Nonlinear Dynamics of a Body with Two Elastic Supports on an Inclined Rough Plane

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Summary. The features of behavior of a dynamically asymmetric heavy body with two elastic supports on an inclined rough plane are studied. The classical Coulomb model of dry friction is used. The supports can move without friction along parallel guides fixed in the body and interact with the body by means of elastic springs. It is shown that the equilibrium position of the system is isolated. There is no stagnation zone by the tilt angle of the guides. Depending on the relationship between friction and the angle of inclination of the plane, the body can slide on one or both supports. The equations of motion represent a dynamic system of variable structure. The range of inclination angles of the supporting plane close to the critical one is considered. For inclination angle values smaller than the critical one, the body on rigid supports does not start sliding from rest along the given inclined plane; for angle values greater than the critical one, the body on rigid supports starts sliding from rest. For elastic supports, it is shown that if the initial position of the body is not equilibrium, then, starting from a state of rest, the center of mass of the system acquires a non-zero longitudinal velocity even if the angle of inclination of the supporting plane is less than the critical one.

Introduction

To ensure the stability of building structures when supported on a rough foundation, it is necessary to take into account various factors: deviations in the dimensions of finished structures from design ones, compliance of structures, deviation of the supporting surface from the horizontal, etc. In particular, it is of interest to model the behavior of structures for a case of various additional disturbances, such as an earthquake (see, for example, [1]). The difficulties of modeling mechanical systems with dry friction were actively considered at the end of the 19th century ([2]). At the beginning of the 20th century, the phenomenon of "friction impact" was substantiated ([3]).
Modeling the interaction of dry friction with elastic forces leads to the formation of dynamic systems of variable structure. Such systems, even after the linearization of the equations, have properties characteristic of nonlinear systems. Such equations have several singular points depending on the mode of motion. In [4], the sliding of a flat rigid body supported on a horizontal rough plane by two elastic telescopic supports was considered. The alternation of body sliding on one and two supports is described depending on the coefficient of friction.
In this paper, some effects of the behavior of a dynamically asymmetric heavy body on two elastic supports on an inclined rough plane are considered. The equations of plane-parallel motion of the system are obtained for the cases of sliding of one or two supports. The equilibrium position has been found. Conditions for falling one of the supports into a friction cone are obtained. A numerical calculation of motion from a state of rest in a position close to equilibrium has been carried out. It is shown that the center of mass of the system acquires a non-zero longitudinal velocity even when the angle of inclination of the reference plane is less than that required for the beginning of sliding of a solid (inelastic) body.

Problem statement

Consider a heavy body $ABCD$ with mass m (Fig. 1a) supported by supports AA_1 and BB_1 on an inclined rough plane. We assume that the angle α between the plane and horizontal H_1H_2 is small. For simplicity, we assume that the body is rectangular ($AB = 2a$, $AD = 2b$).The support rods can slide along parallel guides without friction. To simulate the compliance of the supports, we introduce identical springs of sufficiently high rigidity between the body and the rods. When the springs are not stressed, $AA_1 = BB_1 = l_0$, $l_0 < b < a$. The center of gravity G_1 of the rectangle is displaced from the center of the rectangle G along the straight line DC by a distance d. Consider $d > 0$, the right support is more loaded than the left one. Support springs act on the body by forces $F_{el1} = -k(l_1 - l_0), F_{el2} = -k(l_2 - l_0)$, where k is the spring stiffness coefficient, l_1 is the length of the left support AA_1, l_2 is the length of the right support BB_1.

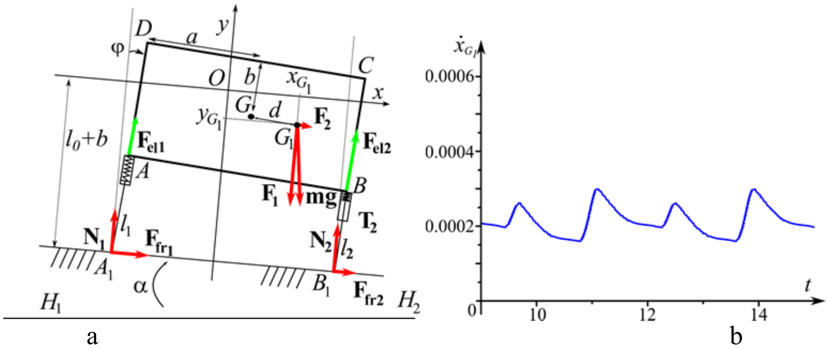

Figure 1: The body on a rough inclined plane. a) Mechanical system in consideration; b) Time dependence of the longitudinal velocity $\dot{x}_{G_1}$ of the center of mass in the steady regime of motion for the angle of inclination $\alpha < \alpha\_crit$.

