Two-stroke single-cylinder engine with elastic hinges with preset force characteristics

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Abstract. The work is devoted to balancing a two-stroke single-cylinder engine by installing two elastic hinges with a given characteristic (the dependence of the restoring moment on the angle of rotation) between the strut and the crank and between the crank and the connecting rod. The hinge is an elastic element (spring or pneumatic spring) that moves between the guides of the calculated shape. The characteristic of the elastic hinge between the crank and the connecting rod is such that the lateral force acting on the piston decreases many times during engine operation. With certain parameters of the engine under consideration with a hinge between the crank and the connecting rod, the lateral force acting on the piston is zero at any angle of rotation of the crank. The characteristic of the elastic hinge between the strut and the crank is such that the angular velocity of the crank will be constant. In this case, you can either completely abandon the flywheel in the engine design or significantly reduce its weight. Flywheel inertia can now account for up to 80 percent of all moving parts in an engine. Such parameters of the proposed two-stroke single-cylinder engine are selected, at which it becomes fully balanced. At the moment, the considered problem has been solved for a certain constant angular velocity of the crank. For a different angular velocity, a different characteristic of the elastic hinge will be required. A possible solution to this problem can be the use of a pneumatic spring as an elastic element of the proposed hinges. By changing the pressure in the pneumatic spring in an appropriate way with a change in the angular velocity of the crank, it is possible to achieve the proper coefficient of uneven operation of the considered two-stroke engine. The approach proposed here for balancing a two-stroke single-cylinder engine can also be applied to balancing four-stroke engines.

The problem of balancing internal combustion engines (ICE) based on a crank mechanism (CM) has been the subject of many works, for example [1-4]. This work is devoted to a two-stroke single-cylinder engine (TSSCE) with two elastic hinges with preset force characteristics [5] located between the crank and connecting rod at point A (with characteristic $M_{12}(\varphi)$) and between the crank and connecting rod at point 0 with characteristic $M_1(\varphi)$ (Fig. 1, *(a)*). The dependence $M_{Cr.}(\varphi)$ on the TSSCE crank shaft was taken from [6] and was approximated by the analytical function (1), where φ is in degrees.

$$\begin{gathered}
M_{Cr.}(\varphi) = -2{,}693510959090909 \cdot 10^{7} + 1009131{,}066530486 \cdot (\varphi \cdot \pi/180) - \\
-16831{,}236170949003 \cdot (\varphi \cdot \pi/180)^{2} + 164{,}56930773216035 \cdot (\varphi \cdot \pi/180)^{3} - \\
-1{,}0445902415421795 \cdot (\varphi \cdot \pi/180)^{4} + 0{,}004497285554988163 \cdot (\varphi \cdot \pi/180)^{5} - \\
-0{,}000013297457442994316 \cdot (\varphi \cdot \pi/180)^{6} + 2{,}665200570792428 \cdot 10^{-8} \cdot (\varphi \cdot \pi/180)^{7} - \\
-3{,}463864860887725 \cdot 10^{-11} \cdot (\varphi \cdot \pi/180)^{8} + 2{,}636647972125913 \times 10^{-14} \cdot (\varphi \cdot \pi/180)^{9} - \\
-8.947485203612654 \cdot 10^{-18} \cdot (\varphi \cdot \pi/180)^{10} + (2{,}693510959090909 \cdot 10^{7} - \\
-1008993{,}066791191 \cdot (\varphi \cdot \pi/180) + 16823{,}278538256654 \cdot (\varphi \cdot \pi/180)^{2} - \\
164{,}3809582389641 \cdot (\varphi \cdot \pi/180)^{3} + 1{,}0422601817406472 \cdot (\varphi \cdot \pi/180)^{4} - \\
-0{,}0044809411801945455 \cdot (\varphi \cdot \pi/180)^{5} + 0{,}000013232168767022518 \cdot (\varphi \cdot \pi/180)^{6} - \\
-2{,}651459239985029 \cdot 10^{-8} \cdot (\varphi \cdot \pi/180)^{7} + 3{,}452203947414538 \cdot 10^{-11} \cdot (\varphi \cdot \pi/180)^{8} - \\
-2{,}636647972125913 \cdot 10^{-14} \cdot (\varphi \cdot \pi/180)^{9} + 8{,}947485203612654 \cdot 10^{-18} \cdot (\varphi \cdot \pi/180)^{10}) \cdot \mathrm{Sign}[180 - \varphi].
\end{gathered} \tag{1}$$

In this formulation of the problem, a CM with a counterweight on the connecting rod is considered. We consider that the weight of the counterweight is P_4, and its length is ℓ_0. Rod weight AC_4 was not taken into account (Fig. 1).

