The hysteretic dynamics of a harmonically excited snap-through oscillator in the presence of additive noise excitations

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Summary. The noisy dynamics of the system called the smooth and discontinuous (SD) oscillator in its frequency domain is studied. The nonlinearity is smooth (continuous) or nonsmooth (discontinuous) depending on the value of the smoothness parameter α. It was observed that the hysteretic region decreases with a higher correlation in noise. Furthermore, an increase in the smoothness parameter in the presence of noise also has the effect of reducing the hysteretic region.

Introduction

The study of discontinuous nonlinear systems subjected to stochastic excitations using non-smooth transformations, approximate analytical methods, numeric-analytical methods and numerical methods are few [1]. Most of these studies were carried out with the assumption of noise being uncorrelated in nature. However, white noise is a mathematical abstraction and the noise occurring in real physical systems often has a finite bandwidth. Furthermore, the problem is more involved in the vicinity of a bifurcation as the presence of a subcritical bifurcation in a deterministic dynamical system leads to the formation of a pair of stable solutions separated by an unstable branch, thus creating a hysteretic loop when a forward and a backward bifurcation is carried out. The dynamics follows a particular bifurcation branch until the critical value of a control parameter is reached, followed by a sudden transition to another branch. However, if the integration is carried out using the varying the same control parameter in the reverse manner, the critical transition occurs at a different value. Noise has the potential to influence these critical parameters [2]. The dependence of the size of the hysteresis loop on the intensity and correlatedness of noise is thus important to demarcate the stability regime of the dynamical system. The complexity of the problem is increased when there is an interplay of nonsmoothness in the dynamical system. To decipher the underlying dynamics of such noise-induced transitions, an archetypal snap-through truss oscillator has been considered. The dynamics of the system can be considered to be smooth or non-smooth depending on the smoothing parameter α in the problem. The non-dimensionalized equations of motion of this system are given by Eq. 1. In the smooth regime, the system is observed to bear significant resemblance to the Duffing oscillator, exhibiting the standard dynamics governed by the hyperbolic structure associated with the double well. At the discontinuous limit however, the dynamics diverges substantially. The loss of local hyperbolicity leads to a jump in the velocity flow when crossing from one well to another. The system has coexisting attractors in the presence of damping and external excitations.

$$\ddot{x} + 2\zeta\dot{x} + x\left(1 - \frac{1}{\sqrt{x^2+\alpha^2}}\right) = f_0\cos\omega t + n\lambda(t), \tag{1}$$

where ζ is the damping coefficient, f_0 is the amplitude of forcing and ω is the corresponding forcing frequency. $\lambda(t)$ depicts the Ornstein Uhlenbeck noise and n denotes its intensity.

Results and discussions

The results of direct numerical simulations using forward integration has been shown in Fig. 1. It can be observed that in the presence of noise, the upper branch near $\simeq \omega = 0.8$ is continued for a larger magnitude of frequency before there is a transition to the lower amplitude branch. The transition is depicted by a saddle-node bifurcation of limit cycles, leading to the creation of a stable and and unstable branch of bifurcations [4]. It is also observed that the lower amplitude branch has a marked increase in the observed maximum amplitude of oscillations post the saddle-node bifurcation. Thereafter, the effect of correlatedness in noise on the transition to the lower branch is studied. The intensity of noise, mean and variance are kept constant at 0.5,0 and 0.1 and the mean reverting speed is varied, here denoted by θ. The system is assumed to be ergodic and the maximum amplitude is computed with one simulation from an ensemble of realizations where a long time history of 10^3 cycles is considered. It was observed that the transition occurred for a lower magnitude of frequency when the mean reversion speed of the Ornstein Uhlenbeck process was higher *i.e.* higher correlatedness in noise led to a proponent of transition to the lower amplitude branch when a forward numerical integration is carried out; see Fig. 2.

Furthermore, the effect of varying the smoothing parameter in the SD oscillator on the transition to the lower branch is studied. Fig. 3. is a depiction of the variation of nonlinear restoring force as a function of α. It can be observed that the restoring force becomes stiff as α decreases and is discontinuous at $\alpha = 0$ [3]. Fig. 4 depicts the variation in the control parameter leading to the occurrence of the nonsmooth saddle-node bifurcation. As α decreases, the bifurcation is delayed to a higher parameter of ω. Also, a corresponding increase in the maximum amplitude of the response has been observed post occurrence of the bifurcation. For the simulations, the intensity of noise n is kept constant at 0.5 and the process parameters of the Ornstein Uhlenbeck process are $0, 0.5, 0.3$ respectively. The step size of integration is fixed at 0.001.

