Dynamics of Temporal Localized States in Time-Delayed Optically Injected Kerr Gires-Tournois Interferometers

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Summary. We study theoretically the formation and dynamics of phase-locked temporal localized states (TLSs) and frequency combs that can be generated from a high finesse Fabry-Perot microcavity containing a Kerr medium that is coupled to a external cavity in the presence of optical injection. These TLSs possess a strongly asymmetrical oscillating tail which results from third order dispersion induced by the cavity. Using a first principle model based on delay algebraic equations and applying a combination of direct numerical simulations and path continuation methods, we disclose sets of multistable dark and bright TLSs coexisting on their respective bistable homogeneous backgrounds. In particular, we show that the detuning of the injection with respect to the microcavity resonance controls the region of existence of TLSs and its change can lead to a period-doubling route to chaos. Further we discuss a transformation of the original delay model to a normal form given by a partial differential equation using a rigorous multiple time scale analysis and a functional mapping approach.

Introduction

Time-delayed systems (TDS) describe a large number of phenomena in nature and they proposed for hosting localized structures, fronts and chimera states [1, 2, 3, 4, 5, 6, 7]. They also have been analyzed from the perspective of their equivalence with spatial extended systems [8]. In general, the expansion of a delayed term leads to drift and diffusion. However, it was shown [9] that considering a more general class of singularly perturbed TDS allows to cancel the generic diffusive behavior which leads to a dispersive response. Here, an example of TDS giving rise to second and third order dispersion (TOD), which combined with Kerr effect and optical injection is able to generate temporal localized states (TLSs), as well as molecules induced by third order dispersion [10].

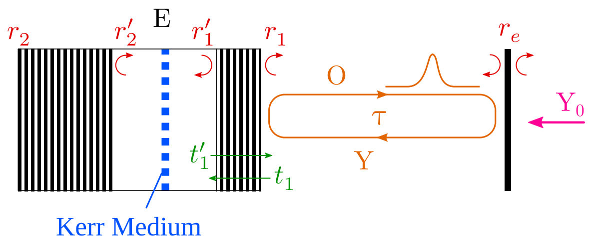

Figure 1: A schematic of the coupled cavity configuration. E denotes the field amplitude in the Kerr region. The output and injection fields in the external cavity are represented by O and Y, respectively. The round-trip time is τ

Model

Our schematic setup is depicted in Fig. 1 in which we show i) the intra-cavity field E and ii) the external cavity field Y. We follow the approach of [11] that consists in solving the field propagation in the linear sections of the micro-cavity. That way one obtains a dynamical model linking the two fields E and Y. Their coupling is achieved considering the transmission and reflection coefficients of the top Distributed Bragg Mirror (DBR). After normalization, one obtains the rate equations for the fields E and Y

$$\frac{dE}{dt} = \left[-1 + i\left(\gamma\left|E\right|^2 - \delta\right)\right]E + hY \quad , \quad Y = \eta\left[E\left(t-\tau\right) - Y\left(t-\tau\right)\right] + \sqrt{1-\left|\eta\right|^2}Y_0\,. \tag{1}$$

The field cavity enhancement can be conveniently scaled out using Stockes relations allowing E and Y to be of the same order of magnitude. This leads to the simple input-output relation $O = E - Y$. The cavity-enhanced complex nonlinear coefficient is $\gamma = \gamma_r + i\gamma_i$, where γ_r stands for the self-phase modulation and γ_i models the two-photon absorption. The effects of the external mirror and signal extraction (e.g., a beam-splitter or transmission through the mirror itself) are combined in the attenuation factor $\eta = r_e \exp\left(i\varphi\right)$. The coupling between the intra-cavity and the external cavity fields E and Y is given in Eqs. (1) by a delay-algebraic equation (DAE). The latter takes into account all the multiple reflections in a possibly high finesse external cavity for which $|\eta| \lessapprox 1$ [9]. Note that in the limit of a very low external mirror reflectivity $\eta \ll 1$, one would obtain $Y = \eta E\left(t-\tau\right) + \mathcal{O}\left(\eta^2\right)$ leading to the so-called Lang-Kobayashi model [12].

Results

In **[SCH19]**, the DAE system (1) was successfully used to prove the existence of multistable dark and bright TLS coexisting on their respective bistable homogeneous continuum wave (CW) backgrounds for fixed parameter values, see Fig. 2

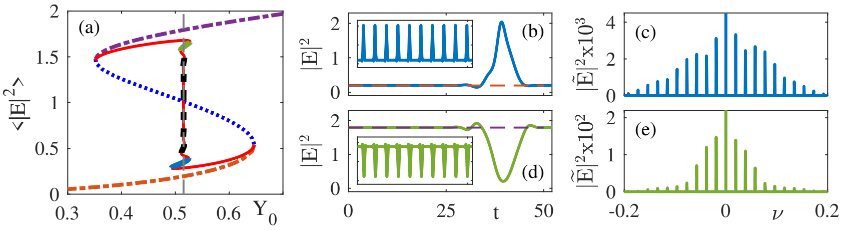

Figure 2: (a) Bistable cavity response under CW injection (dotted line). The lower and upper snaking branches (full lines) correspond to the trains of bright (b) and dark (d) TLSs observed as a function of time t. The inset shows the dynamics over many roundtrips. The respective frequency combs are depicted in panels(c) and (e), where the injection frequency was cut for clarity.

Here, we conduct extensive direct numerical simulations of DAEs (1) in the combination with path continuation methods. In particular, the influence of the detuning δ is analyzed in details. We show that it controls the region of existence of TLSs and its change can lead to a period-doubling route to chaos.

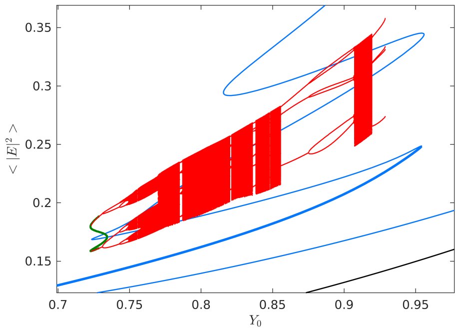

Figure 3: A part of the branch of the TLS (blue) of DAEs (1) as a function of the injection Y_0, obtained with the contunuation methods together with the results of the direct numerical simulations (red). A period-doubling route to chaos is visible. Green line indicate a branch of period-doubled solutions.

Further, the influence of attenuation factor η will be studied. In particular, the regimes of low and high η will be assessed. We demonstrate that in the so-called good cavity limit ($\eta \to 1$), the dynamics is dominated by TOD leading to a wide region of multistability between bright and dark TLS.
Finally we discuss a transformation of the original DAE model (1) to a normal form at the onset of optical bistability given by a partial differential equation using a rigorous multiple time scale analysis and a functional mapping approach.

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