# Low Voltage Operation of Vilnius Chaotic Oscillator

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*Summary*. The paper is dedicated to the numerical study of nonlinear oscillations exhibited by the Vilnius chaotic generator. The need for practical applicability of the mentioned generator in chaotic communications defined the necessity to study the low-power operation of the circuit, designed initially to be powered from high voltage sources. The bifurcation map and corresponding one-parameter diagrams reveal complex nonlinear dynamics and regions of robust chaotic oscillations.

## 1. Introduction

The use of chaotic signals generated by various electronic systems has been growing for the last several decades, and it is considered a major candidate for future technologies. Chaotic oscillators have found applications in communications [1]-[3], random number generators [4], chaotic computing [5] and other fields. The main requirements for the practical application of a chaotic oscillator are the circuit's simplicity, the variety of different chaotic modes, and robustness.

The rapid expansion of the Internet of Things (IoT) creates new challenges to electronic circuit design. Numerous applications like environmental sensing and healthcare monitoring impose the need for long-term autonomous operation of electronic modules. In [6], the authors conclude that batteries are the most promising energy source for IoT applications, particularly wireless sensor networks. Energy harvesting techniques suffer from insufficient power levels and often are unpredictable and thus insecure. This, in turn, implies DC voltage levels available to power electronic circuits being in the range of several volts.

Furthermore, industry trends and the natural need for miniaturization of sensor nodes create new challenges in power circuit design. The battery life may determine the useful lifespan of the device in cases where the batteries are not replaceable for any number of reasons. The requirement of long autonomous operation of IoT devices dictates additional restrictions on chaotic generators, including energy efficiency and low power operation.

The current study is dedicated to a comprehensive analysis of the low-voltage operation of the Vilnius chaos oscillator. First presented in 2004 [7], this circuit has been intended to operate from a 20 V source. However, several attempts have been made to adapt this oscillator to IoT applications [3], [8]. Thus, there is a need to study the nonlinear dynamics of the system operated at much lower voltages than it originally was supposed to.

This paper is organized as follows. The second section is devoted to the description of the schematic and analytical model of the Vilnius oscillator. The third section presents the nonlinear analysis of the system dynamics under study at low voltage operation. The last section is devoted to the overall conclusions and suggestions on the applicability of this type of chaotic oscillator.

## 2. Vilnius Oscillator Schematic and Model

The schematic of the Vilnius chaotic oscillator is depicted in Fig.1. The circuit is easy to implement and modify, as it includes no unique components, just the off-the-shelf operation amplifier, diode, capacitors, inductors and resistors.

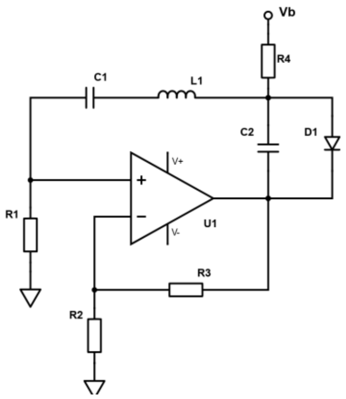


Figure 1: The schematic diagram of the Vilnius oscillator

This oscillator exhibits complex behaviour under specific component parameters despite being relatively simple. The frequency of the waveforms observed in the circuit is determined by reactive components *C1*, *L1*, and *C2*. Thus, it is possible to adapt the scheme for the frequency range of interest. Diode *D1* is the mandatory nonlinear element needed for the chaotic oscillator. It can be a general purpose silicon diode like 1N4148 or Schottky diode. Also, no special requirements apply for the operational amplifier. A reader can use, for example, an LTspice computer simulation program to get quick insight into the operation of the circuit and to observe typical waveforms. However, even such a brief study reveals that the number of parameters that affect the system's dynamics is too large to adopt a trial and error approach when the robust chaos is of interest. That is why the comprehensive study of nonlinear dynamics using bifurcation diagrams must be performed to identify the regions of chaotic behaviour for further practical implementation of the Vilnius oscillator.