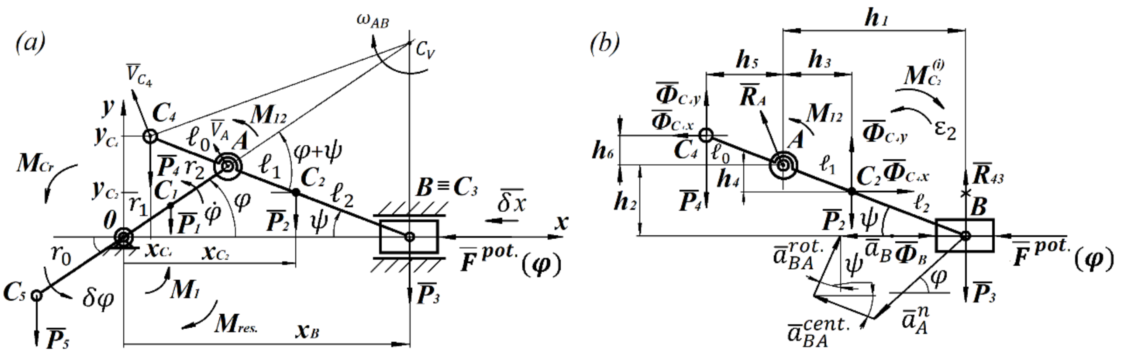


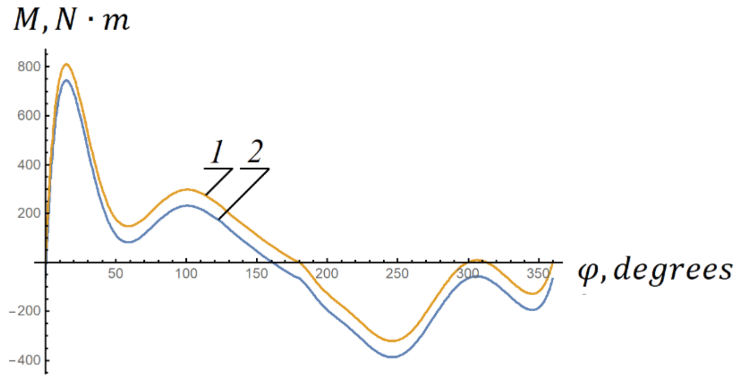
Fig. 1.

To determine the dependence $M_{12}(\varphi)$ in steady state, taking into account the operating resistance $M_{res.}$, the horizontal force $F^{pot.}$ acting on the piston from condition (2) was determined.

$$F^{pot.}(\varphi)dx = M_{Cr.}^{pot.}(\varphi)d\varphi, \tag{2}$$

where $M_{Cr.}^{pot.}(\varphi) = M_{Cr.}(\varphi) - \int_0^{2\pi} M_{Cr.}(\varphi)d\varphi/2\pi$ - potential component of the moment on the shaft of the ICE ($\int_0^{2\pi} M_{Cr.}^{pot.}(\varphi)d\varphi = 0$) in the steady state of the engine, taking into account the resistance (Fig. 1, *(a)*).

It is assumed that dependence (1) and, consequently, dependence (2) can be determined for any TSSCE. Figure 2 shows the dependencies $M_{Cr.}(\varphi)$ (dependence 1 according to formula (1)) and $M_{Cr}^{pot.}(\varphi)$ (dependence 2) at one turn of the crank.



1 - $M_{Cr.}(\varphi)$; 2 - $M_{Cr}^{pot.}(\varphi)$.

Fig. 2.

Omitting calculations for dependence (2), we write an expression for $F^{pot.}(\varphi)$ ($\oint F^{pot.}dx = 0$).

$$F^{pot.} = M_{Cr.}^{pot.}/((r_1 + r_2) \cdot \sin\varphi \cdot \left(1 + \lambda \frac{\cos\varphi}{\sqrt{1-\lambda^2 \cdot \sin^2\varphi}}\right)), \tag{3}$$

where r_1 – distance from point 0 to the center of mass of the crank; $(r_1 + r_2)$ – crank length; $\lambda = \frac{(r_1+r_2)}{(\ell_1+\ell_2)}$; ℓ_1 – distance from point A to the center of gravity of the connecting rod AB; $(\ell_1 + \ell_2)$ – crank length AB.

To determine the dependence $M_{12}(\varphi)$ (4), in which the lateral force R_{43} acting on the piston is equal to 0 at any angle of rotation of the crank (Fig. 1, *(b)*), the sum of the moments of active forces (P_2, P_3, P_4, $F^{pot.}$), inertia forces ($P_3 \cdot \ddot{x}_B/g$, $P_2 \cdot \ddot{x}_{C2}/g$, $P_2 \cdot \ddot{y}_{C2}/g$, $P_4 \cdot \ddot{x}_{C4}/g$, $P_4 \cdot \ddot{y}_{C4}/g$), torque inertia of the connecting rod ($I_{C2}^{(2)} \cdot \varepsilon_2$) relative to point A. It was assumed that the angular velocity of the crank φ is constant. The direction of rotation of the crank is counterclockwise (Fig. 1, *(a)*).

$$M_{12} = -P_4 \cdot h_5 + P_3 \cdot h_1 + P_2 \cdot h_3 + I_{C2}^{(2)} \cdot \varepsilon_2 + \frac{P_3}{g} \cdot \ddot{x}_B \cdot h_2 + F^{pot.} \cdot h_2 +$$

$$+ \frac{P_2}{g} \cdot \ddot{x}_{C2} \cdot h_4 + \frac{P_2}{g} \cdot \ddot{y}_{C2} \cdot h_3 - \frac{P_4}{g} \cdot \ddot{x}_{C4} \cdot h_6 - \frac{P_4}{g} \cdot \ddot{y}_{C4} \cdot h_5, \qquad (4)$$

where $h_1 = (\ell_1 + \ell_2) \cdot (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)$; $h_2 = (\ell_1 + \ell_2) \cdot \lambda \cdot \sin\varphi$; $h_6 = \ell_0 \cdot \lambda \cdot \sin\varphi$;

$h_3 = \ell_1 \cdot (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)$; $h_4 = \ell_1 \cdot \lambda \cdot \sin\varphi$ $h_5 = \ell_0 \cdot (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)$;

$\varepsilon_2 = \lambda \cdot \dot{\varphi}^2 \cdot \sin\varphi \cdot (1 - (\lambda \cdot \cos\varphi / (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi))^2) / (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)$;