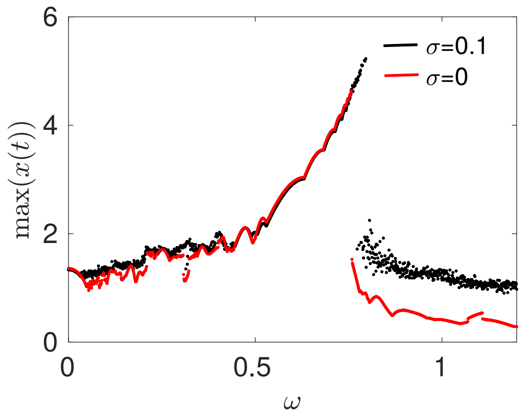

Figure 1: The frequency response of the SD oscillator for $\alpha = 1.1, f_0 = 0.25, \zeta = 0.0141$ using numerical integration with stochastic Runge-Kutta method when $\sigma = 0, 0.1$.

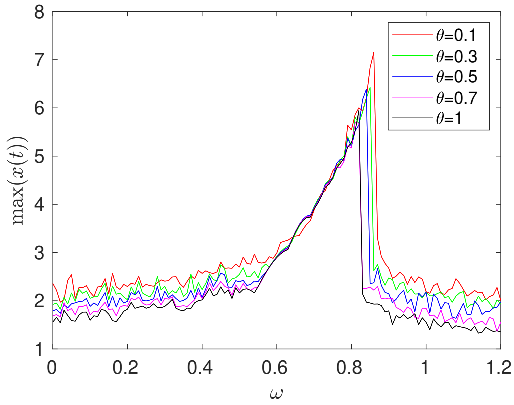

Figure 2: The frequency response of the SD oscillator with Stochastic Runge-Kutta method with different correlated noises.

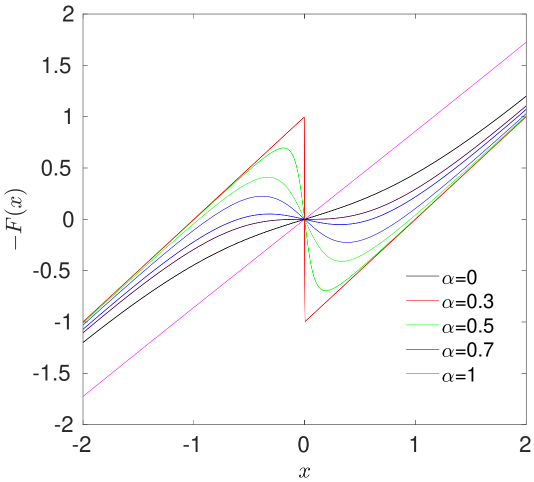

Figure 3: Nonlinear restoring force for different values of α.

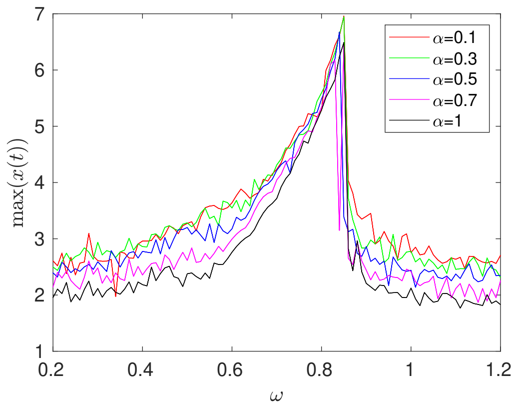

Figure 4: The frequency response of the SD oscillator with Stochastic Runge-Kutta method with different values of the smoothing parameter α.

A systematic semi-analytical and numerical approach to determine the effect of additive noise on the region of co-existence of attractors in a SD oscillator is currently under investigation. The noise is appended to the harmonic excitation in the system. The effect of the degree of nonlinearity given by α, the intensity of noise n, correlatedness of noise $\lambda(t)$, damping ζ are explored.

References

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