In this study, a system of equations initially developed in [1] is used to describe the dynamics of the system:

$$\frac{dx}{dt} = y \tag{1}$$

$$\frac{dy}{dt} = ay - x - z \tag{2}$$

$$\varepsilon\frac{dz}{dt} = b + y - c(e^z - 1), \tag{3}$$

where $x = \frac{V_{C1}\cdot q}{k_B\cdot T}$; $y = \frac{I_{L1}\cdot q\cdot\sqrt{\frac{L}{C_1}}}{k_B\cdot T}$; $z = \frac{V_{C2}\cdot q}{k_B\cdot T}$; $a = \frac{(k-1)\cdot R_1}{\sqrt{\frac{L}{C_1}}}$; $b = \frac{I_{R4}\cdot q\sqrt{\frac{L}{C_1}}}{k_B\cdot T}$; $c = \frac{I_S\cdot q\cdot\sqrt{\frac{L}{C_1}}}{k_B\cdot T}$; $\varepsilon = \frac{C_2}{C_1}$;

$k = 1 + \frac{R_2}{R_1}$; $I_{R4} = \frac{V_b}{R4}$; $k_B$ is Boltzmann's constant; T is the temperature in Kelvins.

The system's parameters of interest are *a*, *b* and *ε*, which could be adjusted by input voltage, variable capacitor $C_2$ and variable resistors $R_1$, $R_2$, $R_3$, and $R_4$.

The study of the nonlinear dynamics of the Vilnius oscillator will be provided based on one and two-parameter bifurcation diagrams that allow estimation of the system's mode of operation for various combinations of component values. However, this approach requires obtaining the discrete-time model of the original oscillator. The models could be constructed by application of the Poincare map. In the case of the Vilnius oscillator, $y=0$ is selected as the Poincare plane. Thus, the trajectories crossing this plane from one side will define the sampled model and provide the required information on the periodicity of the regimes under study (see Fig.2.).

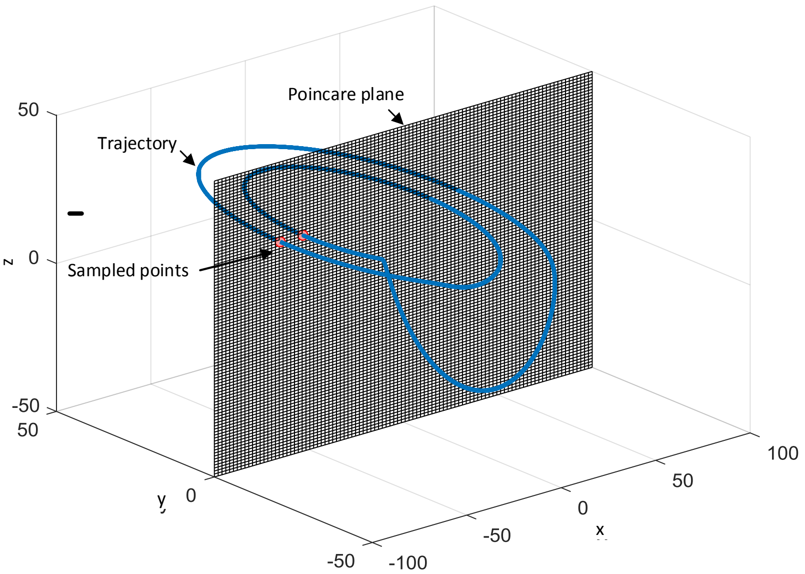


Figure 2. The introduced Poincare plane for obtaining a sampled model of the Vilnius oscillator

All the calculations are made utilizing specially prepared MATLAB scripts, including the solution of the equations with the Runge-Kutta (4,5) method, implemented in the *ode45* function. As the construction of bifurcation diagrams in the wide range of system parameters could be a time-consuming task, the Parallel Computing Toolbox functionality has been intensively utilized to efficiently distribute the computation tasks between all available physical cores of the computer. The results of calculations and the analysis of the obtained diagrams are provided in the next section.

## 2. Nonlinear Dynamics at Low Voltage Operation

The main goal of the current research is to study the dynamics of the Vilnius oscillator, operating in the voltage range viable for practical applications in wireless sensor networks. Thus, one of the parameters under study is *b*, which is directly connected to the current $I_{R4}$ and voltage $V_b$. The task is to identify the lowest border of supply voltage at which the system could exhibit robust chaotic oscillations and study the qualitative changes in the dynamics as *b* is varied.