$\ddot{x}_B = -(r_1 + r_2) \cdot \dot{\varphi}^2 \cdot (\cos\varphi + \lambda \cdot \cos 2\varphi)$; $\ddot{x}_{C_2} = -(r_1 + r_2) \cdot \cos\varphi \cdot \dot{\varphi}^2 - \ell_1 \cdot \lambda^2 \cdot \cos 2\varphi \cdot \dot{\varphi}^2$;

$\ddot{y}_{C_2} = -\ell_2 \cdot \lambda \cdot \dot{\varphi}^2 \cdot \sin\varphi$; $\ddot{x}_{C_4} = -(r_1 + r_2) \cdot \cos\varphi \cdot \dot{\varphi}^2 + \ell_0 \cdot \lambda^2 \cdot \cos 2\varphi \cdot \dot{\varphi}^2$;

$\ddot{y}_{C_4} = -(r_1 + r_2 + \ell_0 \cdot \lambda) \cdot \sin\varphi \cdot \dot{\varphi}^2$.

In the absence of an elastic hinge at point *A*, the lateral force R_{430} will be determined by the dependence (5).

$$R_{430} = M_{12} / ((\ell_1 + \ell_2) \cdot (1 - 0{,}25 \cdot \lambda^2 + 0{,}25 \cdot \lambda^2 \cdot \cos 2\varphi)) \qquad (5)$$

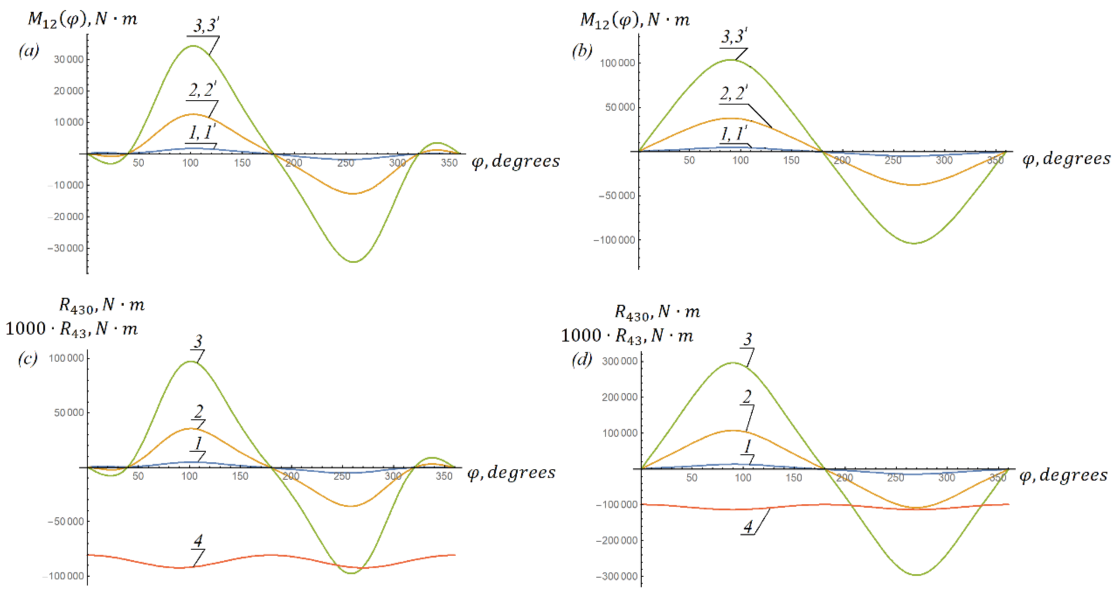
Dependence (4) turned out to be such that $\int_0^{2\pi} M_{12}(\varphi) d\varphi \neq 0$. It should be noted that $\int_0^{2\pi} M_{12}(\varphi) d\varphi$ does not depend on the angular velocity of the crank. To obtain such dependence $M_{12}^{pot.}(\varphi)$ that $\int_0^{2\pi} M_{12}^{pot.} d\varphi = 0$, dependence (6) can be used.

$$M_{12}^{pot.} = M_{12} - \int_0^{2\pi} M_{12} d\varphi / 2\pi \qquad (6)$$

When the elastic hinge is located with characteristic (6), the lateral force acting on the piston $R_{43}(\varphi)$ (Fig. 1, *(b)*), is determined by the following expression.

$$R_{43} = \int_0^{2\pi} M_{12} d\varphi / (2\pi \cdot (\ell_1 + \ell_2) \cdot (1 - 0{,}25 \cdot \lambda^2 + 0{,}25 \cdot \lambda^2 \cdot \cos 2\varphi)) \qquad (7)$$

Fig. 3 shows the dependencies $M_{12}(\varphi)$, $M_{12}^{pot.}$, $R_{430}(\varphi)$, $R_{43}(\varphi)$ obtained by dependences (4), (5), (6), (7), respectively. Options *(a)* and *(c)* were obtained with $P_4 = 0$ and $\ell_0 = 0$. Variants *(b)* and *(d)* were obtained with such values of P_4 and ℓ_0 ($P_4 = 20$ N, $\ell_0 = 0.4\ m$) that the center of mass of the system "counterweight at point C_4-rod-piston" is at point A.

