Flight Behaviour of a Two-Line, Four-Point Disk Kite

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Summary. Existing work on kite behaviour mainly considers single-line kites and associated dynamics are limited to quasi-static analyses. Mathematical descriptions rely on generalised aerodynamic models which have not been validated experimentally and thus lack being suitable for design optimization purposes. Kites are utilized in both professional sporting activities and potential new energy generation applications. Understanding the dynamics of these kites allows for innovative improvements for such applications. Furthermore, derivations of existing models are typically based on energy methods which are not immediately helpful to inform design optimisation guidelines. In this work we derive the fully non-linear governing dynamic equations of motion for a two-line four-attachment point kite using a four degree of freedom Newton-Euler formulation. We implement an aerodynamic model which has been previously tested and validated (CFD/wind-tunnel) by our group. The analysis considers and focuses on stability regions of selected flight scenario and the maneuverability of the kite for relevant kite design parameters.

Background

A kite is an atypical aerodynamic object and knowledge about its flight dynamics is rare in literature. Over the last decade few articles have been published about two-line kites, even fewer include dynamic analysis [1, 2]. Quasi-static analysis was performed by Dawson [3] to investigate the turning maneuver of a kite. Other existing work has been done on the dynamics of two-line kites by Sánchez et al. [1]. Their work used the Lagrangian formalism to investigate the stability of the kite for different wind speeds and one control parameter, but did not investigate the reason for turning and restorative mechanisms of the kite flight. Research on the topic has recently increased with new applications of kites for energy generation and ship propulsion [4], but the underlying, inherently nonlinear, dynamics is still under-studied. The kite considered in this work consists of two lines, one for each hand (as depicted in new applications) which connect to each side of the kite. These lines, each split to connect to four attachment points on the kite surface, introduce the necessary moments to control the kite. The kite is modelled with circular disks to represent the aerodynamic surfaces as suggested by Stevenson [5] to be able to validate theoretical models with experimental investigation and observation. This is a first approach implementing a validated model; additional, more complex geometries can be introduced in future investigations. The kite system suggests two modes of behaviour: the first representing the stalling behaviour of the kite, the other representing the expected (typical) kite flight. The dynamics of the considered kite shows restorative behaviour along the elevation and pitch variables, but requires attention to control significant changes of the azimuthal angle. In this work we focus on the roll control of a user and its stability regions for a selected set of parameters and ranges.

Model

This kite system is described by five coordinate systems: **global**, **line**, **kite**, and two **disks** (Figure 1). Euler angles are used to define the kinematics of the kite with a 3-2-1 body-fixed rotation [6] resulting in four degrees of freedom, with θ_1 being the elevation of the kite, θ_2 the azimuthal deviation of the kite, θ_3 the pitching of the kite, and θ_4 the yaw of the kite. The nonlinear set of governing state-space equations is of the form

$$\mathbf{A}_{8\times 8}\, \dot{\boldsymbol{q}}_{8\times 1} = \mathbf{f}_{8\times 1}\left(\boldsymbol{M}^{L}, \boldsymbol{M}^{K}, \boldsymbol{q}, t\right) \tag{1}$$

$\mathbf{A}_{8\times 8}$ represents the inertia matrix, $\mathbf{f}_{8\times 1}$ is the forcing vector containing the lift and drag moments as well as the kinematic constraint relations, and $\boldsymbol{q}_{8\times 1}$ is defined in Eq. (2a). The moment equations are derived using the **line** and **kite** systems where i denotes the disk number.

$$\boldsymbol{q}_{8\times 1} = \left[\theta_1, \dots, \theta_4, \dot{\theta}_1, \dots, \dot{\theta}_4\right]^T \quad \text{(a)} \qquad \boldsymbol{M}_{\text{Lift \& Drag}} = \left(\boldsymbol{r}_{\mathrm{CP}_i} + \boldsymbol{r}_{\mathrm{OC}}\right) \times \boldsymbol{F}_i \quad \text{(b)} \tag{2}$$

$$\boldsymbol{M}^{K} = \left[\underbrace{\sum_{i=1}^{2} \boldsymbol{r}_{\mathrm{CP}_i} \times \boldsymbol{F}_i}_{\text{Lift and Drag}}\right]^{K} \tag{3}$$

$$\boldsymbol{M}^{L} = \left[\underbrace{\sum_{i=1}^{2} \boldsymbol{r}_{\mathrm{OC}} \times \boldsymbol{F}_i}_{\text{Lift and Drag}} + \underbrace{\boldsymbol{r}_{\mathrm{OC}} \times m_k \boldsymbol{g}}_{\text{Weight Force}}\right]^{L} \tag{4}$$

Parameter values for the centre of pressure and aerodynamic forces are taken from Dawson's work [3]. The kinematic constraint originates from the lines being straight resulting in a limitation of the moment along the roll axis and the magnitude of the pitching angle with respect to the lines.