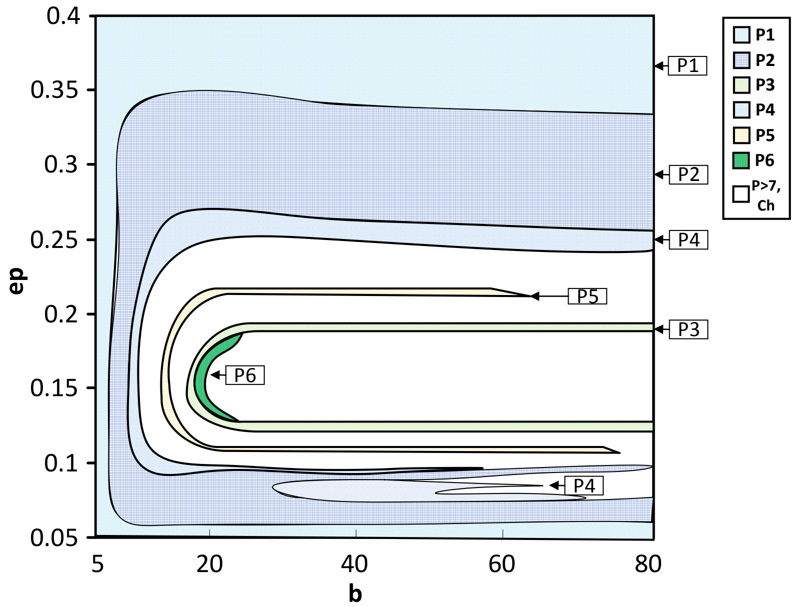


Figure 3: Two parameter bifurcation diagram for *a*=0.3; *b*=5-89; *ε*=0.05-0.4.

The subsequent study of the nonlinear dynamics of the oscillator is based on the construction of a two-parameter bifurcation diagram of the system (bifurcation map) and analysis of the corresponding one-parameter bifurcation diagrams as the cross-sections of the map.

The second parameter, chosen to be varied for the bifurcation map, is *ε*, physically dependent on the capacitor $C_1$ and $C_2$ values. The obtained map for *ε*=0.05-0.4 and *b*=5-80 is shown in Fig. 3. Periodic operation modes are depicted up to period-7, and other high-periodic regimes and chaos are shown as white regions.

The bifurcation map demonstrates that for low values of *b* (defined by input voltage), the system's dynamics are mainly periodic- exhibiting period-1 to period-4 oscillations for all values of *ε*. From a practical point of view, the system could not be used as the generator of chaotic oscillations for extremely low voltages (defined by *b*<12). This is also illustrated in Fig.4. The brute-force bifurcation diagram shows the clear transition from P1 to P4 and back to P1 through subsequent period doublings without any signs of chaotic oscillations in the main branches. No coexisting chaotic attractors could be detected either.

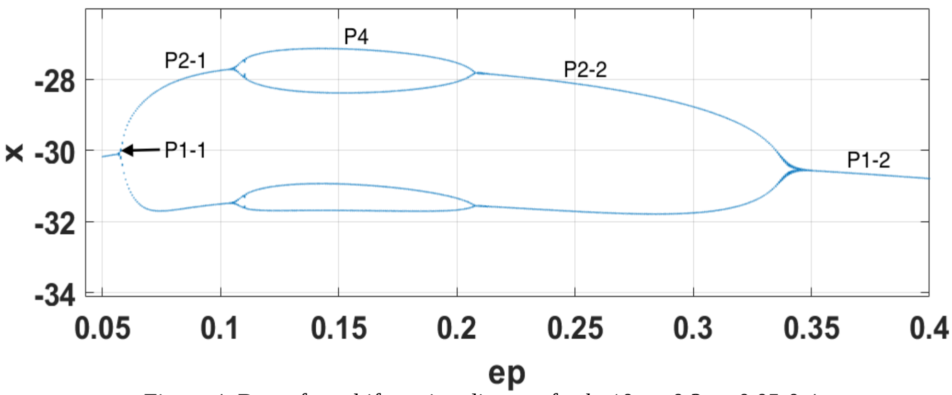


Figure 4: Brute-force bifurcation diagram for *b*=10; *a*=0.3; *ε*=0.05-0.4.