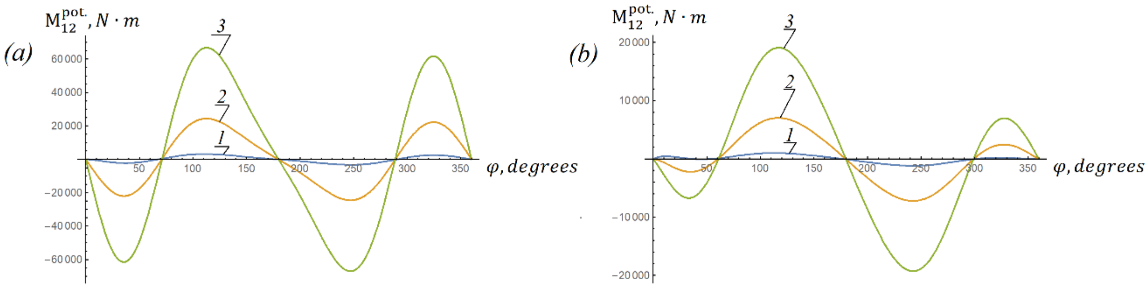


$r_1 = 0{,}1\ m$; $r_2 = 0{,}1\ m$; $\ell_1 = 0{,}2\ m$; $\ell_2 = 0{,}2\ m$; $P_2 = 10\ N$; $P_3 = 15\ N$;

1 - $\dot{\varphi} = 100\ s^{-1}$; 2 - $\dot{\varphi} = 300\ s^{-1}$; 3 - $\dot{\varphi} = 500\ s^{-1}$; 4 – dependence $R_{430}(\varphi)$ according to (5).

Fig. 3.

As can be seen from Figure 3, the dependences $M_{12}(\varphi)$ (without a prime) and $M_{12}^{pot.}$ (with a prime) practically coincide. Installing an elastic hinge with a given characteristic between the crank and the connecting rod reduces the lateral force acting on the piston $R_{43}(\varphi)$ hundreds of times. It should be noted that the reaction R_43 (φ) is practically independent of the angular velocity of the crank for these parameters. It was possible to select such parameters of the system under consideration, in which the lateral force $R_{43}(\varphi)$ is equal to zero at any angle of rotation of the crank. Figure 4 shows such dependencies. Option *(a)* obtained with $P_4 = \ell_0 = 0$. Option *(b)* with $P_4 = 10\ N$ и $\ell_0 = 0.1$ m. For the case when the center of mass of the system "counterweight at point C_4-rod-slider" is at point A, select such parameters for which $R_{43}(\varphi) = 0$ at any angle φ, failed.



$$r_1 = 0{,}1\ m;\ r_2 = 0{,}1\ m;\ 1 \text{ - } \dot{\varphi} = 100\ s^{-1};\ 2 \text{ - } \dot{\varphi} = 300\ s^{-1};\ 3 \text{ - } \dot{\varphi} = 500\ s^{-1};$$

$$a)\ \ell_1 = 0{,}2\ m;\ \ell_2 = 0{,}2\ m;\ P_2 \approx 54{,}40\ N;\ P_3 = 79\ N;$$
$$b)\ \ell_1 = 0{,}4\ m;\ \ell_2 = 0{,}4\ m;\ P_2 = 10\ N;\ P_3 \approx 18{,}16\ N.$$

Fig. 4.

The case was considered when the angular velocity of the crank is constant. To determine the characteristics of the elastic hinge at point 0 ($M_1(\varphi)$, Fig. 1, *(a)*) the Lagrange equation of the second kind was compiled.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} = Q_{\delta\varphi}, \tag{8}$$

where $T = \frac{1}{2} I_{giv.} \cdot \dot{\varphi}^2$ – kinetic energy of the mechanism in Fig. 1, *(a)*);

$$I_{giv.} = I_0^{(1)} + \frac{P_2}{g} \cdot (r_1 + r_2)^2 \cdot [(\sin\varphi - \frac{\ell_1 \cdot \lambda}{2(\ell_1 + \ell_2)} \cdot \sin 2\varphi)^2 + \frac{\ell_2^2 \cdot \cos^2\varphi}{(\ell_1 + \ell_2)^2}] + I_{C2}^{(2)} \cdot \frac{\lambda^2 \cdot \cos^2\varphi}{(1 - \frac{1}{4}\lambda^2 + \frac{1}{4}\lambda^2 \cos 2\varphi)^2} +$$

$$+ \frac{P_3}{g} \cdot (r_1 + r_2)^2 \cdot (\sin\varphi + \frac{1}{2}\lambda \cdot \sin 2\varphi)^2 + \frac{P_4}{g} \cdot \frac{\lambda^2 \cdot \cos^2\varphi}{(1 - \frac{1}{4}\lambda^2 + \frac{1}{4}\lambda^2 \cos 2\varphi)^2} \cdot ((\ell_0 + \ell_1 + \ell_2)^2 +$$

$$+2(\ell_0 + \ell_1 + \ell_2) \cdot ((r_1 + r_2) \cdot cos\varphi + (\ell_1 + \ell_2) \cdot (1 - \tfrac{1}{4}\lambda^2 + \tfrac{1}{4}\lambda^2 cos 2\varphi)) \cdot tg\varphi \cdot \lambda \cdot \sin\varphi) + \frac{P_5}{g} \cdot r_0^2;$$

