Escape of two-DOF dynamical system from the potential well

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Summary. The escape of initially excited dynamical system with two degrees of freedom from three different one-dimensional benchmark potential wells is considered. Main challenge is revealing the basic mechanisms that govern the escape in different regions of the parametric space, and constructing appropriate asymptotic approximations for analytic treatment of these mechanisms. In this study numerical and analytical tools are used in order to classify and map the different escape mechanisms for a variety of initial conditions, and to offer the analytic criteria predicting the system's behavior for those cases.

Introduction

Escape from a potential well is a classic problem, relevant in many branches of physics, chemistry and engineering. Among many examples of possible applications, one encounters dynamics of molecules and absorbed particles, celestial mechanics and gravitational collapse [1,2], energy harvesting [3] responses of Josephson junctions [4], various aspects of capture into and escape from the resonance [5], as well as the capsizing of ships [6]. Even more profound, and widely explored manifestation of the escape phenomenon is a dynamic pull-in in microelectromechanical systems (MEMS) [7,8]. In this current study, we will expand the discussion and analyze two DOF systems. For a better understanding of the problem and a more reliable model in our work, three one-dimensional examples of potentials are considered: inverse hyperbolic potential, cubic potential and biquadratic potential. General setting explored in the present work can be presented as two equal particles attached to each other by a spring, and imbedded into the potential well (Fig 1).

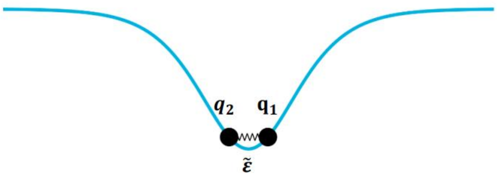

Figure 1: sketch of the general setting

Main challenge is revealing the basic mechanisms that govern the escape in different regions of the parametric space, and constructing appropriate asymptotic approximations for analytic treatment of these mechanisms.

Results

In this study numerical and analytical tools, such as Poincare maps and grid classification, are used in order to classify and map the different escape mechanisms for various initial conditions. Further investigation was performed on two different sets of initial conditions. For the first set of conditions, only one of the particles was excited by a nonzero initial velocity. For the second set of conditions, both particles were excited. For convenience, in this case other coordinates were used: center of mass R and interparticle displacement W. When only W coordinate is excited, one can reveal an additional escape mechanism - dissociation, in which the distance between the particles eventually diverges. In order to investigate the classic escape mechanisms in which both particles escape in the same direction, we add minor disturbance to the center of mass velocity $\dot{R}$.

$$\textit{set}\ \mathrm{I} \quad q_1(0) = q_2(0) = \dot{q}_2(0) = 0\ ;\ \dot{q}_1(0) = v_0$$

$$\textit{set}\ \mathrm{II} \quad R(0) = W(0) = 0\ ;\ \dot{W}(0) = v_0\ ;\ \dot{R}(0) = 0\ /\ \dot{R}(0) = \delta$$

Variety of analytical and dynamical methods were used to derive the dependence between the minimum energy required for different escape mechanisms of the system and the spring stiffness ε. The resulting relations were validated by the numerical results, as presented in (Fig 2).

