Dynamics and minimalistic control of a flexible structure containing bi-stable elements

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Summary. This study aims to design and analyze a system of mechanically-coupled bistable elements such that transitions between states (different equilibrium branches) take place in a desirable order by controlling a single degree of freedom. Such systems may be useful in a range of applications, e.g. soft robots or foldable structures, where a complex sequence of movements or configurations needs be achieved by minimal control. The theoretical part involves analytical and numerical investigation of the non-linear dynamic response of an array of bistable elements connected in series. The model accounts for the non-linear behavior of the bistable elements assuming linear damping and negligible inertia where the only control input is the displacement at the end of the chain (u_0). The quasi-static and dynamic response of a chain of two elements is studied. Focus is put on the multiplicity of equilibrium states, stability, and conditions for transition between stable states. Special attention is given to identifying critical rates that separate between different transition sequences, and how they are influenced by the properties of the bistable elements and the variability between them. We show, and demonstrate experimentally, that by clever design of the bistable elements, one can robustly control the order of transitions between states by merely varying the rate (speed) of the applied overall extension.

Introduction

A bi-stable mechanical element has two stable equilibrium states for the same external load and allows transition between states. The behavior of a chain of bistable elements has been studied extensively in various contexts. For example, in [1] the quasi-static behavior and stability of equilibrium configurations of a chain of identical bistable elements was investigated. Recently, robotic systems with bistable elements have been proposed for achieving controlled transitions using minimalistic control [2]. In another recent work [3], such concept was demonstrated on a system of identical hyper-elastic thin-walled balloons connected in series, where a desirable state is attained by a sequence of quasi-static inflations and deflations using a single flow control input.

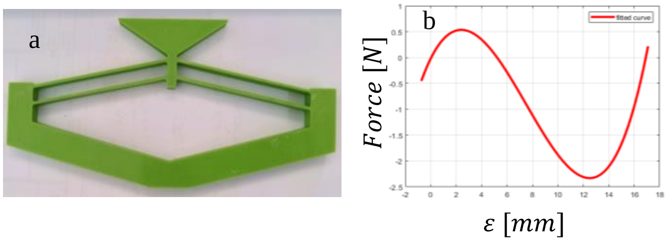


Figure 1: **(a).** A 3D-printed bistable element made of ABS material. **(b).** The measured force–elongation relation of the 3D-printed bistable element.

Results

The dynamic equations of a system comprising two bistable elements connected in series were formulated by assuming linear damping and negligible inertial forces. A thorough investigation revealed that the sequence of transitions between stable equilibrium branches can be dictated by merely controlling the input (elongation) speed.

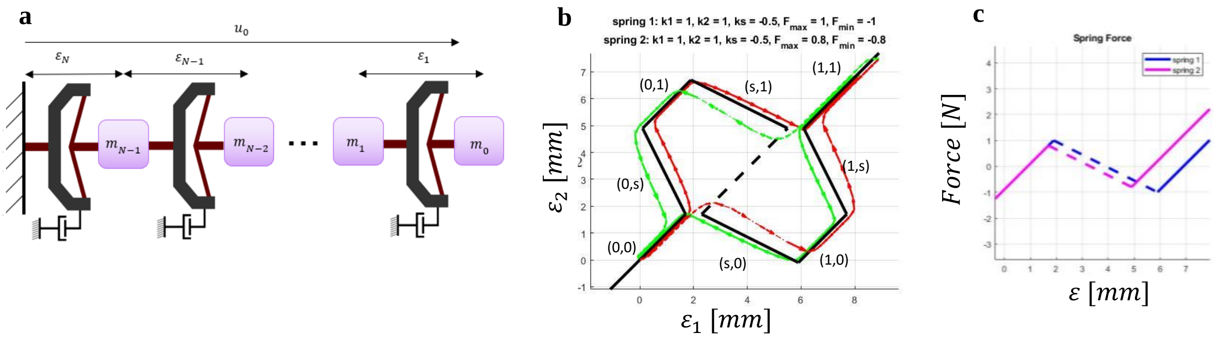
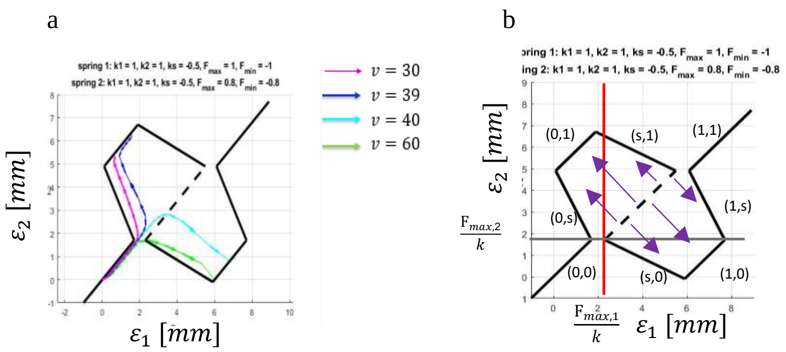


Figure 2: **(a).** Dynamic model; u_0 is the input control and ε_i is the elongation of element i. **(b).** Equilibrium curves (dashed – unstable) and dynamic trajectories on the map of the elongation of the elements. **(c).** The force – elongation relation of the elements in the system.