$I_{giv.}$ – reduced moment of inertia of the mechanism in Fig. 1, *(a)*);

$$\frac{\partial T}{\partial \varphi} = \left(\frac{1}{2}\dot{\varphi}^2\right) \cdot [-\frac{2I_{C2}^{(2)} \cdot \lambda^2 \cdot \cos\varphi \cdot \sin\varphi}{(1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)^2} + \frac{(I_{C2}^{(2)} \cdot \lambda^4 \cdot \cos\varphi^2 \cdot \sin 2\varphi)}{(1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)^3} +$$

$$+ \frac{(2P_3 \cdot (r_1 + r_2)^2 \cdot \cos\varphi + \lambda \cdot \cos 2\varphi) \cdot (\sin\varphi + 0{,}5\lambda \cdot \sin 2\varphi)}{g} -$$

$$- \frac{1}{g} \cdot P_2 \cdot (r_1 + r_2)^2 \cdot (((2I_{C2}^{(2)} \cdot \cos\varphi \cdot \sin\varphi)/(\ell_1 + \ell_2)^2) + 2(\cos\varphi - \frac{\lambda \cdot \ell_1 \cdot \cos 2\varphi}{(\ell_1 + \ell_2)}) \times$$

$$\times (sin\varphi - \frac{\lambda \cdot \ell_1 \cdot \sin 2\varphi}{2(\ell_1 + \ell_2)})) - (2\lambda^2 \cdot P_4 \cdot \cos\varphi \cdot \sin\varphi \cdot ((\ell_0 + \ell_1 + \ell_2)^2 + 2\,\lambda \cdot (\ell_0 + \ell_1 + \ell_2) \cdot ((r_1 + r_2) \times$$

$$\times \cos\varphi + (\ell_1 + \ell_2) \cdot (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)) \cdot \sin\varphi \cdot tg\varphi))/(g \times$$

$$\times \cdot (1 - 0{,}25\lambda^2 + 0{,}25\lambda^2 \cdot \cos 2\varphi)^2) + (\lambda^4 \cdot P_4 \cdot (\cos\varphi)^2 \cdot \sin 2\varphi \cdot ((\ell_0 + \ell_1 + \ell_2)^2 + 2\lambda \cdot (\ell_0 + \ell_1 + \ell_2) \times$$

$$\times ((r_1+r_2)\cdot\cos\varphi + (\ell_1+\ell_2)\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi))\cdot\sin\varphi\cdot tg\varphi))/(g\times$$

$$\times(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi)^3) + (\lambda^2\cdot P_4\cdot(\cos\varphi)^2\cdot(2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot((r_1+r_2)\cdot\cos\varphi +$$

$$+(\ell_1+\ell_2)\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi))\cdot\sin\varphi + 2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot((r_1+r_2)\cdot\cos\varphi +$$

$$+(\ell_1+\ell_2)\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi))\cdot\sec\varphi\cdot tg\varphi + 2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot\sin\varphi\cdot(-(r_1+r_2)\times$$

$$\times\sin\varphi - 0{,}5\lambda^2\cdot(\ell_1+\ell_2)\cdot\sin 2\varphi)\cdot tg\varphi))/(g\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi)^2)];$$

$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}}\right) = \dot{I}_{giv.}\cdot\dot{\varphi} + I_{giv.}\cdot\ddot{\varphi};$

$$\dot{I}_{giv.} = -(\frac{2I_{C2}^{(2)}\cdot\lambda^2\cdot\cos\varphi\cdot\sin\varphi\cdot\dot{\varphi}}{1-0{,}25(\lambda^4+0{,}25(\lambda^2\cdot\cos 2\varphi)^2} + \frac{(I_{C2}^{(2)}\cdot\lambda^4\cdot(\cos\varphi)^2\cdot\sin 2\varphi\cdot\dot{\varphi})}{(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi)^3} -$$

$$-(2\lambda^2\cdot P_4\cdot\cos\varphi\cdot\sin\varphi\cdot((\ell_0+\ell_1+\ell_2)^2 + 2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot((r_1+r_2)\cdot\cos\varphi +$$

$$+(\ell_1+\ell_2)\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi))\cdot\sin\varphi\cdot tg\varphi)\cdot\frac{\dot{\varphi}}{g\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi)^2} +$$

$$+(\lambda^4\cdot P_4\cdot(\cos\varphi)^2\cdot\sin 2\varphi\cdot((\ell_0+\ell_1+\ell_2)^2 + 2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot((r_1+r_2)\cdot\cos\varphi +$$

$$+(\ell_1+\ell_2)\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi))\cdot\sin\varphi\cdot tg\varphi)\cdot\varphi)/((1-0{,}25\lambda^2+\dot{0},25\lambda^2\cdot\cos 2\varphi)^3) +$$

$$+(1/g)\cdot 2P_3\cdot(r_1+r_2)^2\cdot(\sin\varphi+0{,}5\lambda\cdot\sin 2\varphi)\cdot(\cos\varphi\cdot\dot{\varphi}+\lambda\cdot\cos\varphi\cdot\dot{\varphi}) + (1/g)\cdot P_2\cdot(r_1+r_2)^2\times$$

$$\times(-\left(\frac{2\ell_2^2\cdot\cos\varphi\cdot\sin\varphi\cdot\dot{\varphi}}{(\ell_1+\ell_2)^2}\right) + 2(\sin\varphi - \frac{\lambda\cdot\ell_1\cdot\sin\varphi}{2(\ell_1+\ell_2)})\cdot(\cos\varphi\cdot\dot{\varphi} - (\lambda\cdot\ell_1\cdot\cos\varphi\cdot\dot{\varphi})/(\ell_1+\ell_2))) +$$

$$+(\lambda^2\cdot P_4\cdot(\cos\varphi)^2\cdot(2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot\big((r_1+r_2)\cdot\cos\varphi + (\ell_1+\ell_2)\cdot\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi\big))\cdot\sin\varphi\cdot\dot{\varphi} +$$

$$+2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot((r_1+r_2)\cdot\cos\varphi + (\ell_1+\ell_2)\cdot(1-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi))\cdot\sec\varphi\cdot tg\varphi\cdot\dot{\varphi} +$$

