Energy model of free vibrations and resonance in elastic bodies

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Mechanics based on the concepts of space, time, and energy with a single elastic modulus and a new average stress scale that takes into account the energy of particles in the initial state is used to analyze the energy features of free vibrations and resonance. When describing motion in the Lagrange form, the elastic energy is determined by the quadratic invariant of the tensor, whose components are partial derivatives of Euler variables with respect to Lagrange variables. The increment of the invariant due to elastic deformation is represented as the sum of two scalars, one of which depends on the average value of the relative lengths of the edges of particles in the form of an infinitesimal parallelepiped, the second is equal to the standard deviation of these lengths from the average value. It is shown that each of the scalars can be represented as two dimensionless kinematic analogs of elastic energy that participate in the implementation of the law of conservation of energy in different ways. One part of the elastic energy passes into kinetic energy and participates in the implementation of the law of conservation of energy for the body as a whole, taking into account external forces. The second part is not converted to kinetic energy, leads to a change in the deformed state of the particles in accordance with the equations of motion, while maintaining the same level of the elastic energy of the particles used for this purpose. Kinematic analogues differ from the corresponding types of energy by a multiplier equal to the elastic modulus, which is directly proportional to the density and heat capacity of the material and inversely proportional to the volume compression coefficient. Transverse, torsional, and longitudinal vibrations are considered in free and resonant conditions. A mechanism for converting forced vibrations into their own after the termination of external influences is proposed. Resonance occurs due to the superposition of free and forced vibrations with the same or similar frequency with the formation of a new free wave on each cycle with an increase in the amplitude and energy involved in the vibrations, which occurs mainly due to internal sources, and not external forces.