Vibration control of underactuated 3-DOF systems inspired by tuned vibration absorbers: the non-linear Euler-Lagrange controller

Jasper Juchem* and Mia Loccufier*
Department of Electromechanical, Systems and Metal Engineering, Ghent University, Ghent, Belgium

Summary. The concept of the non-linear Euler-Lagrange controller aims to combine the advantages of passive and active vibration control. The flexibility and adaptability of active control is combined with the intuitive design of a passive tuned vibration absorber. To verify this statement, an intuitive tuning procedure, which is inspired by the tuned vibration absorber, is stated. An energy-inspired approach is used to proof (asymptotic) stability using Lyapunov's direct method. A three-link planar manipulator with one actuator at the base is controlled to mitigate vibrations in the unactuated links. The controller contains a non-linear damper. Finally, some different experiments should give insight whether it is necessary to capture all modes, with the disadvantage that the number of controller parameters increases drastically. Also, a non-linear damper is compared to a linear one.

System description

The system that needs to be controlled, also referred to as the process, is a planar three-link manipulator. The joints are assumed to be frictionless. Between the links a spring creates a restoring force. At the base of the first link no spring is attached. The equation of motion is given by the general differential equation:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + K(q) = -Mu \tag{1}$$

with the generalized coordinates the relative angles between the links $q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}'$ and $\cdot'$ the transpose. The inertia matrix $D(q) = D(q)' > 0 \in \mathbb{R}^{n_p \times n_p}$ with $n_p = 3$ the number of generalized coordinates of the process, $C(q,\dot{q}) \in \mathbb{R}^{n_p \times n_p}$ the coriolis/gyroscopic and damping terms, stiffness matrix $K(q) \in \mathbb{R}^{n_p}$, and $u \in \mathbb{R}^{n_p}$. The matrices for this system can be found in [3].
The controller consists of two blocks that are placed in parallel: a proportional controller $u_1 = K_p M' q$ and an Euler-Lagrange controller

$$\begin{cases} D_0\ddot{z} + \frac{\partial F}{\partial \dot{z}}(\dot{z}) + K_0(z) = -N_1 M' q - N_2 M' \dot{q} \\ u_2 = \nu_1 z + \nu_2 \dot{z} \end{cases} \tag{2}$$

with $z \in \mathbb{R}^{n_c}$ the generalized coordinates of the controller, $D_0 \in \mathbb{R}^{n_c \times n_c}$ the inertia matrix, $\partial F/\partial \dot{z} \in \mathbb{R}^{n_c}$ non-linear damping function, $K_0(z) \in \mathbb{R}^{n_c}$ stiffness matrix of the controller, $N_1, \nu_1' \in \mathbb{R}^{n_c \times n_p}$ the amplification of the position coupling with the process in the input and output equation respectively, and $N_2, \nu_2' \in \mathbb{R}^{n_c \times n_p}$ the amplification of the velocity coupling with the process in the input and output equation respectively.
The controller effort in (1) is then given by $u = u_1 + u_2$. Notice the similarity with adding a mechanical structure to the system. However, the difference with adding a tuned vibration absorber is the way of connecting the controller and the process. Furthermore, in the differential equations $M = diag(\{0,1\}) \in \mathbb{R}^{n_p \times n_p}$ forces a collocated control strategy.
For the compound system to be an Euler-Lagrange system, the following must hold: $\nu_1 = N_1'$.

Asymptotic stability: Lyapunov's direct method

The stability is proven using Lyapunov's direct method. Let x_e be an equilibrium point for

$$\dot{x}(t) = f(x(t)) \tag{3}$$

where $f : D \to \mathbb{R}^n$ is a locally Lipschitz map from a domain $D \subset \mathbb{R}^n$ into $\mathbb{R}^n$, with $x_e \in D$. Let $V_L : D \to \mathbb{R}$ be a continuously differentiable function, such that

$$V_L(x_e) = 0, V_L(x) > 0 \text{ in } D \setminus \{x_e\}, \dot{V}_L(x) \leq 0 \text{ in } D, \text{ and } \dot{V}_L(x) < 0 \text{ in } D \setminus \{x_e\} \tag{4}$$

with $\dot{V}_L(x) = \frac{\partial V_L(x)}{\partial x} f(x)$. Then x_e is asymptotically stable.
Here, the Hamiltonian $\mathcal{H}$ is chosen to be the Lyapunov function. This leads to four conditions for the controller parameters that need to be fulfilled:

1. $\frac{dK_0}{dz}$ should be continuous, invertible and positive definite,
2. $\frac{dK(q)}{dq} + MK_p - M\nu_1 \frac{dK_0(z)}{dz}^{-1} \nu_1' M' > 0$,
3. $\nu_2' = -N_2$, and
4. $\left(\frac{\partial F}{\partial \dot{z}}\right)' \dot{z} \geq 0$.

