# Nonlinear vibrations of nanoplates based double mode model and the nonlocal elasticity theory

Jan Awrejcewicz*, Olga Mazur**
* *Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, Lodz, Poland*
** *Department of Applied Mathematics*
*National Technical University KhPI, Kharkiv, Ukraine*

*Summary*. Geometrically nonlinear vibrations of the rectangular simply supported nanoplates are investigated. The governing equations are employed in mixed form and used the nonlocal elasticity theory as well as Kirchhoff's hypotheses and the von Kármán theory. Application of the Bubnov-Galerkin method with a double mode model allows to reduce the governing system of partial differential equations (PDEs) to the system of the second-order ordinary differential equations (ODEs) . Analysing obtained ODEs the small-scale effects and force influence are studied.

Particular interest in the study of nanostructures is associated with their wide application in high-tech industry due to excellent mechanical, thermal, electrical properties. Theoretical and experimental research has allowed to observe the small-scale effects that were not detected within the classical theory. This fact led to the development of non-classical continuum theories for study the objects with sizes in nanoscale. The presented work is aimed at the study of geometrically nonlinear vibrations of the small-scale plates. The formulation of the problem is performed based on Kirchhoff's hypotheses, the von Kármán theory. In order to take into account the appearance of the small-scale effects in nanoplates the nonlocal theory of elasticity [1] is applied. It is based on the fact that the stress at a given point is a function of strains at all other points in the body. According to this theory the constitutive relation in differential form [2] has the following form

$$\left(1-\mu\nabla^2\right)\sigma=\sigma', \tag{1}$$

where $\sigma', \sigma$ are local and nonlocal stress tensors, $\mu$ is nonlocal parameter, and $\nabla^2$ is the Laplacian operator.
The nonlocal governing equations are taken in mixed form, introducing the Airy stress function $F$:

$$D\Delta^2 w=\left(1-\mu\nabla^2\right)\left(-N_1\frac{\partial^2 w}{\partial x^2}-N_2\frac{\partial^2 w}{\partial y^2}+L\left(w,F\right)-\rho h\frac{\partial^{2,}w}{\partial t^2}-\delta_0\frac{\partial w}{\partial t}+q\right), \tag{2}$$

$$(1-\mu\nabla^2)\frac{1}{E}\Delta^2 F=-\frac{h}{2}L\left(w,w\right), \tag{3}$$

where differential operators are defined as

$$L\left(w,F\right)=\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}, L\left(w,w\right)=2\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right), \tag{4}$$

and $\Delta^2=(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})^2$, $D=\frac{Eh^3}{12(1-\nu^2)}$ is flexural nanoplate rigidity, $E$ is Young's modulus, $\nu$ is Poisson's ratio, $w$ is deflection of the plate, $N_1, N_2$ are in-plane uniform forces, $q$ is transverse force, $\rho$ is density, $h$ stands for thickness of the plate, whereas $\delta_0$ is damping coefficient. It is assumed that the plate satisfies the simply supposed boundary conditions.
The proposed approach is based on two mode presentation of the deflection $w\left(x,y,t\right)$ of rectangular small-scale plate with sides $a$ and $b$ as follows [3]:

$$w\left(x,y,t\right)=w_1(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}+w_2(t)\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b}, \tag{5}$$

where $w_1, w_2$ are bi-modal amplitudes, $\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$ and $\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b}$ are shape functions, that satisfy the chosen boundary conditions. Substitution (5) into the equation (3) allows to obtain the stress function presentation:

$$F=f_1\cos\frac{2\pi x}{a}+f_2\cos\frac{2\pi y}{b}+f_3\cos\frac{4\pi x}{a}+f_4\cos\frac{4\pi y}{b}+f_5\cos\frac{3\pi x}{a}\cos\frac{\pi y}{b}+f_6\cos\frac{\pi x}{a}\cos\frac{3\pi y}{b}+p_1x^2+p_2y^2, \tag{6}$$

where coefficients depend on the small-scale parameter $\mu$. Applying the Bubnov-Galerkin method, one can get the nonlocal system of ordinary differential equations. Size-dependent analysis of such system allows to study nonlinear vibrations regimes of the considered system. The numerical calculations are performed for graphene nanoplate with various excitation parameters.

## References

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