Non-smooth nonlinearities as restoring forces in a mass-in-mass cell

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Summary. The energy exchanges between particles of a mass-in-mass cell with nonlinear restoring forcing function are studied. Two types of non-smooth compound nonlinearities are considered as restoring forcing function acting on the inner mass. The governing equations of the system are treated analytically with the time multiple scale method in order to find the slow invariant manifold (SIM) as well as singular and equilibrium points of the system. Finally, quasi-analytical system responses are confronted with numerical ones obtained by direct time integration. Interestingly, the SIM of the system possesses several unstable zones and the frequency response curves exhibit isolated solutions.

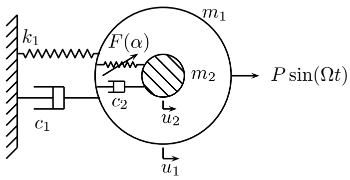

Figure 1: Model of the mass-in-mass cell.

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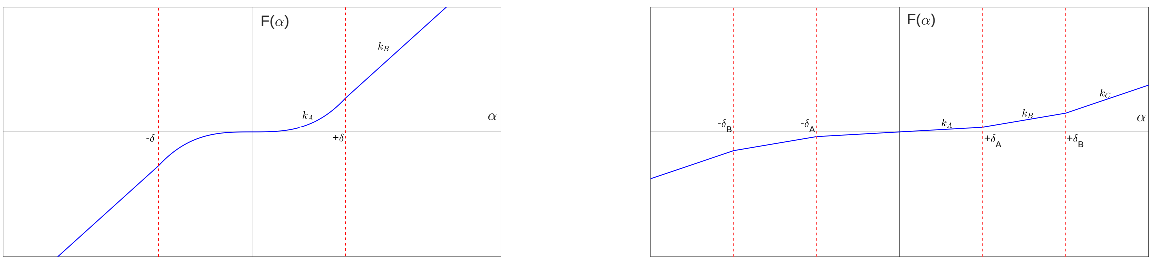
The studied system is represented in Fig. 1. It is composed of an outer mass, m_1, grounded by a constant stiffness k_1 and a damping coefficient c_1. The outer mass is coupled to an inner mass, m_2, with a damping coefficient c_2 and a nonlinear restoring force $F(\alpha)$ which depends on the relative displacement of the two masses. The general governing equations of a 2-dof meta-cell systems are similar to a main system coupled with an attached nonlinear absorber [1].

$$\begin{cases} m_1\ddot{u}_1 + k_1u_1 + c_1\dot{u}_1 + F(u_1-u_2) + c_2(\dot{u}_1-\dot{u}_2) = P\sin(\Omega t) \\ m_2\ddot{u}_2 + F(u_2-u_1) + c_2(\dot{u}_2-\dot{u}_1) = 0 \end{cases} \tag{1}$$

Two types of non-smooth compound nonlinearities are considered for $F(\alpha)$, which are plotted in Fig. 2:

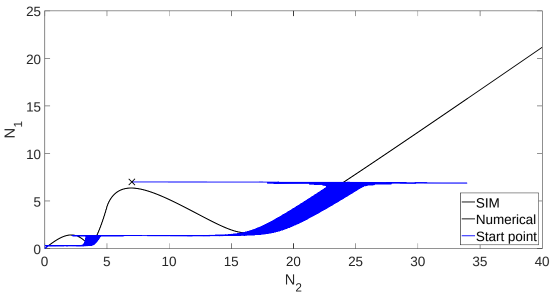


(a) (b)

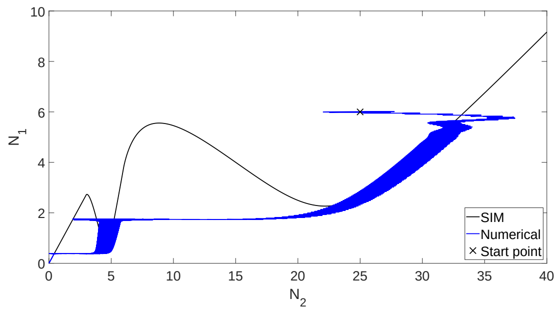
Figure 2: Considered nonlinearities for the restoring force $F(\alpha)$: a) Pure cubic and linear; b) Piece-wise linear.