Controller tuning

This research focusses on mitigating vibrations due to impulse impacts on the third link. After impact, the open-loop system will vibrate infinitely, as there is no damping present. It is not possible to apply a controlled torque to the second or third joint, what complicates diminishing these vibrations. The topology of the controller allows the introduction of tuning techniques from passive vibration control. One can try to make the controller sensitive to the main system, such that easy energy transfer occurs from the process to the controller. Once the energy is transferred, it can be dissipated in this controller [1].

Eigenfrequency matching

To facilitate an easy energy flow from the process to the controller, eigenfrequency matching is well-known strategy [2]. In this work, this is done in two steps. Firstly, the eigenfrequencies of the substructures are matched by tuning K_p in the case that $\nu_1 = \nu_2 = \mathbf{0}$. Secondly, the eigenfrequency of the process and controller are matched. Let $\Omega^2 = diag(\omega_1^2, \omega_2^2, \omega_3^2)$ with ω_i an eigenfrequency of the process. Now, the controller is tuned to have the same eigenfrequency

$$K_0(z) = D_0 \Omega^2 z \tag{5}$$

Notice that depending on n_p the number of modes that are captured can be altered. To achieve stability, $\omega_i \neq 0$, as all conditions on $\frac{dK_0}{dz}$ are then fulfilled. The second condition leads to the expression $k_{p,1} - \nu_{111}^2/\omega_1 - \nu_{112}^2/\omega_2 - \nu_{113}^2/\omega_3 > 0$ with $K_p = diag(k_{p,1}, k_{p,2}, k_{p,3})$ and $\nu_1 = \begin{bmatrix} \nu_{111} & \nu_{112} & \nu_{113} \\ \nu_{121} & \nu_{122} & \nu_{123} \\ \nu_{131} & \nu_{132} & \nu_{133} \end{bmatrix} = \begin{bmatrix} \nu_{111} & \nu_{112} & \nu_{113} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, which is equivalent as only the first link can be actuated.

Optimization

Without loss of generality it can be assumed that D_0 is an identity matrix [4]. This leaves us with three unknown matrices: ν_1, ν_2, and $\frac{\partial F}{\partial \dot{z}}$. As ν_2 introduces a conservative coriolis force and due to non-linear damping, the analysis is too complex to be carried out analytically [5, 6]. Therefore, the remaining parameters will follow from an optimization. The optimization algorithm will use four objective functions:

$$f_1 = T_{s,1}\omega_1; f_2 = T_{s,2}\omega_1; f_3 = T_{s,3}\omega_1; f_4 = \int \frac{\mathcal{H}(t)}{\max \mathcal{H}} dt \tag{6}$$

with settling time $T_{s,i}$ of link i the time after which the time signal stays within 5% of the maximal deviation around equilibrium, and ω_1 the slowest eigenfrequency.
As mentioned before, the controller's damping will be a non-linear function

$$\frac{\partial F}{\partial \dot{z}} = c_1 \dot{z} + c_2 \arctan\left(\frac{c_0 - c_1}{c_2}\dot{z}\right) \approx \begin{cases} c_0 \dot{z} & \text{if } \dot{z} \approx 0 \\ c_1 \dot{z} + cst & \text{if } |\dot{z}| \to \infty \end{cases} \tag{7}$$

with $c_i > 0$. Then $\left(\frac{\partial F}{\partial \dot{z}}\right)' \dot{z} > 0$ if $c_0 > c_1$.

Results

In this section different controllers are compared. First of all, increasing the number of generalized coordinates of the controller, n_c, leads to capturing all modes of the process. However, it also increases the number of controller parameters drastically, thus increasing the optimization time. Next to that, a non-linear Euler-Lagrange controller will be compared to a linear one. All simulations will be the result of an impulse on the tip of the third link.

Conclusions

A planar underactuated three-link manipulator will be controlled with a non-linear Euler-Lagrange controller. Next to stability, a tuning strategy based on tuned vibration absorbers will be validated. The number of generalized coordinates of the controller (n_c) will be varied from one to three to observe whether the increase in tuning parameters leads to an significantly improved result. Also, the difference between linear and non-linear damping is examined.

References

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