Non-smooth Reduced Interface Models for Co-simulation of Mechanical Systems

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Summary. In a co-simulation setup, an entire system is divided into multiple subsystems modelled separately which may be simulated with different solvers and time scales suited to the subsystems. These subsystems are then interfaced and coupled in order to simulate the whole system. For interactive rates, the non-iterative coupling schemes are used which are prone to instability. This problem can become more challenging when non-smoothness is introduced into the modelling. In this work, an efficient model-based coupling approach is introduced which is able to account for contact state transmissions between the communication points, resulting in more stable and accurate results compared to other coupling methods. To further clarify the advantages of the proposed approach, an illustrative example is also presented.

Introduction

Co-simulation is generally used to simulate systems consisted of different sub-systems with different nature; like multibody systems interacting with hydraulics, electronics or even other mechanical sub-systems which are modelled with different phenomena. The main challenges for modelling such complex systems are related to the interfacing of these subsystem components. One challenge will raise when contact interactions are included in the modelling which introduces non-smoothness to the dynamics and increases the complexity of the model. Contact interactions which are also known as unilateral constraints, transform the underlying dynamics problem by adding inequalities into the dynamic formulation and turn it to a linear or a non-linear complementarity problem. In real-time applications, efficiency of the simulation is also a requirement, where we are usually interested in non-iterative co-simulation because there is not enough time to go back in time and resolve for a time step. Stability is a key element in non-iterative co-simulation which is related to the size of the macro time step. To make a co-simulation more stable, we can estimate the subsystem variables between the communication points and couple the subsystems. One promising way to estimate these variables is to use model-based coupling in which the mechanical system is reduced into an interface model that emulates the dynamics of the mechanical system at the interface. However, there still exist some aspects in model-based co-simulation which can be further improved.

In this work, we are looking at non-smooth interface models. The idea is that to add some information about the unilateral contacts of the full model in the interface model. This information would help us to estimate contact state changes during macro time step using an interface model. As will be illustrated, this will enable us to successfully simulate some dynamic behaviours which cannot be captured otherwise.

Dynamic formulation of the interface model

Consider a multibody system subjected to unilateral and bilateral constraints where friction is also neglected. In a general form, the dynamic equations of a multibody system can be written in the impulse-momentum level as

$$\hat{\mathbf{M}}\mathbf{v}^{+} + h\hat{\mathbf{c}} = \mathbf{M}\mathbf{v} + h\mathbf{f}_{\mathrm{a}} + h(\mathbf{A}_{\mathrm{i}}^{\mathrm{T}}\boldsymbol{\lambda}_{\mathrm{i}}^{+} + \mathbf{A}_{\mathrm{b}}^{\mathrm{T}}\boldsymbol{\lambda}_{\mathrm{b}}^{+} + \mathbf{A}_{\mathrm{u}}^{\mathrm{T}}\boldsymbol{\lambda}_{\mathrm{u}}^{+}) \tag{1}$$

where the contents of the *modified mass matrix* $\hat{\mathbf{M}}$, and *modified Coriolis and centrifugal terms* $\hat{\mathbf{c}}$ are determined with the time discretization method used [1]. $\mathbf{q} = \mathbf{q}(t_{\mathrm{k}})$ and $\mathbf{v} = \mathbf{v}(t_{\mathrm{k}})$ are the generalized coordinate and velocity of the system which are related by the transformation $\dot{\mathbf{q}} = \mathbf{N}\mathbf{v}$ and are known at the instant t_{k}. Moreover, $\mathbf{f}_{\mathrm{a}}$ and $\boldsymbol{\lambda}_{\mathrm{i}}$ are the applied and interface forces and $\mathbf{A}_{\mathrm{i}}$ is the corresponding interface Jacobian matrix. Similarly, $\boldsymbol{\lambda}_{\mathrm{b}}$ and $\boldsymbol{\lambda}_{\mathrm{u}}$ are the bilateral and unilateral constraint forces and their corresponding Jacobian matrices are $\mathbf{A}_{\mathrm{b}}$ and $\mathbf{A}_{\mathrm{u}}$. Then, considering a time step of size h, its configuration, velocity and unknown constraint forces in the next time-step are shown with $+$ sign. The constraint and the interface velocity arrays are related to the generalized velocity array $\mathbf{v}$ as $\mathbf{w}_{\mathrm{j}} = \mathbf{A}_{\mathrm{j}}\mathbf{v}$ $(\mathrm{j} \equiv \mathrm{i}, \mathrm{b}, \mathrm{u})$, which can be discretized through time using the first-order Taylor series expansion as

