Constrained input modulation for impulse-based motion control

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Summary. We revisit the impulse-based motion control, introduced in [9], from a viewpoint when the control signal is constrained by the actuator limits. We demonstrate how the impulsive control [9] is modulated for the weighted input impulses, and what the consequences it has for the control performance. In addition, the state transients as a jumping map are discussed, when the amplitude-bounded (modulated) impulses can no longer guarantee the asymptotic convergence, and the stable limit cycles can appear instead.

Introduction

Hybrid control systems (see for example [6, 7] and references therein) allow for both continuous and discrete dynamics, while the discrete control actions may appear in form of the continuously or event- or state-depended switching of the bounded control value or, alternatively, in form of the discrete impulses with finite energy content. Intensive research on the possibly uniform and generalized description of the hybrid dynamical systems and, especially, on their stability analysis and stabilization was performed already one and half decades ago. Here we exemplary refer to the several available seminal (correspondingly tutorial) works like [4], and special issues like introduced in [1].
Impulsive dynamics (see e.g. [3] for basics) appear quite often in evolution of the natural processes, where some short-term perturbations (or generally stimuli) act instantaneously in the form of impulses and, thus, give rise to instantaneous jumps of the thereby affected dynamic states. A classical academic example is the bouncing ball (see for example in [4]), while in engineering, the various vibro-impact [3] systems can be found, for example, in the mechanical play-pairs also known as backlash [10]. When it comes to impulsive control actions, then the input impulse, or impulsive force cf. [3],

$$p_0 = \lim_{\Delta t \to 0} \int_{t_0}^{t_0+\Delta t} u(\tau) d\tau \tag{1}$$

at the controlling instant t_0 is, to say, scaling the Dirac measure δ_{t_0}, so that the impulsive control effort $u = p_0\,\delta_{t_0}$ cannot be a function of time. Analysis of such control systems may require to properly denote and handle the impulsive differential equations (see e.g. [5]). At the same time, introduction of a supplementary jump (or jumping) map (cf. the unified modeling framework provided in [2]) of the state transitions at all $t = t_0$ enables the further use of conventional system notations for all $t \neq t_0$ by means of e.g. ODEs or, correspondingly, state-space equations. In applications, one cannot expect that the impulse magnitude will keep a well-specified control action $u(\cdot)$ always below some finite actuator constraint. Therefore, once the input signal is inherently bounded $|u(\cdot)| < U$ by some positive system constant U, an impulsive control cannot be directly implemented, no matter which particular control strategy is lying behind.
Hybrid impulsive motion control, addressed in this work, was introduced in [9] while some preliminary formulation, including an experimental case study, was demonstrated before in [11]. The impulsive control was proposed for systems of the second order with uncertain upper-bounded damping, while the state axes represent the guard conditions that trigger a dedicated impulsive control action as soon as one of the both is crossed. One can notice that other impulsive controls were also proposed formerly in [8] and [12], for the mechanical systems with friction. In either approach, however, an impulsive action occurs first when a motion trajectory falls on the position axis, which implies several zero velocity (and subsequent re-acceleration) phases before it converges to the origin. In the following, we will briefly summarize the impulse-based control [9], for convenience of the reader, and then address the impulse modulation for bounded inputs.

Impulse-based motion control

The impulse-based motion control [9] is given by

$$m\ddot{x} + d\dot{x} + kx = \underbrace{-\alpha \operatorname{sign}(\dot{x}) \frac{\mathrm{d}}{\mathrm{d}x} \operatorname{sign}(x) - \beta \operatorname{sign}(x) \frac{\mathrm{d}}{\mathrm{d}\dot{x}} \operatorname{sign}(\dot{x})}_{\equiv u}, \tag{2}$$

where the continuous system dynamics is shaped by the inertial mass $m > 0$ and uncertain (or in the worst case unknown) stiffness and damping constants $k, d > 0$, respectively. Note that the upper-bound of the damping coefficient $d < D$ needs to be known. The discrete control value u is parameterized by $\alpha, \beta > 0$ and is acting only when the motion trajectory crosses one of the state axes, i.e. at $(0, \dot{x}_0)$ or $(x_0, 0)$. This provides a disjoint jump set $\mathcal{D} = \dot{X}_0 \cup X_0 = \{(x, \dot{x}) \in \mathbb{R}^2 \,|\, x = 0 \cup \dot{x} = 0\}$ and makes both control actions (on the right-hand side of (2)) respectively disjunctive and, therefore, simultaneously appearing only in zero equilibrium $(x, \dot{x}) = \mathbf{0}$, while $\operatorname{sign}(0) = 0$ is defined. The proposed control system (2) is well fitting into the autonomous-impulse hybrid systems framework [2] and, thus, can be fully described by $\dot{\mathbf{x}} = f(\mathbf{x})$ if $\mathbf{x} \in \mathcal{C}$ and $\mathbf{x}^+ \in J(\mathbf{x})$ if $\mathbf{x} \in \mathcal{D}$, where the flow and jump maps are f and J, respectively. The belonging flow and jump sets are disjoint so that $\mathcal{C} = \mathbb{R}^2 \backslash \mathcal{D}$. The parametric conditions for the gains are $0.5m|\dot{x}_0| \leq \alpha < m|\dot{x}_0|$ and $\beta = 0.5|x_0|D$, while a state value during last crossing of the orthogonal axis is denoted by the subindex zero, i.e. x_0 and $\dot{x}_0$ correspondingly. For further details on and analysis of the impulse-based motion control we refer to [9].

