coefficients (α, β) and an additional parameter that depends on the slow half wave plate rational frequency and the electric field magnitude (Ω). An investigation of the nonlinear dynamical response of ellipsoidal particles, with varying dimension and material properties, was carried out using linearization methodology yielding an eigenvalue problem. The analysis reveals a diverse local bifurcation structure (Fig.2 left) that includes coexisting bi-stable equilibria and a narrow range of limit cycle oscillations (Fig.2 right) due to subcritical Hopf threshold (depicted by the red line in Fig.2 left).

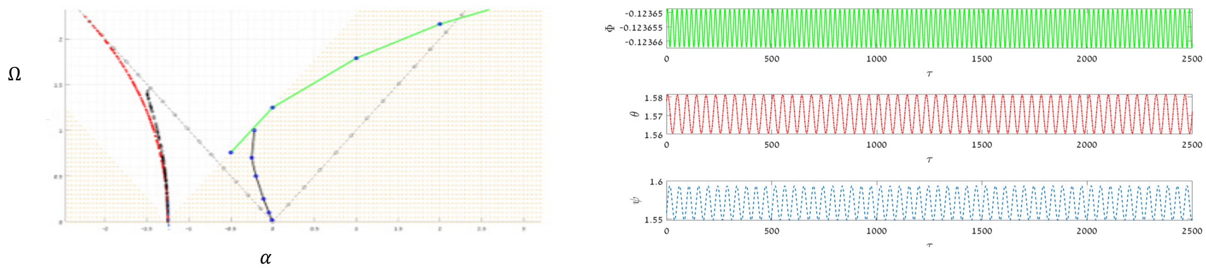

Fig.2: Stability map for a gold nano-ellipsoid with exciting frequency Ω as a function of a material parametre α (left) and time histories of limit cycle oscillations in the Eulerian frame where $\Phi=\phi-\Omega\tau$(right).

Discussion

The classification of different solutions is portrayed in the Cartesian physical state space (Figs. 3,4). It is shown that equilibrium in the Euler angle autonomous system corresponds to a periodic solution in Cartesian space, whereas a periodic solution corresponds to a quasiperiodic Cartesian state space (Fig.3 blue triangles). Furthermore, nonstationary chaotic solutions were found between two secondary bifurcation thresholds (Fig.3 red squares). Moreover, additional quasiperiodic solutions were found for $\Omega > 1$ for which no equilibrium is reached in the Eulerian state space. It is noteworthy that the limiting case of a nano-spheroid is governed by a single non-dimensional parameter (Ω) and its bifurcation structure at steady state (Fig.1 right) does not include chaotic rotations. We note possible excitation with two frequencies may yield a region of chaotic interactions for incommensurate frequency ratios.

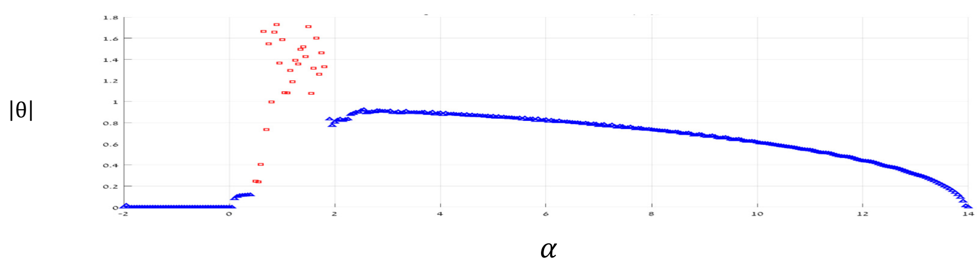

Fig. 3: Bifurcation diagram for a hybrid nano-ellipsoid where $\Omega = 0.5, \beta = 1$.

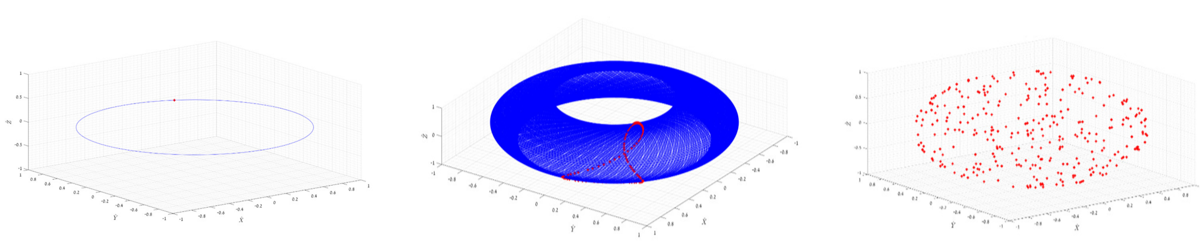

Fig.4: Poincare' maps overlaid on the lab frame physical state space (Z(X,Y)) of periodic (left) and quasiperiodic (center) solutions. Poincare' map of a chaotic solution (right).

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