Frequency-Energy Plot of Unsymmetrical Nonlinear Energy Sink

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Summary. Studying the nonlinear energy sink (NES) attachment with a linear dynamical oscillator (LO) on the frequency-energy plot (FEP) reveals the underlying nonlinear dynamical behavior of the LO-NES system. In addition, the FEP reveals different kinds of periodic motions on NNMs backbone branches. The unsymmetrical force nonlinear energy sink (UNES) incorporates a cubic stiffness element at one side of its equilibrium position where a weak linear restoring coupling stiffness acts in both directions. The obtained FEP has revealed several unsymmetrical backbones of 1:1 resonance between the UNES and the LO periodic oscillations at different nonlinear frequency levels.

Introduction

The frequency energy plots (FEPs) analysis has a significant impact on revealing the underlying nonlinear dynamics of the structure-NES nonlinear motion. The periodic motion of stiffness-based structure-NES systems have been extensively studied on harmonic NNM backbone curves and their associated branches/tongues of subharmonic and super harmonic periodic motions in the FEPs. The FEPs accompanied with the superimposed wavelet frequency spectrum content have confirmed the existence of different kinds of resonance captures between the primary system and the NES oscillations at the backbones and their associated subharmonic branches and tongues. The resonance captures on the FEPs have explained the rapid and passive targeted energy transfer (TET) from the primary system into the NES attachment [1–5].

UNES Description

The UNES attachment with the LO in [6] is shown in Figure 1 where the UNES mass is coupled with the LO by a cubic stiffness element that acts only to the left-side of the equilibrium position. In addition, the UNES incorporates a weak linear restoring spring that acts in both directions as shown. The physical parameters in [6] are used here for generating the frequency energy plot of the Hamiltonian version of the LO-UNES system (i.e, $\lambda_1 = \lambda_2 = 0$). The other parameters are; LO mass of $M = 1\,\text{kg}$, LO coefficients of stiffness of $k_1 = 1\,\text{N/m}$, NES mass of $m = 0.05\,\text{kg}$, NES coefficients of cubic nonlinear stiffness of $k_{nl} = 1\,\text{N/m}^3$, and the UNES coefficient of weak linear restoring stiffness $k_{res} = 0.03$ N/m.

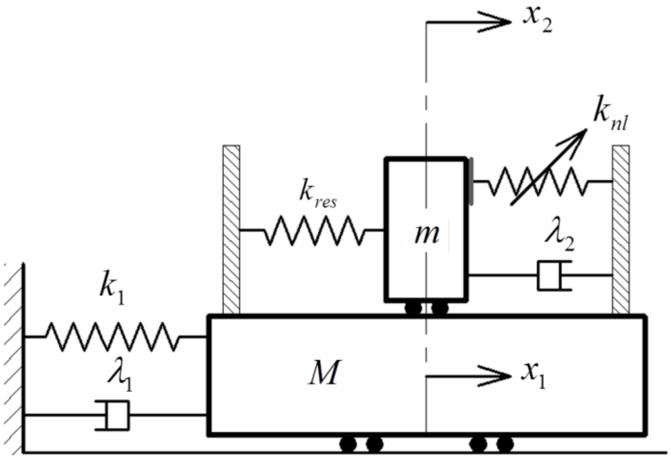

Figure 1: Unsymmetrical NES attachment with a linear oscillator

Frequency Energy Plot of the LO-UNES System

The FEP backbones are generated here by applying the numerical continuation method in [4, 5] with the considered LO-UNES system in Figure 1. The free-response of the Hamiltonian equations of motion of the system is obtained at the given physical parameters in the previous section for zero velocities and nonzero displacements. Therefore, the underlying nonlinear dynamical behaviour due to the influence of the unsymmetrical coupling stiffness force is investigated on the obtained FEP. Accordingly, seven backbone curves (*A1, A2, B1, B2, C, D* and *E*) are obtained in the FEP as shown in Figure 2. The backbones are named according to the ratio of the frequency content in the oscillation response between the LO and UNES masses. Therefore, $U11\pm$ denotes to an unsymmetrical NNM periodic motion at which the NNM does not pass through the origin in the NNM configuration space where the $+$ and $-$ indices indicate to the in and out of phase periodic motions, respectively. $U11+$ branch of the backbone indicates to 1:1 resonance frequency ratio of the in phase periodic motion between the LO and UNES masses. As shown from the FEP, the backbones, *A2, B1, B2*, and *C*, approach 1, 1/2, 1/3 and 1/4 of the natural frequency of the LO at high energy levels. Unlike other kinds of NESs, all backbones are at 1:1 resonance for different values of the nonlinear frequency.

The NNM, the related periodic time response of the UNES mass and the associated mass of the LO, and the corresponding nonlinear UNES coupling force are plotted for selected data points *a* and *b*, respectively, as shown in Figure 3. For the data point *a*, the NNM does not pass through the origin where this NNM is located at the $U11-$backbone *A1*. Therefore a 1:1 out of phase resonance takes place between the UNES and LO periodic motions. The data point *b* is located $U11-$ backbone *B1*. For this data point, also 1:1 out of phase resonance takes place between the UNES and LO periodic motions. In Figures 3c and 3f, the strong nonlinear effect to the right-side of the equilibrium position of the UNES and the weak linear coupling effect in the left-side are shown.