However, there is a definite border (*b*>12), where the system becomes chaotic for a relatively wide range of *ε* values. There could be intermittent chaotic dynamics (with various periodic windows) or robust chaos without interrupting periodic modes. Fig. 4 shows classical period-doubling routes to chaos enclosing several intervals of robust chaotic oscillations (RCh1-RCh4). It can be inferred that setting system parameters within the ranges indicated - ε=0.1-0.23 - should guarantee stable chaotic oscillations without the issue of transitioning to some periodic mode due to external noise or fluctuations in the component's values.

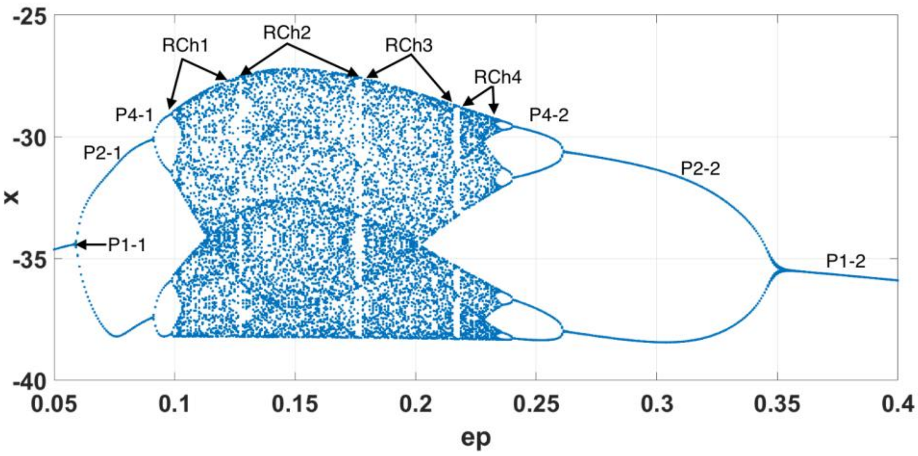


Figure 5: Brute-force bifurcation diagram for $b$=15; $a$=0.3; $\varepsilon$=0.05-0.4.

The system's behaviour remains similar for higher values of $b$, as shown in Fig. 6. However, the amplitude of $V_{C1}$ (variable $x$) increases. Compared to the previous diagram, the system exhibits a non-smooth transition to chaos from the left side. This phenomenon could be explained by the presence of a diode in the circuit, defining the non-smooth switchings as the voltage rises and a certain threshold is reached. It has been noticed that the systems with non-smooth bifurcations could exhibit robust chaotic oscillations. However, in this case, we still observe some periodic windows in the chaotic regions and the transition to stable periodic regimes as $\varepsilon$ reaches 0.24.

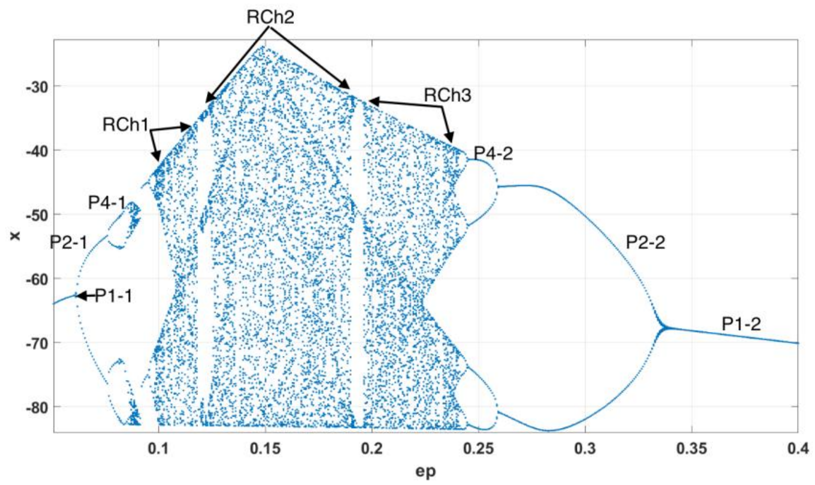


Figure 6: Brute-force bifurcation diagram for $b$=50; a=0.3; ε=0.05-0.4.