Modulation of bounded control input

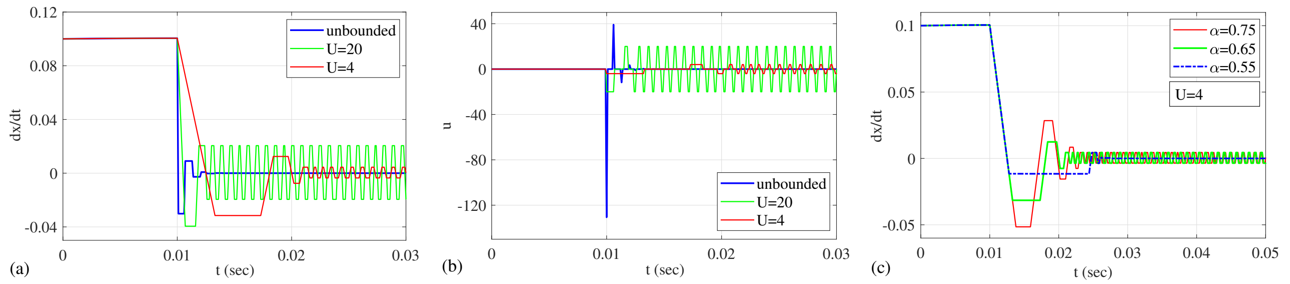
The impulsive control action in (2) requires a control effort $|u| = 2\alpha\delta(\dot{x}_0) \vee 2\beta\delta(x_0)$, where $\delta(\cdot)$ is the Dirac delta function, cf. with eq. (1). Recall that the Dirac delta function can be seen as distributional derivative of the sign-function, weighted by factor 2, and can then be defined and constrained to satisfy the identity as follows:

$$\delta(y) = \begin{cases} \infty, & \text{if } y = 0, \\ 0, & \text{if } y \neq 0; \end{cases} \qquad \int_{-\infty}^{\infty} \delta(y)\mathrm{d}y = 1; \qquad \delta(y_0) = \lim_{\Delta t \to 0} p_{\Delta t}(t_0). \tag{3}$$

Note that the last expression is (3) relates the Dirac delta function to the square pulse p of duration Δt and amplitude $(\Delta t)^{-1}$, cf. with impulsive force in (1). Since the square pulse has unity 'strength' (or 'weight'), it is evident that for a constrained actuator (with $\max u = U$) the pulse duration $T \equiv \Delta t$ is required to be $T = 2(\alpha \vee \beta)U^{-1}$. That leads to an inevitable modulation of the discrete (impulsive) control as $u \mapsto u[x_0, \dot{x}_0](t_0 \leq t \leq t_0 + T)$. In order to analyze the impact of such control modification on the convergence performance of (2), cf. [9], one needs to evaluate

$$\mathbf{x}^{+} = \exp\bigl(A(t_0 + T)\bigr)\, \mathbf{x}_0(t_0) \mp \int_{t_0}^{t_0+T} \exp\bigl(A(t-\tau)\bigr) B\, U d\tau, \tag{4}$$

which provides inhomogeneous (particular) solutions for $\mathbf{x}_0 = [0, \dot{x}_0]^T \vee [x_0, 0]^T$ at the time instant t_0 of the control pulse. Here $A \in \mathbb{R}^{2\times 2}$ and $B \in \mathbb{R}^{2\times 1}$ are the associated system matrix and input distribution vector resulting from the left-hand side of (2). Note that the sign before the integral in (4) changes depending on the quadrants in which the trajectory undergoes zero-crossing. The "$-$" sign captures either $x = 0$ crossing from the II-nd to the I-st quadrant or $\dot{x} = 0$ crossing from the I-st to the IV-th quadrant. And the $+$ sign appears for the corresponding zero-crossings from the IV-th to the III-rd or from the III-rd to the II-nd quadrant. For an asymptotic convergence of the state trajectory towards zero equilibrium, driven by a sequence of the control impulses $u(t_{0,n})$ with $n = 1, \ldots, N$ where $N \to \infty$, it is sufficient to demonstrate constant decrease of Euclidean norm of the state vector after each executed pulse, i.e. $\|\mathbf{x}^{+}\|_2 < \|\mathbf{x}_0\|_2$. A symbolic solution of (4) is computable, yet cumbersome, so that solely several numerical observations are shown and discussed below. For the sake of simplicity, no linear damping is assumed, i.e. $d = D = 0$, so that the left-hand side of (2) represents a harmonic oscillator for the assigned $m = 0.1$ and $k = 10$. The initial values are assigned to be $[x, \dot{x}](t = 0) = [-0.001, 0.1]$, and the forward Euler solver with $\Delta t = 0.0001$ sec is used. The difference between the



converging unbounded control and that U-bounded, which runs into stable limit cycles, is demonstrated in the diagrams (a) and (b). An avoidance of limits cycles and, thereupon, further convergence towards zero equilibrium is demonstrated in the diagram (c) with variation of the α-parameter. A more detailed analysis of the parametric conditions of the occurrence or escape of the limit cycles calls for an explicit solution of (4), equally as for periodic solutions with impulses at $t_{0,n}$.

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