$$\mathbf{w}_{\mathrm{j}}^{+} = \mathbf{A}_{\mathrm{j}}\mathbf{v}^{+} + h\dot{\mathbf{A}}_{\mathrm{j}}\mathbf{v} \tag{2}$$

Knowing that $\mathbf{w}_{\mathrm{b}} = \mathbf{0}$, Eqs. (1) and (2) can be cast into a single matrix form which alongside the complementarity condition of the unilateral constraints will form a *Mixed Linear Complementarity Problem* (MLCP) as,

$$\left\{ \begin{array}{l} \begin{bmatrix} \hat{\mathbf{M}} & -\mathbf{A}_{\mathrm{b}}^{\mathrm{T}} & -\mathbf{A}_{\mathrm{u}}^{\mathrm{T}} & -\mathbf{A}_{\mathrm{i}}^{\mathrm{T}} \\ \mathbf{A}_{\mathrm{b}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{\mathrm{u}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{\mathrm{i}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}^{+} \\ h\boldsymbol{\lambda}_{\mathrm{b}}^{+} \\ h\boldsymbol{\lambda}_{\mathrm{u}}^{+} \\ h\boldsymbol{\lambda}_{\mathrm{i}}^{+} \end{bmatrix} + \begin{bmatrix} h(\hat{\mathbf{c}} - \mathbf{f}_{\mathrm{a}}) - \mathbf{M}\mathbf{v} \\ h\dot{\mathbf{A}}_{\mathrm{b}}\mathbf{v} \\ h\dot{\mathbf{A}}_{\mathrm{u}}\mathbf{v} \\ h\dot{\mathbf{A}}_{\mathrm{i}}\mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{w}_{\mathrm{u}}^{+} \\ \mathbf{w}_{\mathrm{i}}^{+} \end{bmatrix} \\ \mathbf{0} \leq \mathbf{w}_{\mathrm{u}}^{+} \perp \boldsymbol{\lambda}_{\mathrm{u}}^{+} \geq \mathbf{0} \end{array} \right. \tag{3}$$

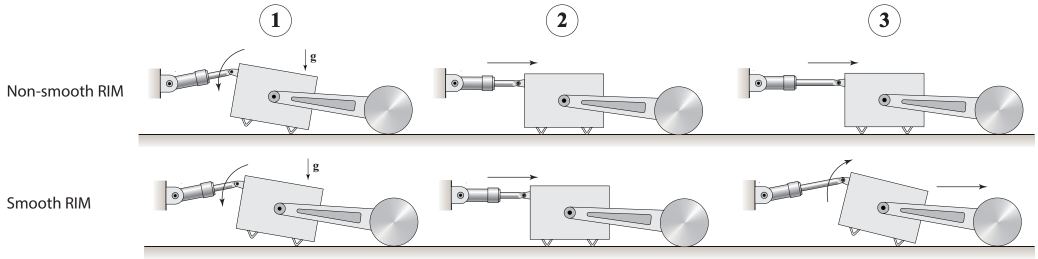

Figure 1: The manoeuvre sequence of the multibody system using smooth and non-smooth RIMs.

