Nonlinear Normal Modes in the pendulum system under magnetic excitation

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Summary. Dynamics of two coupled pendulums under magnetic excitation is considered. Inertial components of the pendulums are essentially different, and a ratio of masses is chosen as a small parameter. Padé approximants is used to for the magnetic forces approximation. The small parameter method and the method of multiple scales are used to construct nonlinear normal modes (NNMs), one of them is localized one. Influence of the initial values and the system parameters, including the pendulum masses, change to the NNMs is studied.

Introduction. The basic model.

The system containing two pendulums under the electromagnetic motor influence is studied in few papers [1-3] where some corresponding mathematical models are constructed, and their validation is discussed after comparison of the numerical simulation and experimental results. Then some important aspects of the system dynamics are analyzed. Here we consider a similar system of two pendulums under magnetic force when inertial characteristics of these pendulums are essentially different. In this case, a localization of energy on the small mass pendulum is possible. To describe the system dynamics, the nonlinear normal modes (NNMs) theory is used. Different theoretical aspects of the NNMs theory and applications of the theory are presented in numerous publications, in particular, in reviews [4,5].

The Padé approximant and the nonlinear least squares method are used for analytical presentation of the magnetic force; such approximation demonstrates a good correspondence with experimental results presented in [2,3] as is shown in Fig.1.

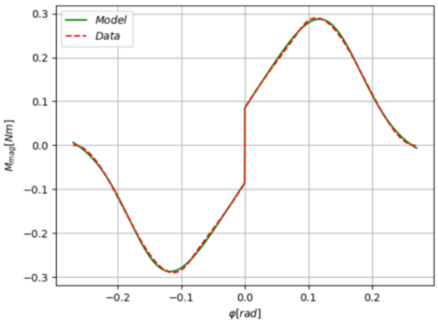

Fig.1. Comparison of the magnetic force Padé approximation with experimental result [2,3].

Equations of motion of the system under consideration with respect to the pendulum angles are the following:

$$\begin{cases} \varepsilon I\ddot{\varphi}_1 = \varepsilon M_{mag_1} - \varepsilon mgs\sin\varphi_1 - k_l(\varphi_1 - \varphi_2), \\ I\ddot{\varphi}_2 = \varepsilon M_{mag_2} - mgs\sin\varphi_2 - k_l(\varphi_2 - \varphi_1), \end{cases} \tag{1}$$

Here I is the inertia moment of the big mass pendulum, rotating masses, $k_l(\varphi_1 - \varphi_2)$ is the moment of the torsional deformation, s is a distance between the pendulum centrum of masses and the axes of rotation. Later $\sin(\varphi) \cong \varphi - \frac{\varepsilon}{6}\varphi^3$. The small parameter ε characterizes the pendulum masses ratio. Then the small parameter method is used. In zero approximation, the relation $\varphi_{10} = \varphi_{20}$ is obtained for the coupled (in phase) vibration mode, and the relation $\varphi_{20} = 0$ is obtained for the localized vibration mode after the following time transformation: $\boldsymbol{t=\varepsilon\tau}$. Without the magnetic influence in a wide range of changes in the system parameters, the error of the analytical solution is insignificant over sufficiently long time intervals and slowly increases with the growth of t up to the values of the pendulum initial angle deviations of the order 0.8727 rad (50°). Numerical simulations show a good exactness of the obtained analytical solution for relatively small values of the parameter ε. But with increasing of the parameter ε there is a gradual transition from the localized vibration mode to the out-of-phase one.

Influence of the initial deviations and the system parameters to the connected and localized nonlinear normal vibration modes under magnetic excitation

Considering numerical simulation of the connected and/or localized NNMs in the system under magnetic excitation we can conclude that at small angles, the influence of the magnetic force is greater, which means that the shape of the NNMs should not be preserved, although a localization of vibration is saved. At the same time, for large angles (for high energies), the both stable NNMs take place. But, if the modulated magnetic effect itself is small, the in-phase mode will be observed even with small values of the initial deviations of the pendulums. The localized NNM for small and relatively large values of initial deviations is presented in Fig. 2б where the simulation is made for the following system parameters: $m = 0.5\,kg, k_l = 0.5\frac{Nm}{rad}, s = 2.5\,m$.

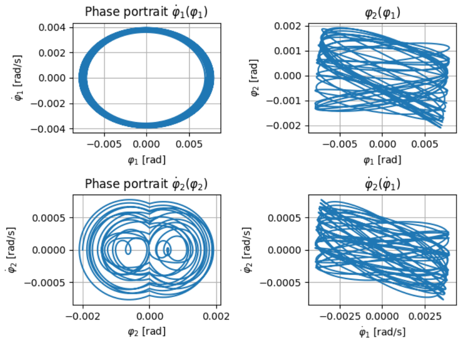

Fig. 2a: $\varphi_1(0) = 0.007rad, \varepsilon = 0.22, \varphi_2(0) = -0.002$ rad

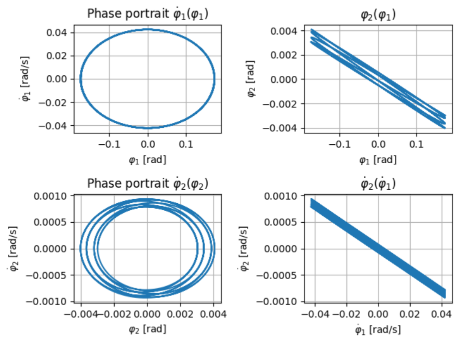

Fig.2b: $\varphi_1(0) = 0.175\,rad, \varepsilon = 0.02, \varphi_2(0) = -0.003\,rad$

Fig.2. Localized NNM for small and relatively large values of the initial angles.

With the moderate increase of the parameter ε, the connected mode shape is observed with small deviations of the trajectories, but with further increase of ε, the deviations increase significantly for both pendulums, and some divergences from the connected mode shape is observed.

With increase of the coupling, the sharps of the in-phase and localized modes becomes more defined. It is true for a system with both a large magnetic effect and a small one.

Increase of the distance from the center of mass of the pendulum to the axis of rotation under both small and large initial conditions, as well as at small and large coupling coefficients, always leads to the clear manifestation of the connected (in-phase) mode. But if the initial angle of deflection of the pendulum is small, and the connection is small, both for a small magnetic effect and for a large magnetic one, then increasing the distance from the center of mass of the pendulum to the axis of rotation *s* does not contribute to the emergence of a stable localized shape.

A change in the distance from the center of mass to the axis of rotation with not too large values of the small parameter (ε <0.2 and with a large value of the coupling coefficient), does not significantly affect the localized shape of the oscillations. With a small connection and a large distance from the center of mass of the pendulum to the axis of rotation, there are significant wanderings of trajectories around the localized mode

Conclusions

An analytical and numerical study of nonlinear vibration modes in a system consisting of two connected pendulums under the influence of electromagnetic forces is carried out, and here the masses of the pendulums differ significantly (*the masses ratio is characterized by the small parameter* ε), which leads to the appearance of a localized vibration mode. Both vibration modes are constructed by the small parameter method. Influence of the system parameters to the connected and localized NNMs is analyzed. In particular, we can conclude that both NNMs are clearly manifested for not small values of the initial angles. With a very small initial angle, the localized mode does not manifest itself.

References

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