Analysis & Results

The analysis considers the full set of non-linear equations. The governing equation (1) is solved using a standard numerical integration scheme, Runge-Kutta $\mathcal{O}(4)$. For the stability analysis, the Jacobian of the forcing term was approximated with a central difference formula; its eigenvalues were computed for a selected parameter range of interest.

Figure 2 depicts stability regions of pitch θ_3 and yaw θ_4 angles for a selected azimuthal angle θ_2 of $0°$ and increasing elevation values θ_1 from $0° - 90°$. In the order from left to right, the top left figure being $\theta_1 = 0°$ and the bottom right being $\theta_1 = 90°$. Blue zones represent stable solutions, while orange crosses represent unstable solutions, and grey zones represent spurious solutions (e.g. referring to negative line tension or collision with the ground). At $\theta_1 = 0°$ there are two columns of stable solutions, the spurious solution referring to the collision with the ground. As the elevation angle increases the spurious solution disappears and the stable blue region becomes a circle-like zone which migrates from negative to positive θ_3 values. This variation in θ_3 creates a restoring moment which brings the system back to it's elevation equilibrium. However, when the azimuthal angle is varied, the resultant twisting behavior acts to further exacerbate the azimuthal deviation — an unstable equilibrium.

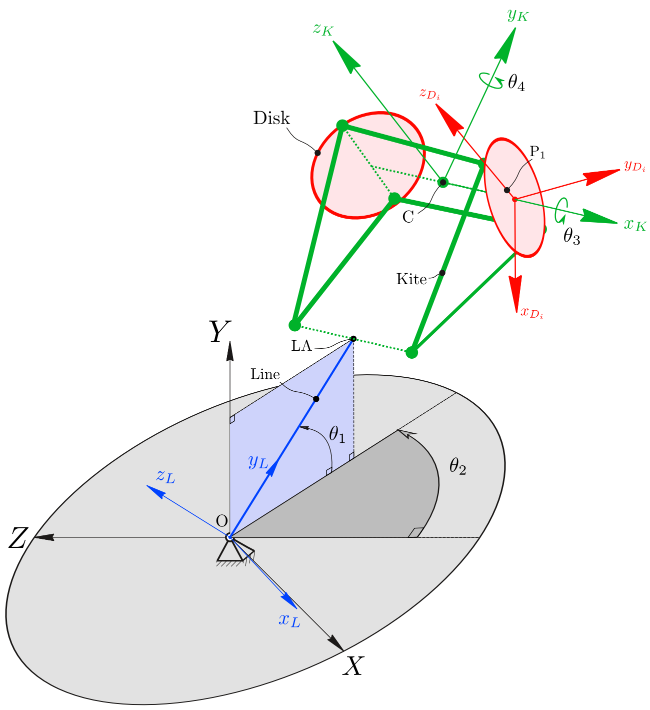

Figure 1: Coordinate systems for the two line kite

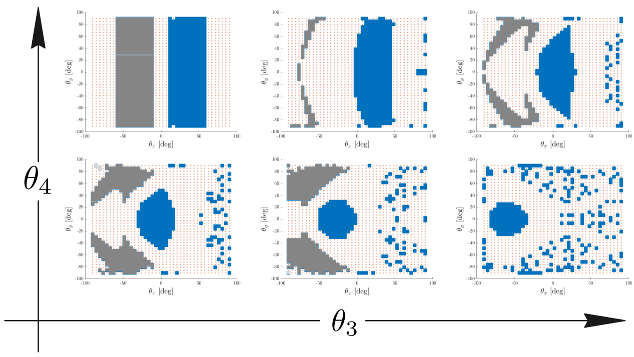

Figure 2: Stability regions of the system as the elevation changes (from left to right — 0,15,30,40,60,90). Blue (filled) → stable, grey (filled) → spurious but stable, orange (cross) → unstable

Conclusion

Kites are being utilised within new applications for energy generation, professional sporting activities, and ship propulsion [4]. Further development of accurate models for these fields will allow for vast and effective design improvements to enhance applications. In this work, a two-line four-attachment point disk kite is modelled using a Newton-Euler formulation. This model utilises validated wind-tunnel data for the aerodynamic forces. The stability of the system is analysed for a specific range of elevation angle values. Results demonstrate how the kite pitches to reach its equilibrium elevation. This technique can be used to optimise multiple design parameters for numerous applications. By observing the stability zones of the kite, the design parameters can be adjusted for specific flight behaviour and characteristics. Additionally, the user control mechanism for operating such kites can be further investigated for better performance.

References

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