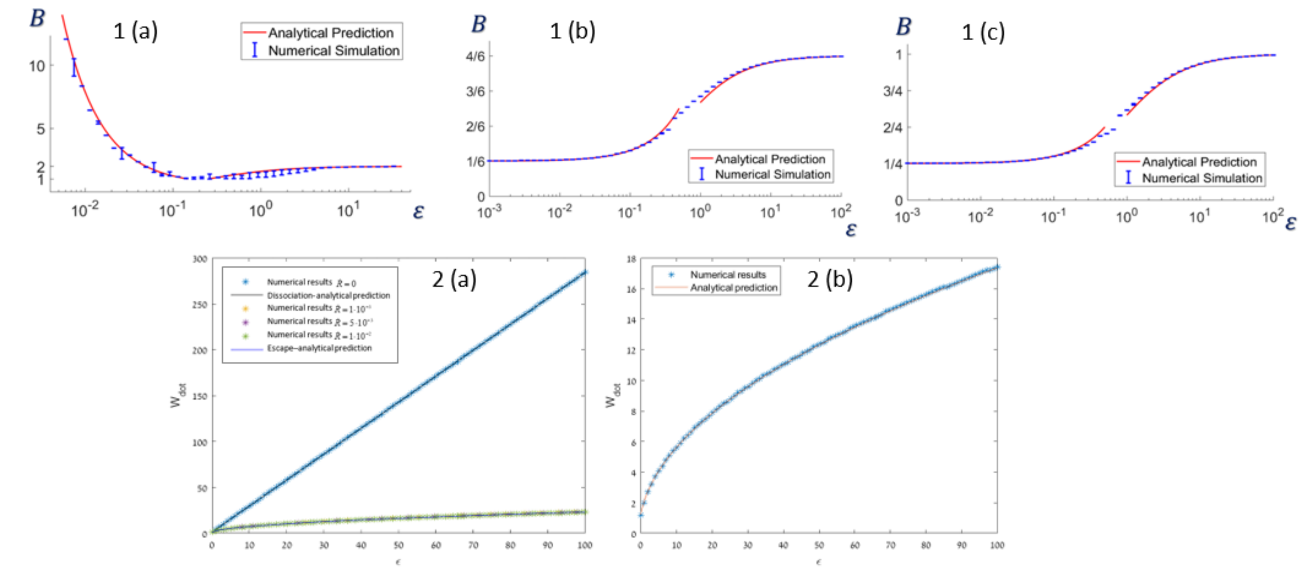

Figure 2: the analytical prediction and numerical results for the different cases (1) set I, minimum energy required to escape as a function of the spring stiffness (a) inverse hyperbolic potential (b) cubic potential (c) biquadratic potential. (2) set II, minimal separation velocity $\dot{W}$ required for each escape mechanism as a function of the spring stiffness (a) biquadratic potential (b) cubic potential

Discussion

For the case of the single particle excitation, a clear distinction between the cases of weak and strong coupling has been revealed. For the small coupling, the escape is achieved when the excited particle has enough energy to pull the remaining particle from the well. For the case of the strong coupling, one can separate the timescale of fast interparticle oscillations; appropriate averaging delivers the modified effective potential for relatively slow evolution of the center of masses. Interestingly, two aforementioned limiting cases faithfully cover almost all space of parameters. To explore the chaotic responses, Lyapunov exponents were calculated, as presented in (Fig 3).

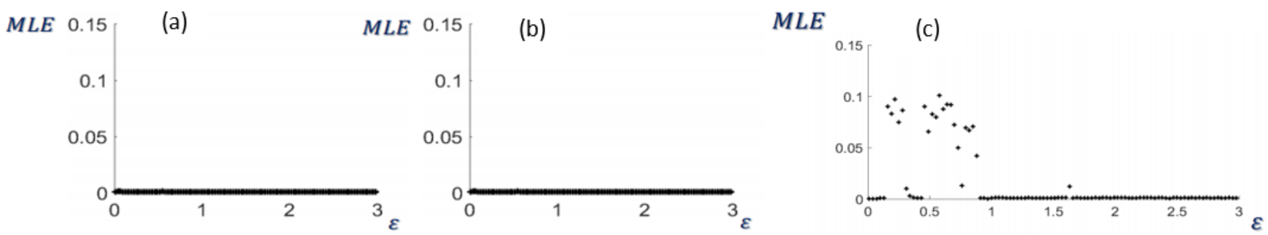

Figure 3: Lyapunov exponents as a function of the spring stiffness (a) Cubic potential (b) Biquadratic potential (c) Hyperbolic potential

Only the inverse hyperbolic potential has shown characteristics of chaotic behavior, mainly for low values of the spring stiffness, while the cubic and biquadratic potentials have not exhibited the chaos-governed escape dynamics. For the other set of initial conditions, a new escape mechanism – dissociation, derived from the symmetry of the initial conditions and the biquadratic potential, was recognized. Along with an additional escape mechanism, related to a parametrically driven acceleration of the center of masses. Those two mechanisms introduced in (Fig 2.2.a)), the dissociation mechanism occurs for higher energy levels while the other is described by lower levels. For those cases no separation of the parameters plane was needed and the same regime described the system for all ranges of the stiffness.

References

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