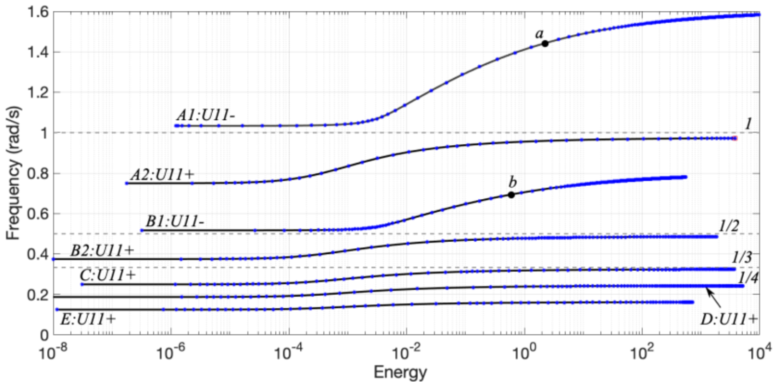

Figure 1: The obtained unsymmetrical backbone curves of the FEP of the LO-UNES system.

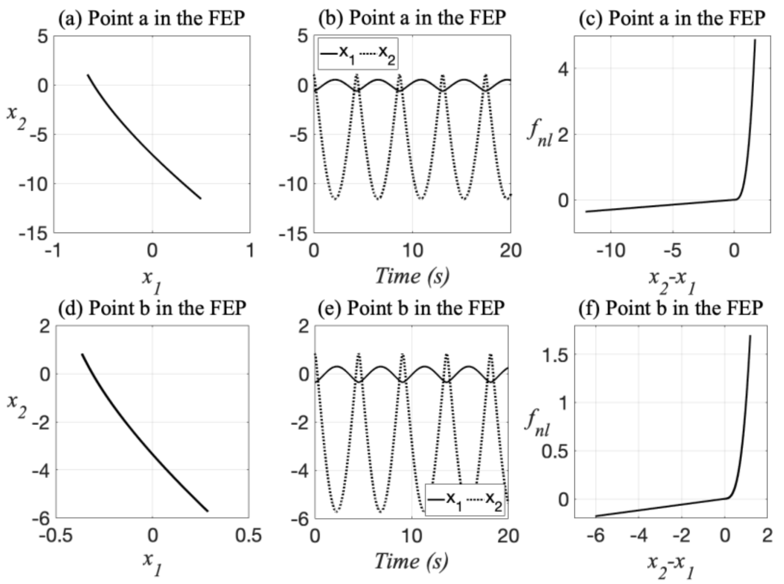

Figure 3: The NNMs in (a) and (d), their corresponding oscillation displacements in (b) and (e), and the corresponding nonlinear coupling forces in (c) and (f) for the data points *a* and *b* in the FEP.

Conclusions

In this study, the frequency energy-dependence of LO-UNES system is studied on the frequency energy plot. The underlying nonlinear dynamical behavior of the LO-UNES is revealed on the FEP. Unlike other existing NESs, the UNES is found to be associated with several backbones of unsymmetrical NNMs of 1:1 in phase and out of phase resonance between the UNES and LO oscillations for wide range of nonlinear frequency levels. The UNES shows a unique capability of functioning at 1:1 resonance at a broadband frequency-energy fashion.

References

[1] Vakakis A.F., Gendelman O.V., Bergman L.A., McFarland D.M., Kerschen G., Lee Y.S. (2008) Nonlinear targeted energy transfer in mechanical and structural systems, Springer Science & Business Media,

[2] Lee Y.S., Kerschen G., Vakakis A.F., Panagopoulos P., Bergman L., McFarland D.M. (2005) Complicated dynamics of a linear oscillator with a light, essentially nonlinear attachment. *Physica D* **204**: 41-69.

[3] Quinn D.D., Gendelman O., Kerschen G., Sapsis T.P., Bergman L.A., Vakakis A.F. (2008) Efficiency of targeted energy transfers in coupled nonlinear oscillators associated with 1:1 resonance captures: Part I. *J. Sound Vib.* **311**: 1228-1248.

[4] Kerschen G., Peeters M., Golinval J-C., Vakakis A.F. (2009) Nonlinear normal modes, Part I: A useful framework for the structural dynamicist. *Mech Syst Signal Process* **23**: 170-194.

[5] Peeters, Maxime, Viguié, Régis, Sérandour, Guillaume, Kerschen, Gaëtan and Golinval, J-C. (2009) Nonlinear normal modes, Part II: Toward a practical computation using numerical continuation techniques. *Mech Syst Signal Process* **23**: pp. 195-216.

[6] Vakakis A.F., Al-Shudeifat M.A., Hasan M.A. (2014). Interactions of propagating waves in a one-dimensional chain of linear oscillators with a strongly nonlinear local attachment. *Meccanica* **49**: 2375-2397