$$+2\lambda\cdot(\ell_0+\ell_1+\ell_2)\cdot\sin\varphi\cdot tg\varphi\cdot(-(r_1+r_2)\cdot\sin\varphi\cdot\dot{\varphi} - 0{,}5\lambda^2\cdot(\ell_1+\ell_2)\cdot\sin\varphi\cdot\dot{\varphi})))/(g\cdot(1-$$

$$-0{,}25\lambda^2+0{,}25\lambda^2\cdot\cos 2\varphi)^2)$$

$$Q_{\delta\varphi} = F^{pot.}\cdot\left((r_1+r_2)\cdot sin\varphi + \frac{(r_1+r_2)^2\cdot sin2\varphi}{2\cdot(\ell_1+\ell_2)}\right) + M_1 - P_1\cdot r_1\cdot Cos\varphi + P_5\cdot r_0\cdot\mathrm{Cos}\,\varphi - \frac{P_2\delta y_{C_2}+P_4\delta y_{C_4}+M_{12}\delta(\varphi+\psi)}{\delta\varphi};$$

P_1, P_2, P_4, P_5 – the weights respectively of the crank, connecting rod, counterweight at point C_4 and counterweight at point C_5;

r_1 is the coordinate that determines the position of the center of mass of the crank;

r_0 is the coordinate that determines the position of the counterweight at point C_5;

y_{C_2}, y_{C_4} – ordinates of points C_2 and C_4 respectively (Fig. 1); $\delta y_{C2} = \ell_2\cdot\ \lambda\cdot cos\varphi\delta\varphi$;

$\delta y_{C4} = (r_1+r_2)\cdot cos\varphi\delta\varphi + \ell_0\cdot(r_1+r_2)/(\ell_1+\ell_2)\cdot cos\varphi\delta\varphi$;

$\delta\psi = (r_1+r_2)\cdot cos\,\varphi\,\delta\varphi/(\ell_1+\ell_2)\cdot(1-0{,}25\cdot(\frac{r_1+r_2}{\ell_1+\ell_2})^2 + 0{,}25\cdot(\frac{r_1+r_2}{\ell_1+\ell_2})^2\cdot cos2\varphi)$;

$$Q_{\delta\varphi} = F^{pot.}\cdot\left((r_1+r_2)\cdot sin\varphi + \frac{(r_1+r_2)^2\cdot sin2\varphi}{2\cdot(\ell_1+\ell_2)}\right) - P_1\cdot r_1\cdot cos\varphi +$$

$$+P_5\cdot r_0\cdot cos\,\varphi + M_1 - P_2\cdot\ell_2\cdot\frac{r_1+r_2}{\ell_1+\ell_2}\cdot cos\varphi - P_4\cdot((r_1+r_2)\cdot cos\varphi +$$

$$+\ell_0\cdot\frac{r_1+r_2}{\ell_1+\ell_2}\cdot cos\varphi) - M_{12} -$$

$$-M_{12} \cdot \frac{(r_1 + r_2) \cdot cos\,\varphi}{(\ell_1 + \ell_2) \cdot (1 - 0{,}25 \cdot (\frac{r_1 + r_2}{\ell_1 + \ell_2})^2 + 0{,}25 \cdot \left(\frac{r_1 + r_2}{\ell_1 + \ell_2}\right)^2 \cdot cos2\varphi)};$$

Let us rewrite equation (8) in the following form.

$$\dot{I}_{giv.} \cdot \dot{\varphi} + I_{giv.} \cdot \ddot{\varphi} - \frac{1}{2}\dot{\varphi}^2 \cdot \frac{\partial I_{giv.}}{\partial \varphi} = Q_{\delta\varphi}. \tag{9}$$

From equation (9) we obtain the dependence of the moment on the crank on its angle of rotation ($\ddot{\varphi} = 0$).

$$M_1 = \dot{I}_{giv.} \cdot \dot{\varphi} - \frac{1}{2}\dot{\varphi}^2 \cdot \frac{\partial I_{giv.}}{\partial \varphi} - F^{pot.} \cdot \left((r_1 + r_2) \cdot sin\varphi + \frac{(r_1 + r_2)^2 \cdot sin2\varphi}{2 \cdot (\ell_1 + \ell_2)}\right) +$$

$$+P_1 \cdot r_1 \cdot cos\varphi - P_5 \cdot r_0 \cdot cos\,\varphi + P_2 \cdot \ell_2 \cdot \frac{r_1+r_2}{\ell_1+\ell_2} \cdot cos\varphi + P_4 \cdot ((r_1 + r_2) \cdot cos\varphi - \tag{10}$$

