Long Time Trapping of Particles in a Rotating Sinai Billiards System

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Summary. We consider a rotating Sinai Billiards system in this paper and consider a square wall boundary with a circular scatterer. We subsequently consider the effects of rotation on this system for a number of particles. The question of ergodicity is considered from the point of view of whether or not in long time the particle can explore the complete phase space. The system is numerically simulated and tested for a range of rotational speed, indicating non-ergodic responses.

Introduction

A two-dimensional billiard with reflective square boundary and a single circular disk obstacle at the center is considered in this paper. In absence of external force, a point mass in this standard classical Sinai billiard system would have straight line trajectory until it collides either with the outer wall or the inner wall. Upon elastic collision it will change the direction of the velocity in accordance with angle of incidence is equal to angle of reflection. In this paper, the billiard system is considered under rotating conditions without an external force. Subsequently, we intend to investigate how non-ergodicity is manifested in the system as a function of rotational speed and the radius of the disk at the centre, the two main parameters governing the outcome of the system.

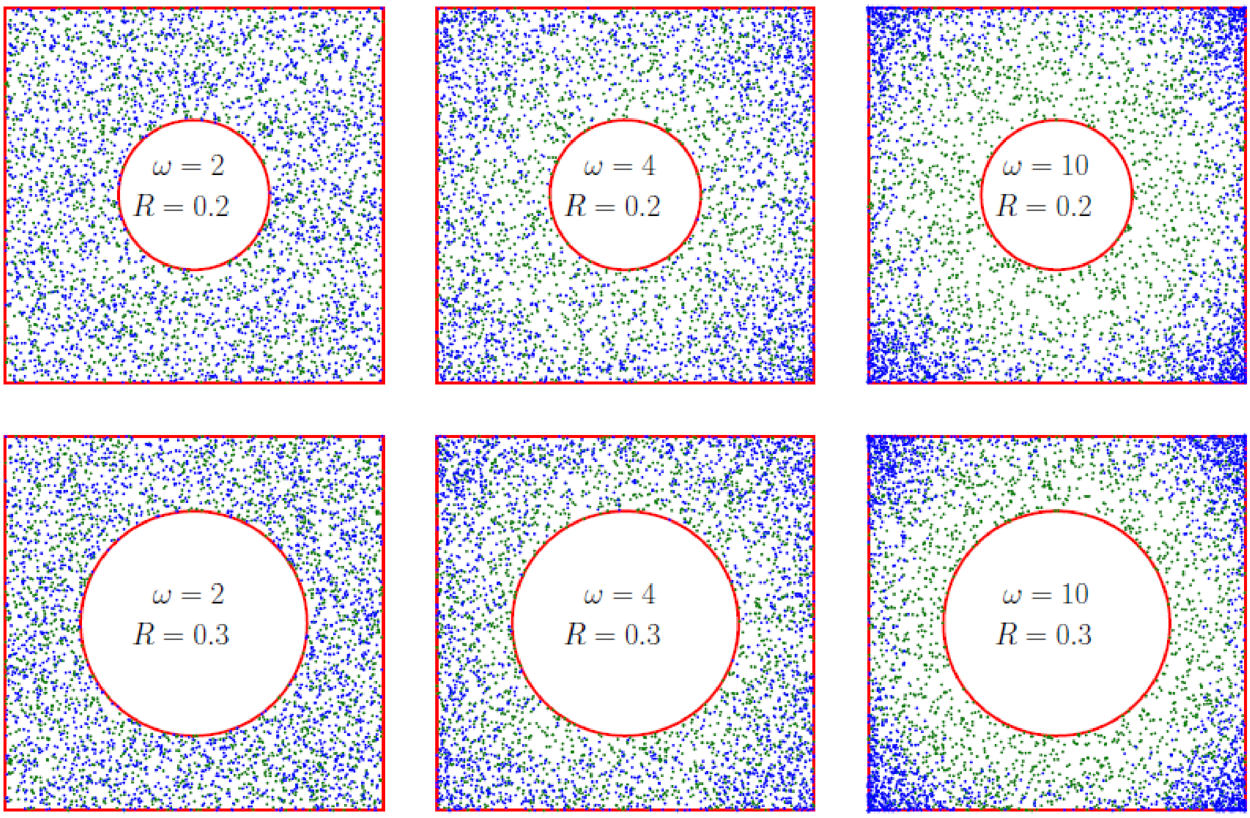

Fig.1. An ensemble of the initial and final positions of particles for different rotational speeds and disk radius. The green points indicate N=2000 initial positions of the mass point for N independent simulations. The blue points indicate their final positions after an equal amount of time has elapsed.

Results and discussions

Particle trajectories starting from different sets of initial conditions defined by position and velocity are simulated. The effect of increasing the rotational speed of the table for several cases of the radius of the circular disk on those trajectories are observed. For a certain set of rotational speed and radius of disk, 2000 initial conditions of location and velocity were considered drawing from uniformly distributed values between the inner and outer boundaries. After a sufficient time has elapsed (t = 10), the locations of the particles are noted for all of these independent runs, as observed in Fig 1.

From Fig.1, it is observed that as rotational speed increases, particles spend more time away from the disk and for high value of rotational speed the trajectories are trapped at the corners of the outer wall of the billiard making the system highly non-ergodic.

References

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