For example, during stretching, the condition for generating a sequence of transition events that differs from that of the quasi-static (or slow) loading is that the first element reaches the negative-stiffness branch (right of the red vertical line in Fig. 3b) before the second element does (above the horizontal gray line in Fig. 3b).



c

$$\varepsilon_1 = \left(\frac{c}{4k_1}\left(1-e^{-\frac{2k_1 t}{c}}\right)+\frac{t}{2}\right)v = \frac{F_{\max 1}}{k_1}$$

$$\varepsilon_2 = \left(\frac{c}{4k_1}\left(e^{-\frac{2k_1 t}{c}}-1\right)+\frac{t}{2}\right)v < \frac{F_{\max 2}}{k_1}$$

Figure 3: **(a).** It can be seen in the graph that there is a critical speed which changes the trajectory of the movement from the trivial trajectory to the non-trivial trajectory. **(b).** Shows the requirements for the transition to the non-trivial route on a force – elongation profile. **(c).** Algebraic transition conditions according to the conditions in graph b.

Let us define the following non-dimensional parameters: $\tau = \frac{k_1}{c}t, \bar{u} = \frac{cv}{2(F_{\max 1}+F_{\max 2})}, \beta = \frac{(F_{\max 1}-F_{\max 2})}{(F_{\max 1}+F_{\max 2})}$. The condition for changing transition sequence is $\beta + \bar{u}\left(e^{-\frac{1}{\bar{u}}}-1\right) > 0$. We find that for relatively small values of β, even up to β=0.18, the critical speed may be well approximated by $v > \frac{2(F_{\max 1}-F_{\max 2})}{c}$.

To test the theoretical predictions, we designed and built an experimental setup. The bistable elements were manufactured by 3D-printing, and the details of their bistable behavior (including the desired difference between the two) were tailored by careful design of the geometry of the parallel bending beams. Damping was introduced by strong magnets attached to each element and hovering over a copper plate (see Fig. 4).

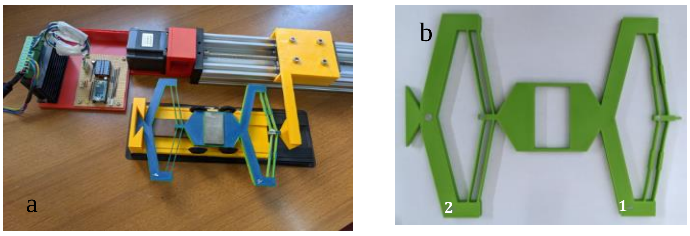


Figure 4: **(a).** Experimental system. **(b).** Two bistable elements are printed on a 3D printer made of ABS material.

The dynamic response was recorded using a digital camera and then analyzed by means of standard image processing tools. The experimental results, in terms of elongation of each of the two bistable elements, are shown in Fig. 5, and display a very good agreement with the theoretical predictions.

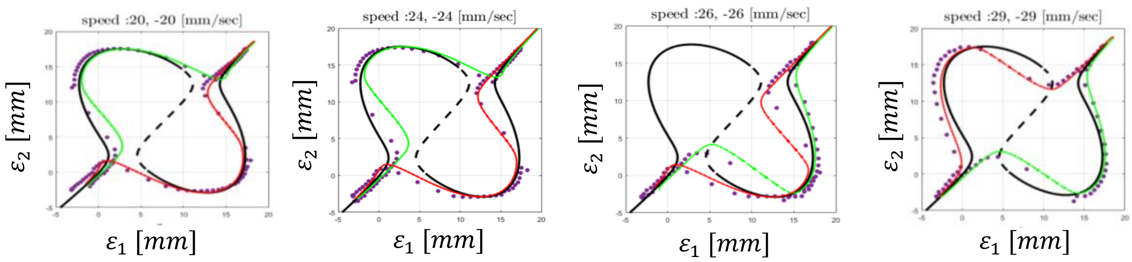


Figure 5: Comparison of the results of the experiment measured in image processing to the results of the simulation. Purple dots (experiment results), red line (stretching simulation) green line (compression simulation), dashed line black (unstable equilibrium), black line (stable equilibrium).

References

[1] I. Benichou and S. Givli, "Structures undergoing discrete phase transformation," *J. Mech. Phys. Solids*, vol. 61, no. 1, pp. 94–113, 2013, doi: 10.1016/J.JMPS.2012.08.009.

[2] E. Ben-Haim, L. Salem, Y. Or, and A. D. Gat, "Single-Input Control of Multiple Fluid-Driven Elastic Actuators via Interaction between Bistability and Viscosity," *Soft Robot.*, vol. 7, no. 2, pp. 259–265, Apr. 2020, doi: 10.1089/soro.2019.0060.

[3] D. Melancon, B. Gorissen, C. J. García-Mora, C. Hoberman, and K. Bertoldi, "Multistable inflatable origami structures at the metre scale," *Nat.* |, vol. 592, p. 545, 2021, doi: 10.1038/s41586-021-03407-4.