The second part of the investigation is dedicated to constructing the diagrams for fixed values of $\varepsilon$ and varying the parameter $b$. For $\varepsilon < 0.12$, a wide diversity of dynamical patterns could be observed, as $b$ is varied. Fig. 7. shows period doublings, intermittent chaos and wide periodic windows. These regimes could not be relevant for practical applications, as any slight supply voltage variations could cause unpredicted transitions between different modes, compromising the whole system's security. Chaotic attractors observed in the corresponding regions (see Fig.7- Ch) are not dense enough, indicating the insufficient level of diversity required by practical communication systems.

However, the further increase in parameter $\varepsilon$ leads to the formation of several robust chaotic regions (see Rch1 and RCh2 in Fig.8.) with acceptable characteristics and durability to parameter changes. But some periodic windows are still present in the defined parameter range (see, e.g. P3 window in Fig.8.). As we are interested in the low-voltage operation of the system, the region of $b$=12-28 is the most appropriate for the proposed applications.

Setting the $\varepsilon$=0.15 in the middle of the predicted chaotic region in the bifurcation map (see Fig.3), it is possible to obtain the diagram where all periodic windows shrink, and the continuous robust chaotic area is formed. This is demonstrated in Fig.9.

As it could be deduced from Fig.3, the further increase of ε leads to the deterioration of chaotic dynamics and diagrams, similar to those shown in Fig.7 and Fig.8 could be obtained.