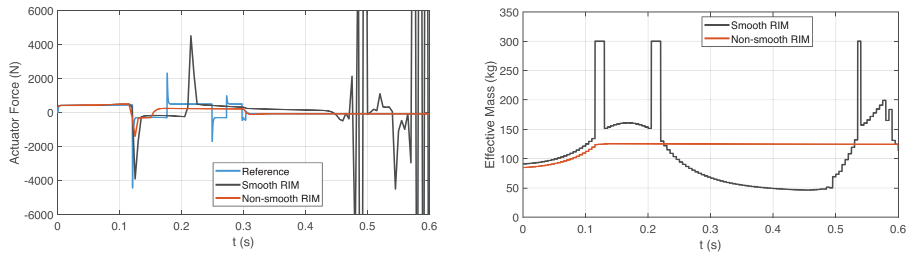

Figure 2: Actuator force (left) and effective mass of the reduced model (right).

The final goal is to obtain a reduced interface model that represents dynamics of the mechanical system associated with interface degrees of freedom parameterized by $\mathbf{w}_\mathrm{i}$. From the first and second rows in Eq. (3), the generalized velocities $\mathbf{v}^+$ and bilateral constraint impulse $h\boldsymbol{\lambda}_\mathrm{b}^+$ can be determined and be substituted into the third and forth rows. Then, the MLCP in Eq. (3) can be rewritten as

$$\left\{ \begin{array}{c} \begin{bmatrix} \mathbf{H}_\mathrm{uu} & \mathbf{H}_\mathrm{ui} \\ \mathbf{H}_\mathrm{ui}^\mathrm{T} & \mathbf{H}_\mathrm{ii} \end{bmatrix} \begin{bmatrix} h\boldsymbol{\lambda}_\mathrm{u}^+ \\ h\boldsymbol{\lambda}_\mathrm{i}^+ \end{bmatrix} + \begin{bmatrix} \mathbf{b}_\mathrm{u} \\ \mathbf{b}_\mathrm{i} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_\mathrm{u}^+ \\ \mathbf{w}_\mathrm{i}^+ \end{bmatrix} \\ \mathbf{0} \leq \mathbf{w}_\mathrm{u}^+ \perp \boldsymbol{\lambda}_\mathrm{u}^+ \geq \mathbf{0} \end{array} \right. \tag{4}$$

Eq. (4) is the *reduced interface model* (RIM) of non-smooth multibody systems which includes the complementarity condition and is able to capture the contact state transformation between the communication points. This reduced model may be referred to as *non-smooth RIM*. The advantages of this formulation will be demonstrated in the next section through numerical simulation.

Example and discussion

A planar model of a hydraulically actuated box with two bottom wedges connected to a disc through a revolute joint was used to illustrate the efficiency of the proposed RIM in a co-simulation setup. The hydraulic actuator is the first subsystem and set of the box and the disc are regarded as the second subsystem. Also, the box and the disc are allowed to slide without friction. In the manoeuvre shown in the Fig. 1, the box is initially a bit rotated around the right wedge. Then, the box will be pushed forward by the actuator under the effect of gravity. According to the reference solution (co-simulation using *zero order hold* setup with macro step size of $h = 0.2$ ms), as the multibody setup moves forward, the box rotates counterclockwise so that bot of its wedges touches the ground and the system continues to move forward horizontally. However, when the *smooth RIM* method introduced in [2] is used for model-based co-simulation, the contact points of the box are treated as bilateral constraints during the macro step which makes the system over-constrained when both wedges touch the ground. This can be also seen in Fig. 2 where the effective mass of the reduced model is depicted and, at the instances that both wedges are in contact with the ground, the effective mass shows a sudden rise in value due to the over-constraining. The over-constraining and the resulting high effective mass values will store excessive pressure in the hydraulic actuator and makes the system unstable so that in some instances, the box jumps off the ground and the whole simulation becomes unstable. The sudden rise in actuator force is also depicted in Fig. 2. However, by employing the proposed method, the contact detachment is taken into account in the reduced model and as it is evident from Fig. 2, the behaviour captured by the non-smooth RIM is similar to the reference solution. It should be mentioned that both model-based simulations were simulated with $h = 5$ ms.

References

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[2] Peiret A., Gonzalez F., Kovecses J., Teichmann M. (2020) Co-Simulation of Multibody Systems With Contact Using Reduced Interface Models. *J. Comput. Nonlinear Dynam* **15(4)**:041001.