Stabilisation of Rayleigh-Plateau modes on a liquid cylinder

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Summary. We show stabilisation of unstable Rayleigh-Plateau(RP) modes on a liquid cylinder by subjecting it to a radial oscillatory body force. The proposed stabilsation was short lived as shown earlier in our inviscid study [5]. Viscous analysis is performed which has importance for this stabilisation. Linear stability predictions are obtained via Floquet analysis [3]. We also solve the linearised, viscous initial-value problem for free-surface perturbations obtaining an integro-differential equation governing the amplitude of Fourier mode. This equation represents the cylindrical analogue of its Cartesian counterpart [1]. Present study [4] shows that RP stabilisation can be extended longer in time using radial oscillatory forcing. Predictions from the numerical solution to this equation demonstrates RP mode stabilisation upto several hundred forcing cycles and shows excellent agreement with direct numerical simulations(DNS) of the incompressible, Navier-Stokes equations using Basilisk [7]. An expanded version of present study is under review in a journal for publication.

Dynamic stabilisation of RP modes - Linear inviscid theory

Liquid filaments, jets or annular fluid films coating the rods are susceptible to breakup into droplets via classical Rayleigh-Plateau (RP hereafter) instability [6, 8]. In our earlier inviscid study [5], Faraday waves on a liquid cylinder where dynamic stabilisation of RP modes was predicted but found to be extremely short-lived in inviscid simulations. Figure 1, shows an infinitely long, quiescent liquid cylinder of density ρ, surface-tension T, kinematic viscosity ν and radius R_0 being subject to a radial, oscillatory body force $\mathcal{F}(r,t)$. Interface perturbation is expressed as $\eta(\theta,z,t) = a_m(t;k)\cos(m\theta)\cos(kz)$, where k and m represents axial and azimuthal wavenumber.

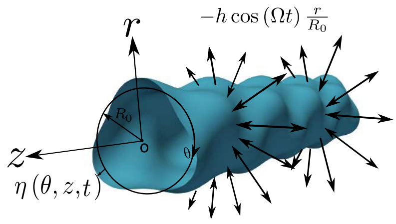
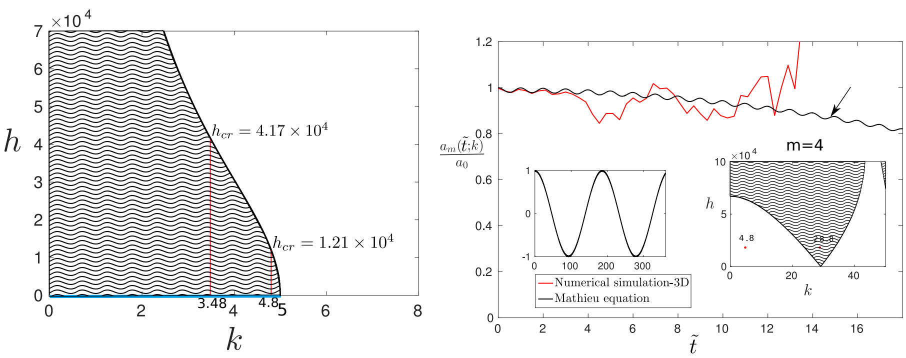


Figure 1: A schematic representation of surface perturbation on a viscous liquid cylinder of radius R_0 subject to a radial body force $\boldsymbol{\mathcal{F}}(r,t) = -h\left(\frac{r}{R_0}\right)\cos(\Omega t)\hat{\mathbf{e}}_r$, where h and Ω are imposed forcing strength and frequency respectively.

Under the linearised, inviscid, irrotational approximation, the equation governing amplitude $a_m(t;k)$ of Faraday waves on the free surface is shown to be Mathieu equation[5],

$$\frac{d^2a_m}{dt^2} + \frac{\mathrm{I}_m'(kR_0)}{\mathrm{I}_m(kR_0)}\left[\frac{T}{\rho R_0^3}kR_0\left(k^2R_0^2+m^2-1\right) + kh\cos(\Omega t)\right]a_m(t;k) = 0, \tag{1}$$

Figure 2a and 2b shows the inviscid stability chart and stabilisation of RP modes through DNS being short lived due to nonlinearity.



(a) Inviscid stability chart
(b) DNS for $k = 4.8, h = 1.8\times10^4$cm/s^2

Figure 2: Shaded and white indicate unstable and stable regions respectively. **Panel (a)** Inviscid stablity chart from equation 1 showing critical forcing strength h_{cr} above which RP unstable mode $k_0 = 4.8$ cm^{-1} will be stablised. Parameters: $\Omega = 600\pi$ rad/s (f=300 Hz), $R_0 = 0.2$ cm, $\rho = 0.957$ gm/cm^3, $T = 20.7$ dynes/cm. **Panel (b)** (Red curve) Time signal from inviscid 3D-DNS [7], ($k_0 = 4.8$ cm$^{-1}, m_0 = 0$) excited at $t = 0$. (Black curve) Solution to equation 1, (Left inset) Zoomed out view of solution to equation 1, (Right inset) Stability chart for $m = 4$ showing unstable non-axisymmetric Fourier mode ($k = 28.8 = 6k_0, m = 4$) at $\tilde{t} \approx 14$ s causing destabilisation.

