A novel iterative procedure for rapid dynamic integrity assessment

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Summary. A new algorithm for estimating the robustness of a dynamical system's equilibrium is presented. Unlike standard approaches, the algorithm does not aim to identify the entire basin of attraction of the solution. Instead, it iteratively estimates the so-called local integrity measure, i.e., the radius of the largest hypersphere entirely included in the basin of attraction of a solution and centred in the solution. The procedure completely overlooks intermingled and fractal regions of the basin of attraction, enabling it to provide a meaningful engineering quantity quickly. The algorithm is tested on various mechanical systems. Despite some limitations, it proved to be a viable alternative to more complex and computationally demanding methods, making it a potentially appealing tool for industrial applications.

Introduction

Local stability is one of the most critical properties of a dynamical state. Engineers heavily exploit this concept. Nevertheless, scientists dealing with dynamical systems are aware that, despite its local stability, a system might diverge from its state if subject to a perturbation sufficiently large to make it cross the boundary of its basin of attraction (BOA). However, the definition of a system's BOAs is computationally very demanding. A few methods for the identification of BOAs of dynamical systems exist [1]. Analytical methods are generally based on Lyapunov functions. However, they are not a feasible option for the majority of real applications. The cell mapping method is probably the most efficient numerical technique for BOA estimation [2]. Experimental methods are almost inexistent, except for a few exceptions [3, 4].
The objective of this study is to develop an algorithm for the robustness assessment of equilibrium points. The procedure reduces the computational cost for global stability analysis by identifying the local integrity measure (LIM) [5] only, overlooking fractal and intermingled portions of the BOA, which are hard to identify and practically less relevant.

Methodology

The algorithm is based on a simple framework. Considering a predefined region of the phase space, initially, the maximal value of the LIM is calculated, being equal to the minimal distance between the equilibrium point of interest and the boundary of the region of the phase space considered. Then, a trajectory of the system in the phase space is computed. If the trajectory does not converge to the desired solution, the LIM is estimated as the minimal distance between the equilibrium point of interest and any point of the non-convergent trajectory. The new estimated value of the LIM (an overestimate of the real LIM value) defines a hypersphere in the phase space denominated hypersphere of convergence, limiting the region of interest. If a simulation converges to the desired solution, then the LIM is not reduced in that iteration. Initial conditions of each simulation are chosen as the farthest point from any other already tracked point within the hypersphere of convergence.
In order to automatically classify the computed trajectories, the phase space is divided into cells. A trajectory is classified as converging or non-converging to the desired solution by analyzing the cells in which points of the trajectory lie. To the reduce computational time, if a trajectory reaches a cell already tracked by a previous trajectory, the simulation is interrupted; all cells containing points of the trajectory are classified according to the reached and already tracked cell. A graphical explanation of the classification procedure adopted is illustrated in Fig. 1.

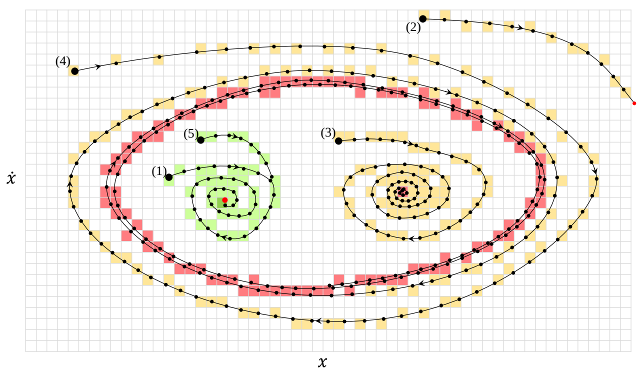

Figure 1: Illustrative examples of trajectory classification. (1) Converging to a known equilibrium; (2) leaving the considered phase space region; (3) converging to an unknown equilibrium; (4) converging to an unknown periodic solution; (5) converging to an already tracked cell.

Results

We implemented the algorithm on systems of various dimensions (up to dimension 8); the analysis illustrated that the algorithm could rapidly and efficiently estimate the LIM value in all cases studied. In particular, the first few iterations already provided a relatively accurate estimate of the real LIM value. The majority of the subsequent simulations converged to the equilibrium of interest, except few ones, which further improved the initial estimate of the LIM. Figure 2a represents the trend of the LIM estimate for the case of a Duffing-van der Pol oscillator with an attached tuned mass damper (4-dimensional system) [6]. The black line in Fig. 2a follows the described path. Light blue lines represent the LIM trend for other repetitions of the algorithm. All curves have a similar tendency. The system under study presents a stable equilibrium point (red cross in Fig. 2b) coexisting with a stable periodic solution (black line in Fig. 2b) for the considered parameter values. We remark that, in Fig. 2b, tracked points are projected on a section of the phase space, which makes it appear that red dots are within the hypersphere of convergence (green dashed line) while they are not.

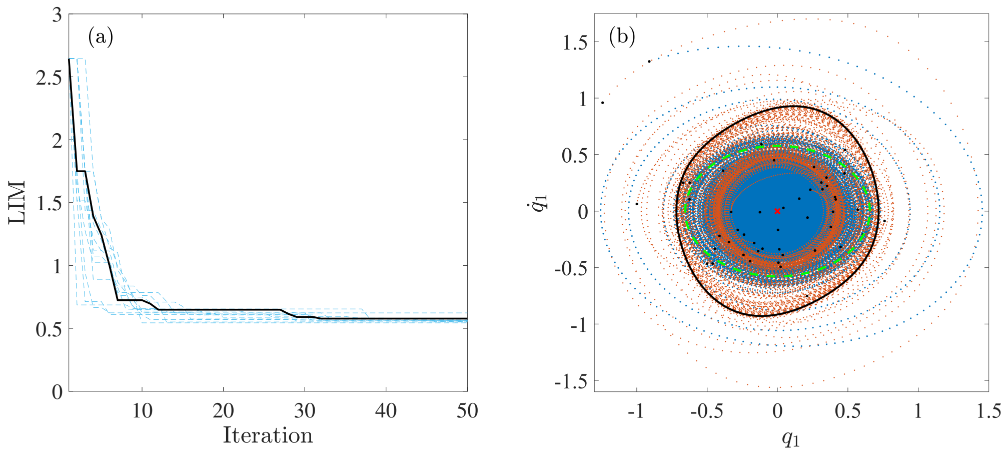

Figure 2: (a) LIM estimated value; (b) projection of the points tracked during the computation; blue and red points: converging and non-converging points, respectively, dashed green line: section of the hypersphere of convergence.

Conclusions

In this study, a new algorithm for estimating the robustness – dynamical integrity – of a stable equilibrium was developed. The algorithm utilizes an approach different from existing numerical methods for global analysis. It does not aim at studying the whole basin of attraction of a solution; instead, it directly tries to estimate the local integrity measure (LIM). From an engineering perspective, this quantity has obvious relevance for the safety of a dynamical system. The obtained results suggest that the proposed algorithm is a viable option for the robustness assessment of an equilibrium point. In particular, thanks to its quickness, it has the potentiality to be utilized in industrial environments, where rapid solutions are generally pursued. Future research developments should aim to make the algorithm utilizable for the robustness estimation of other kinds of solutions, such as periodic motions.

Acknowledgments

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