A nondimensionalized time $\tau = \sqrt{\frac{k_1}{m_1}}t$ and the coordinates of relative displacement and the center of masses of two oscillators are introduced to the system variables. After this, the complex variables of Manevitch [2] are applied. A Galerkin method based on truncated Fourier series, involving the first harmonics of the system, is employed and a time multiple scales method [3] is carried out in order to find the SIM and the dynamical characteristic points of the system [1]. In more details, the SIM is detected at fast time scale, while the equilibrium and singular points are detected at slow time scale, leading to prediction of periodic and non-periodic regimes [4].

For a given set of system parameters, two SIMs of the system corresponding to the restoring forcing functions of Figs. 2a and 2b are illustrated in Figs. 3a and 3b, respectively. In these plots, N_1 and N_2 represent the amplitudes of the center of mass and of the relative displacement of the two oscillators, respectively. These figures are accompanied by free responses obtained by direct numerical time integration of Eq. 1 without external excitation. In both figures, it is seen that, starting from the initial condition, the system follows the SIM and it bifurcates twice before going to the rest position (expected position for free responses of the damped system).



(a)



(b)

Figure 3: SIM and corresponding numerical free responses: a) System with the nonlinearity of Fig. 2a and initial conditions $(w, v, \dot{w}, \dot{v}) = (7, 7, 0, 0)$; a) System with the nonlinearity of Fig. 2b and initial conditions $(w, v, \dot{w}, \dot{v}) = (6, 25, 0, 0)$.

Furthermore, the equilibrium points can be determined for sweeping de-tuning parameter σ. This parameter represents a sweep of the frequency of excitation around the frequency of the outer mass (the system is studied around a $1:1$ resonance). Thus, frequency responses curves can be obtained for a given forcing amplitude, which permits identifying the position of all equilibrium points and predict the amplitude levels of the cell. As an exemple, Fig. 4 shows the frequency response curve of the system corresponding to the nonlinearity of Fig. 2a for a given forcing amplitude. In this figure, the equilibrium points located in the unstable zones of the SIM are in green colour.

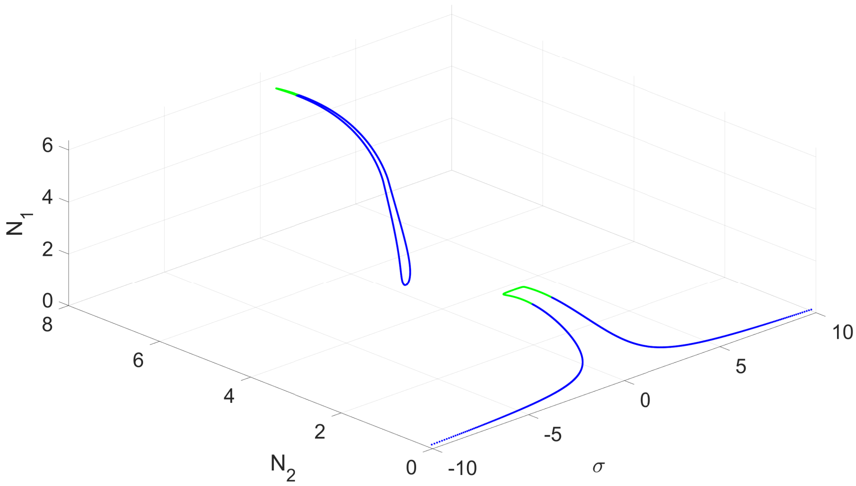


Figure 4: Detected equilibrium points of the system with respect to the de-tuning parameter σ for the system with restoring forcing function of Fig. 2a. σ is a de-tuning parameter representing a sweep of the frequency of excitation around the frequency of the outer mass (the system is studied around a $1:1$ resonance).

The developments presented here provide design tools for tuning the non-smooth nonlinearities of the inner mass in order to control the system, which presents different leves of energy reduction, acting like a gearbox. As a perspective, these theoretical results will be compared with a designed experimental test setup.

References

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