Dynamic stabilisation of RP modes - Linear viscous theory

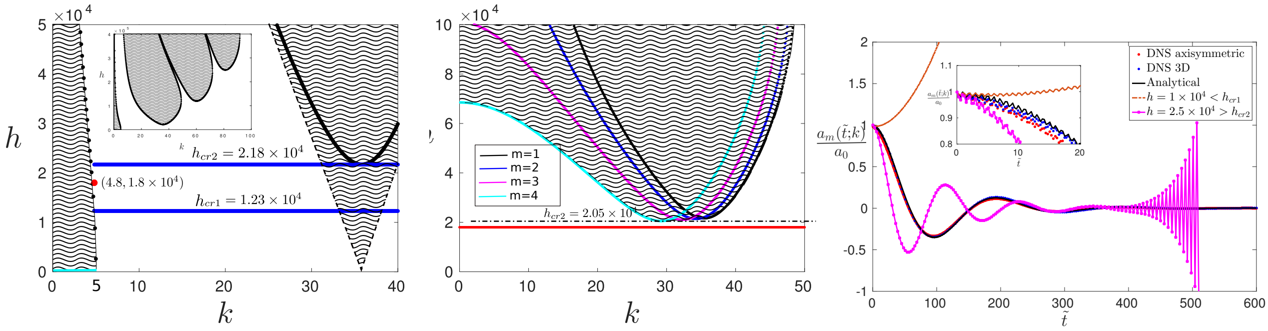
In the *viscous* analysis presented here [4] we performed Floquet analysis following [3] to obtain viscous stability chart and solve the initial-value problem (IVP) following toroidal-poloidal decomposition [2] on a cylinder. We finally obtain an integro-differential equation (2) for $a_m(t;k)$ which has a damping and two memory terms incomparison to Mathieu equation 1. It is shown that, by carefully tuning the strength and frequency of (radial) forcing, RP modes accessible to the system maybe rendered stable thus stabilising the cylinder for long time (many forcing time periods).

$$\frac{d^2 a_m}{dt^2} + 2\nu k^2 \frac{\mathrm{I}_m''(kR_0)}{\mathrm{I}_m(kR_0)}\frac{da_m}{dt} + \int_0^t \hat{\mathbf{L}}^{-1}\left(\tilde{\chi}(s)\right)\frac{\mathrm{I}_m'(kR_0)}{\mathrm{I}_m(kR_0)}\left[\frac{T}{\rho R_0^3}kR_0\left(k^2R_0^2 + m^2 - 1\right)\right.$$
$$\left. + hk\cos\left[\Omega(t-\tau)\right]\right]a_m(t-\tau)d\tau + 4\nu k\frac{\mathrm{I}_m'(kR_0)}{\mathrm{I}_m(kR_0)}\int_0^t \hat{\mathbf{L}}^{-1}\left[\zeta(s)\right]\frac{da_m}{dt}(t-\tau)d\tau = 0 \qquad (2)$$

Stablisation of RP modes: viscous stability chart and DNS comparison

Case	**Fluid**	$a(0)$	m_0	k_0	$\rho^{\mathcal{I}}$	$\rho^{\mathcal{O}}$	$\mu^{\mathcal{I}}$	$\mu^{\mathcal{O}}$	R_0	h	Ω	T
1	Silicone oil	0.01	0	4.8	0.957	0.001	0.1	0.001	0.2	1.8×10^4	600π	20.7

Table 1: Parameters for stability chart and DNS (CGS units)



(a) m=0 (b) m=1,2,3,4 (c) DNS comparison

Figure 3: **Panel a)** Stability chart ($m = 0$) and **panel (b)** non-axisymmetric ($m = 1, 2, 3, 4$) modes with Case 1 parameters in table 1. Figure 3a shows bold black lines → viscous tongue, black dashed line→ inviscid tongue. (Inset) complete chart. The mode ($k_0 = 4.8, m_0 = 0$) is stabilised when $h_{\text{cr1}} < h < h_{\text{cr2}}$ with $h_{\text{cr2}} = 2.05 \times 10^4$ cm/s^2 from $m = 4$ (see figure 3b). We select $h = 1.8 \times 10^4$ (red dot and solid red line in figure 3a and 3b respectively) for stabilisation. **Panel (c)** DNS time signal for case1 with parameters in table 1: (Red and blue dots) from DNS shows excellent agreement with solution to equation 2(refered as Analytical in figure 3c) upto 600 forcing cycles ($\tilde{t} \equiv t\Omega/2\pi$). (Orange line) Destabilisation seen in axisymmetric DNS when $h < h_{\text{cr1}}$ and when (Pink line) $h > h_{\text{cr2}}$. Note that inviscid simulation in figure 2b where for the same k_0, stabilisation is seen for only three forcing cycles.

Conclusions

We solved the initial-value problem (IVP) leading to a novel integro-differential equation governing the (linearised) amplitude of three-dimensional Fourier modes on the viscous liquid cylinder extending our earlier inviscid study. It is demonstrated that by suitably tuning the frequency of forcing and choosing strength $h_{\text{cr1}} < h < h_{\text{cr2}}$, RP mode ($k_0$) can be stabilised with all axisymmetric and three-dimensional modes that may be generated by nonlinear effects, can be prevented from destabilising the cylinder. DNS comparison have shown excellent aggreement with theoretical predictions demonstrating RP stabilisation upto hundreds of forcing cycles.

References

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