$$-\ell_0 \cdot \frac{r_1 + r_2}{\ell_1 + \ell_2} \cdot cos\varphi) + M_{12} +$$

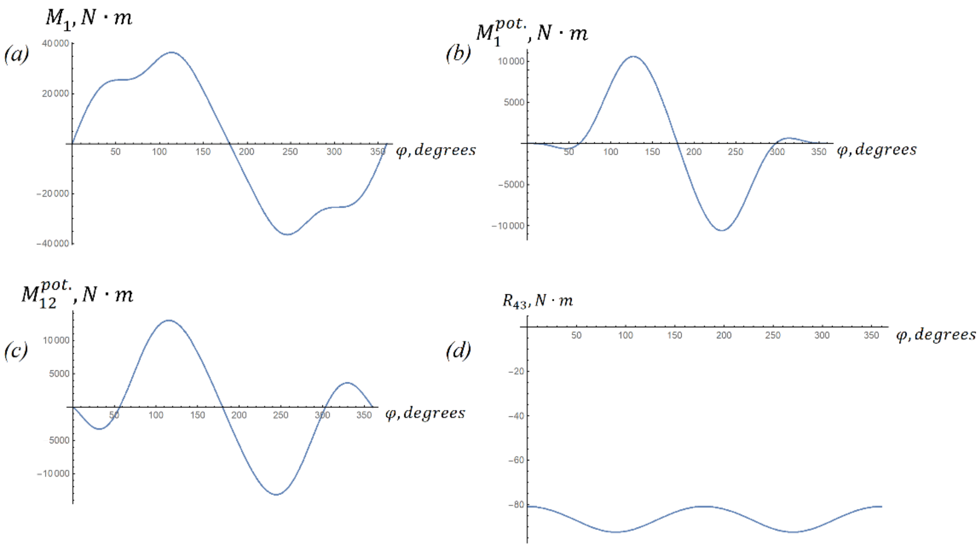
$$+M_{12} \cdot (r_1 + r_2) \cdot cos\,\varphi\,/(\ell_1 + \ell_2) \cdot (1 - 0{,}25 \cdot (\frac{r_1+r_2}{\ell_1+\ell_2})^2 + 0{,}25 \cdot \left(\frac{r_1+r_2}{\ell_1+\ell_2}\right)^2 \cdot cos\;\,2\varphi).$$

Variants were considered in which $M_{12} = M_{12}^{pot.}$, that is, in the presence of an elastic hinge with the characteristic $M_{12}^{pot.}(\varphi)$ between the crank and the connecting rod. It turned out that not for all parameters of the considered system $\int_0^{2\pi} M_1 d\varphi = 0$. For example, for the data on which Fig. 3, *(b)*, $\int_0^{2\pi} M_1 d\varphi \approx 14193.2\,J$ under the condition of vertical balancing (Fig. 5, *(a)*, - according to equation (10)). Vertical balancing of the CM in Fig. 1, *(a)* was determined by the following equation

$$P_5 \cdot r_0 \cdot sin\,\varphi = P_1 \cdot r_1 \cdot sin\,\varphi + P_2 \cdot \ell_2 \cdot sin\,\psi + P_4 \cdot (\ell_1 + \ell_2 + \ell_0) \cdot sin\,\psi. \tag{11}$$

The weight P_5 from equation (11) is given by the following expression

$$P_5 = \left(P_1 \cdot r_1 + P_2 \cdot \ell_2 \cdot \frac{r_1+r_2}{\ell_1+\ell_2} + P_4 \cdot (\ell_1 + \ell_2 + \ell_0) \cdot \frac{r_1+r_2}{\ell_1+\ell_2}\right)/r_0 \tag{12}$$



$\dot{\varphi} = 300\ s^{-1}$; $I_0^{(1)} = 0{,}2\ kg \cdot m^2$; $r_1 = 0{,}1\ m$; $r_2 = 0{,}1\ m$;

a) $r_0 = 0{,}1\ m$ $\ell_0 = 0{,}4\ m$; $\ell_1 = 0{,}2\ m$; $\ell_2 = 0{,}2\ m$;

$P_1 = 5\ N$; $P_2 = 10\ N$; $P_3 = 15\ N$; $P_4 = 20\ N$; $P_5 = 47{,}50\ N$;

b), c), d) $r_0 = 0{,}4\ m$; $\ell_0 = 0{,}2\ m$; $\ell_1 = 0{,}4\ m$; $\ell_2 = 0{,}45\ m$;

$P_1 = 20\ N$ $P_2 = 30\ N$; $P_3 = 30\ N$; $P_4 = 30\ N$; $P_5 = 31{,}47\ N$.

Fig. 5.

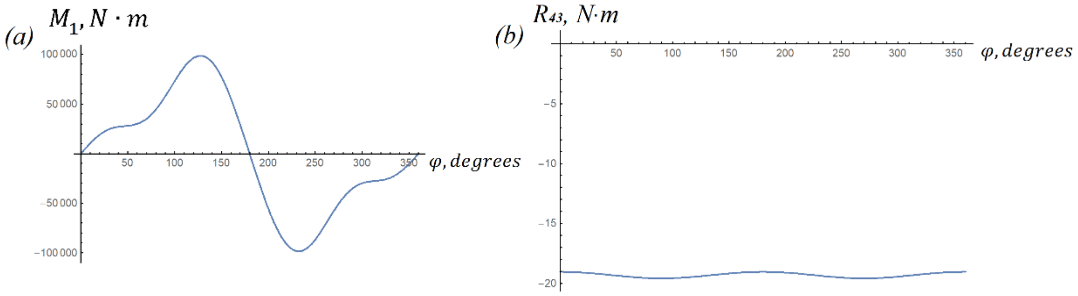
For option *(b)* $\int_0^{2\pi} M_1 d\varphi = 0$, while $\int_0^{2\pi} M_{12}^{pot.} d\varphi = 0$ (*(c)*, - according to (4)). As can be seen from the figure 5, the lateral force acting on the piston (*(d)*, - according to (7)), is quite small.

The condition for finding the center of mass of the system in fig. 1, *(a)* at point 0 the following

$$P_5 = \big((P_2 + P_3 + P_4) \cdot (r_1 + r_2) + P_1 \cdot r_1\big)/r_0, \tag{13}$$

where $P_4 = (P_3 \cdot (\ell_1 + \ell_2) + P_2 \cdot \ell_1)/\ell_0$ (the condition for finding the central mass of the system “counterweight at point C_4-rod-posun” at point A, Fig.1).

