Control-Oriented Modeling of a Planar Cable-Driven Parallel Robot with Non-Straight Cables

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Summary. This communication deals with the control-oriented modeling of cable-driven parallel manipulators when the cables are not considered as straight but show transverse deflections due to fast movements of the platform. The model has been derived from Euler-Lagrange equations with multipliers in order to deal with the kinematic constraints (DAE model). The number of generalized coordinates have been reduced by performing a transformation of the DAE model to an ODE model. The ODE model has been linearized and an $\mathscr{H}_\infty$ controller has been synthesized to control the cable tension and enable the platform to track a reference trajectory.

Introduction

Cable-driven parallel robots (CDPR) are composed of a platform attached to fixed points by cables. The movement of the platform is obtained by winding the cables. One usually distinguishes fully-constrained robots where the number of cables exceeds the number of degrees of freedom and suspended robots. Notwithstanding the commercially available CableCam system[1], CDPR are considered in a number of research projects for applications such as aircraft and ship construction, building facade cleaning and rescue in open air [1].

We are interested here in the modeling of cable robots taking into account cables deflections. More precisely, we take into account the cables inertial effects on the transverse displacements. A planar robot model with three cables made it possible to assess the effects of transverse displacements and the limits of a conventional control strategy [2]. The model was developed from the Lagrangian calculation by approximating the transverse displacements of the cables by parabolic functions.

Albeit this model allows to evaluate the system in simulation, it cannot be readily used to determine the stability properties and properly assess the system performance during a path-following task due to a lack of controllability. This communication discusses three possible methods to obtain such a control-oriented model from a model derived from the laws of physics, taking the example of the planar CDPR.

CDPR description and model

A planar robot with three cables holding a point mass is considered. The dynamic model has been obtained with the Lagrange method with multipliers that suit for dynamic systems with kinematic constraints. Each of the three cables is considered without elongation as depending on three variables: its length l_k (for the cable #k), its transverse displacement (only one mode is considered, corresponding to a parabolic transverse displacement equal to $\frac{w_k}{l_k}x^2$ for cable #k and $x \in [0; l_k]$) and its orientation at the attachment point. The x and y coordinates of the platform are also included, thus yielding a geometric representation based on 11 variables stacked in the generalized coordinates vector q. The three loop-closure equations of the parallel robot provide six kinematic constraint equations $A(q)\,\dot{q} = 0$. This model, described in details in [2], is given as

$$\begin{bmatrix} M(q) & -A^{\mathrm{T}}(q) \\ A(q) & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} F(q,\dot{q},\Gamma) \\ -\dot{A}\,\dot{q} \end{bmatrix} \tag{1}$$

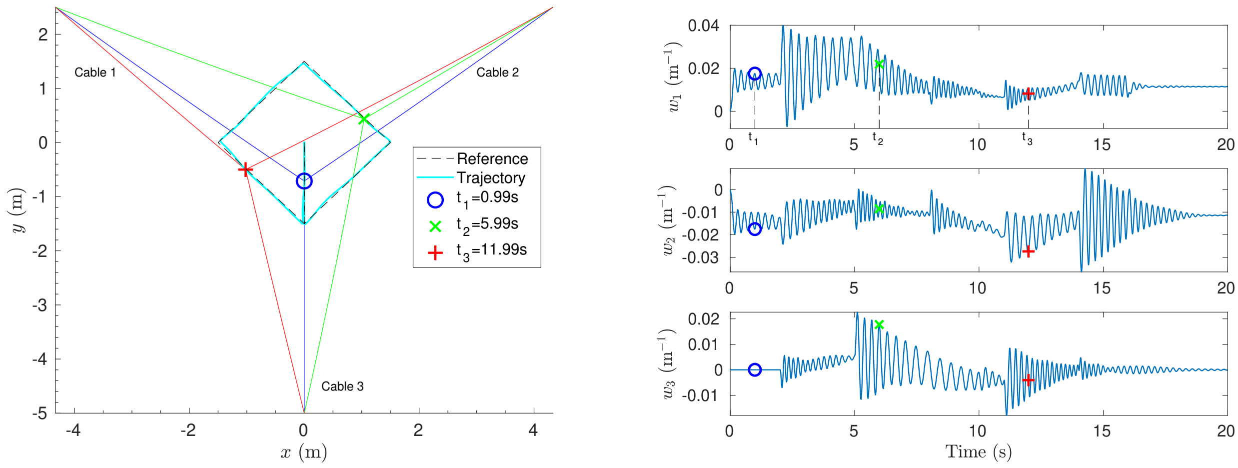
where λ is the multiplier vector, Γ is the vector of the winder torques, M is the kinetic energy matrix and F is the generalized force vector.

From the laws of physics to a control model

Whether based on the Hamiltonian or the Lagrangian [9], the methods of analytical mechanics render possible to determine models of poly-articulated systems with continuous displacements [3]. In addition to the dynamic equations, the model often includes algebraic equations, resulting in a system of differential-algebraic equations (DAE). This is generally the case for parallel robots (including CDPR) when we consider the geometric and kinematic constraints linking the displacements of cables and those of the platform [2, 4].

The control methods allow to study the stability and performance of systems governed by dynamic equations, with the advantage of providing results that are independent from a considered trajectory, thus allowing to reduce the number of simulations to be made in a validation step. Powerful methods allow to evaluate the stability and the performance of ODE

[1] See `http://cablecam.fr`.



a. CDPR configuration at three different times

b. Modal coordinates for the displacements of the cables

Figure 1: Simulation results

systems with uncertainties on parameters. However, a basic prerequisite is that the model is both controllable and observable [5]. Properties can be established globally, for example using methods based on Lyapunov functions which mimic energy functions [6], or more simply locally in which case a linear model, approaching the behavior in a neighborhood of an equilibrium point, is enough [7].

Starting from the nonlinear DAE model, several ways can be followed in order to get a model suitable for control:

A1. The left-hand side matrix in the model (1) can be inverted in order to reveal a second order ODE model $\ddot{q} = f(q, \dot{q}, \Gamma)$ depending on 11 state variables. But it was not possible to conclude on the controllability of the resulting model: the tests on its linearized version failled due to ill-conditioned matrices of the state-space representation. Moreover, standard tools were not able to provide a controllable and observable reduced-order model.

A2. Rather than making the linearization after inversion of the left-hand side matrix as described in A1, model (1) can be directly linearized so as to obtain a linear descriptor model for which analysis and controller synthesis methods are available [8]. However, the linearized descriptor system was also found uncontrollable and unobservable.

A3. When possible, algebraic equations $A(q)\,\dot{q} = 0$ could be solved in order to obtain a description based on a reduced generalized position vector $\tilde{q}$. This leads to a fewer number of equations of higher complexity (transformation of the DEA model to an ODE model).

Controller synthesis

Approach A3 has been implemented and has lead to a model depending on only 5 generalized coordinates. This ODE linear model was found controllable and observable and an $\mathscr{H}_\infty$ control synthesis has been performed in order to control the cables mean tension and the end-effector position (x, y) (refer to [10] for details on the control strategy). Considering a 1 kg platform and a linear density of cables is 0.17 kg/m, the performance of the $\mathscr{H}_\infty$ closed-loop non linear ODE model for a square reference trajectory at a speed of about 1 m/s has been depicted in Fig. 1. The results show a good trajectory tracking and disturbance rejection.

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