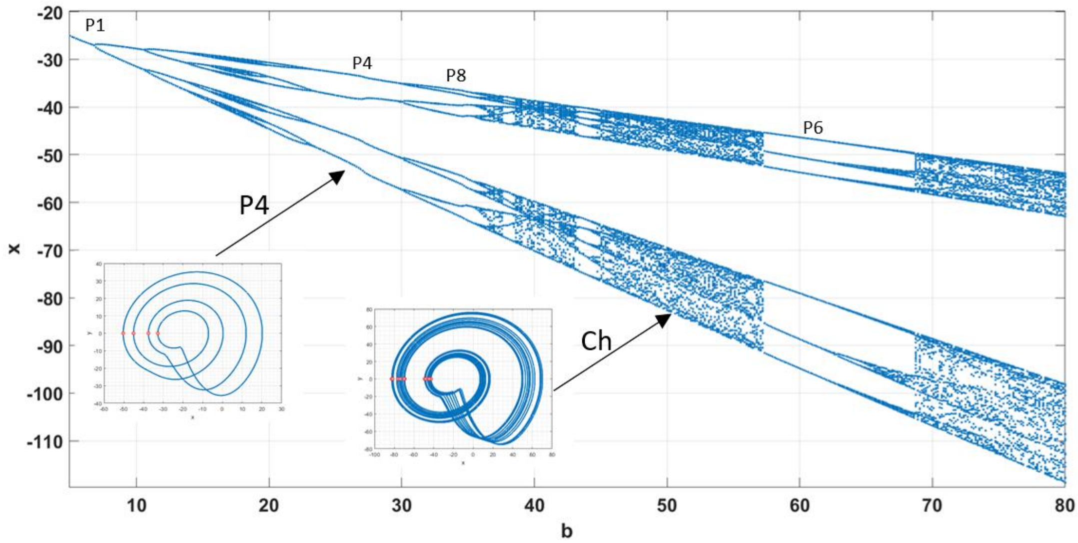


Figure 7: Brute-force bifurcation diagram for a=0.3; ε=0.1; *b*=5-80

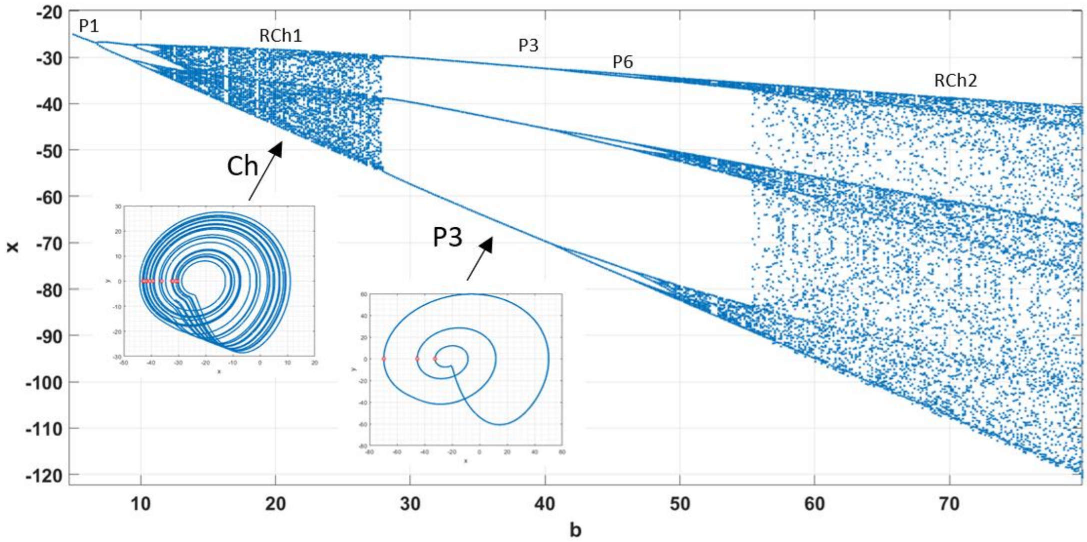


Figure 8: Brute-force bifurcation diagram for a=0.3; ε=0.12; *b*=5-80

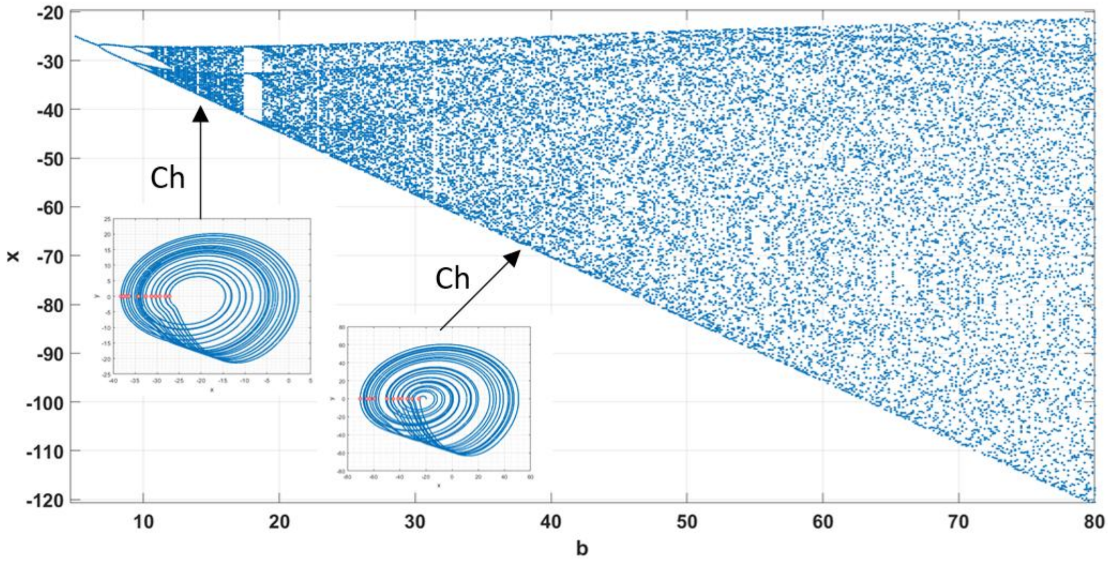


Figure 9: Brute-force bifurcation diagram for a=0.3; ε=0.15; *b*=5-80

## 3. Conclusions

One of the key parameters defining the applicability of the chaotic oscillators to secure communication systems is energy efficiency (possibility to operate in low-power modes) and robustness (insusceptibility to slight parameter variations and noise). Thus, investigating the dependence of nonlinear dynamics of these kinds of generators on the operation voltage is crucial for practical solutions.

The paper demonstrated the numerical study of possible chaotization scenarios and various nonlinear phenomena observed in the Vilnius chaotic oscillator as system parameters vary. It has been shown that the construction of the bifurcation map allows for the convenient identification of the most appropriate parameter ranges to be used for obtaining robust chaotic modes of operation. The main conclusion is that the Vilnius oscillator could robustly generate chaotic signals required in secure communications, even in low-voltage modes of operation. However, the other systems parameters (e.g. $\varepsilon$) should be fine-tuned to exclude transitions to any periodic regime.

Further study could contain the laboratory experiments allowing the verification of numerical simulations and practical estimation of energy efficiency of the proposed robust chaotic regimes.

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