Figure 6, *(a)* shows the dependence $M_1(\varphi)$ in the case when the center of mass of the system in Fig.1, *(a)* is at point 0 (condition (13) is satisfied). Figure 6, *(b)* shows the dependence of the lateral force acting on the piston on the angle of rotation (in the presence of an elastic hinge at point A, Fig. 1, *b*).



$r_0 = 0{,}4\ m;\ r_1 = 0{,}1\ m;\ r_2 = 0{,}1\ m;\ \dot{\varphi} = 300\ s^{-1};\ \ell_0 = 0{,}2\ m;\ \ell_1 = 0{,}4\ m;\ \ell_2 = 0{,}45\ m;$

$\mathrm{I}_0^{(1)} = 0{,}2\ kg \cdot m^2;\ P_1 = 20\ N;\ P_2 = 30\ N;\ P_3 = 30\ N;\ P_4 = 187{,}50\ N;\ P_5 = 128{,}75\ N.$

Fig. 6.

An elastic hinge with a given characteristic is an elastic element, a spring or a pneumatic spring, moving between the guides of the given form (Fig. 7). For option *(a)*, the elastic element is a spring; for option *(b)* the elastic element is a pneumatic spring. The shape of the guides is such that the reactions N_1 and N_2 create a calculated restoring moment depending on the angle of rotation of the elastic element relative to the guides α. For an elastic hinge at point 0, the angle α is equal to the angle of rotation of the crank φ. For a hinge at point A: $\alpha = \varphi + \psi$, or $\alpha = \varphi + \arcsin\,(\lambda \cdot sin\varphi)$. The method for calculating the shape of the guides of an elastic hinge is presented in the article [7]. At first, the guides are calculated assuming that the radius of the rollers in contact with the guides is zero. Then equidistant guides are built to the obtained ones, taking into account the radius of the rollers. For example, according to the dependences $M_{12}^{pot.}(\varphi)$ and $M_1(\varphi)$ derived in this article, it is possible to calculate the guides of the elastic hinges, which will be located between the crank and the connecting rod and between the strut and the crank, respectively. For the steady-state operation of the engine at different constant angular velocities of the crank, elastic hinges with different characteristics are required. From this point of view, elastic hinges are preferable, in which a pneumatic spring is used as an elastic element. Then, with a small change in the magnitude of the angular velocity of the crank, it is possible to achieve the proper coefficient of uneven operation of the considered engine by calculating the pressure change in the air spring [7].

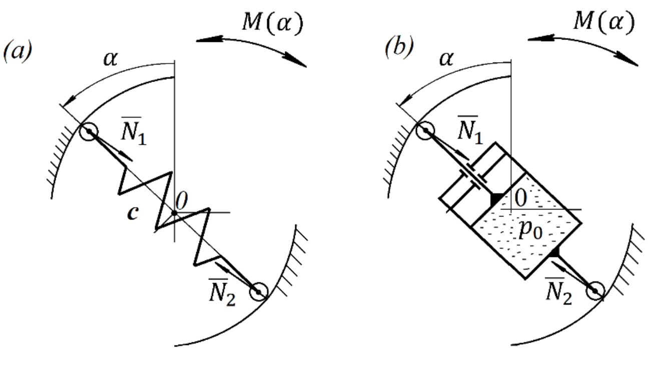


Fig. 7.

Conclusions.

1. Installation of an elastic hinge with a given characteristic between the crank and the TSSCE connecting rod can reduce the lateral force acting on the piston by hundreds of times. For certain parameters of the system under consideration, the lateral force acting on the piston during the operation of the TSSCE is equal to zero at any angle of rotation of the crank.

2. Installation of an elastic hinge with a given characteristic between the strut and the crank, in the presence of an elastic hinge between the crank and the connecting rod, makes it possible to obtain a constant angular velocity of the TSSCE crank with a minimum lateral force acting on the piston.

3. For CM with counterweights on the connecting rod and crank with elastic hinges located between the strut and the crank and between the crank and the connecting rod, it is possible to obtain a fully balanced TSSCE with a constant angular velocity of the crank

REFERENCES

[1] V. Arakelian, S. Briot, «Simultaneous Inertia Force/Moment Balancing and Torque Compensation of Slider-Crank Mechanisms,» Mechanics Research Communications, Elsevier, 37 (2), 2010, pp.265-269.

[2] R. S. Berkof, «Complete force and moment balancing of inline four-bar linkages,» Mechanism and Machine Theory 8, 1973, pp. 397-410.

[3] D. Groza, C. Antonya, «Dynamically Spring Balanced Slider-Crank Mechanism for Reciprocating Machines,» SSRG International Journal of Mechanical Engineering (SSRG – IJME) – Volume 2 Issue 6 – June 2015.

[4] Jih-Lian Haa, Rong-Fong Fungb, Kun-Yung Chenb, Shao-Chien Hsien, «Dynamic modeling and identification of a slider-crank mechanism,» Journal of Sound and Vibration 289 (2006) 1019–1044.

[5] A.N. Zotov, A.S. Sviridov «A Crank Mechanism with Elastic Joints Having Preset Characteristics,» Journal of Machinery Manufacture and Reliability, 2021, Vol. 50, No. 5, pp. 388–395.

[6] V.A. Stopin «Ship's main power equipment. Shipboard diesel engines,» Textbook for course and diploma design, Severodvinsk: Sevmashvtuz, 2003. – p. 64.

[7] A.N. Zotov, A.R. Valeev, A.S. Sviridov « Stabilization of the Torque on the Shaft of a Statically Balanced Crank Mechanism,» 2020 International Conference on Dynamics and Vibroacoustics of Machines (DVM), Samara, Russia, 2020, pp. 1-7.