The following external forces act on the system: gravity **mg**, normal $\mathbf{N_1}, \mathbf{N_2}$ and tangential $\mathbf{F_{fr1}}, \mathbf{F_{fr2}}$ support reactions. The values of normal reactions at the support points are determined from the following relationships: $N_1 = F_{el1}\cos\varphi$, $N_2 = F_{el2}\cos\varphi$. In the case of sliding of the supporting legs, the reactions of the supports are interconnected according to the Coulomb law. In the case when the supporting leg is at rest, the value of the corresponding tangential reactions $F_{fr1} = F_{fr1stat}, F_{fr2} = F_{fr2stat}$ is determined from other motion characteristics.

Let us introduce a coordinate system *Oxy*. The axis *Ox* is parallel to the support plane and is at a distance $l_0 + b$ from it. The axis *Oy* is directed along the normal to the support plane. For convenience, we decompose the force of gravity into two components $\mathbf{mg} = \mathbf{F_1} + \mathbf{F_2}$, where $\mathbf{F_1}, \mathbf{F_2}$ are the projections of the force of gravity on the *Oxy* axes.

As generalized coordinates, we choose the coordinates of the center of mass x_{G_1}, y_{G_1} and the angle φ between the vertical and the sides of the rectangle. We distinguish between longitudinal (along the inclined plane) $\dot{x}_{G_1}$ and transverse $\dot{y}_{G_1}$ velocities of the center of mass.

The equilibrium position $\{\varphi^*, y^*_{G_1}\}$ of the system does not depend on the presence of tangential reactions and can be obtained from the following equations:

$$\begin{cases} y_{G_1} = (F_2 sin\varphi cos\varphi - F_1 cos^2\varphi)/(2k) + (b+l_0)(cos\varphi - 1) - dsin\varphi \\ 2F_2F_1cos^5\varphi + ((F_1^2 - F_2^2)sin\varphi - (2((b+l_0)F_2 + F_1 d))k)cos^4\varphi - \\ -(2k(-F_2 d + F_1(b+l_0))sin\varphi + F_2F_1)cos^3\varphi - 2F_2akcos\varphi sin\varphi + 4a^2k^2 sin\varphi = 0 \end{cases}$$

In the region of small values φ^*, the dependences $\varphi^*(d), y^*_{G_1}(d)$ are single valued. The equilibrium position is isolated. There is no stagnation zone in the angle φ of inclination of the support guides.

Obviously, if friction is absent or sufficiently small, then both points of support slide during motion. If during some time period the point B_1 is motionless, then the number of degrees of freedom has decreased by one, the dynamical system must contain two equations of the second order. The immobility of the point is verified by the inequality: $\Delta = \mu N_2 - abs(F_{fr2stat})$. As the initial conditions we choose the position of rest, at which the guides of the supports are perpendicular to the supporting plane: $\varphi = 0$, $(x_{G_1}, y_{G_1}) = (d, -F_1/2k)$, $\dot{\varphi} = 0, \dot{x}_{G_1} = 0, \dot{y}_{G_1} = 0$. Then the inequality takes the following form: $\mu > \mu_2 = \dfrac{2\left|md(l_0 + b - 0.5F_1/k) - ((a-d)^2 m + J)F_2/(mg)\right|}{(J + m((a-d)^2 + (l_0 + b - 0.5F_1/k)^2)}$, *J* is the central moment of inertia of the body. If the inequality is satisfied, then the right support does not slide. For $\mu < \mu_2$ both supports start sliding. Similar reasoning can be carried out in the case of stopping the supporting point A_1.

Let us single out the value of the angle of inclination $\alpha = \alpha\_crit$ of the plane such that the force F_2 is equal in magnitude to the maximum value of the friction force of rest of the same body with rigid supports: $F_2\_crit = \mu F_1 = \mu$. For smaller inclination angles, the body on rigid supports does not slip. It turned out that for the considered case of elastic supports, the body slides from a state of rest even if the angle of inclination of the plane is less than the critical one. Residual internal oscillations lead to a change in the normal pressure and, accordingly, to a change in the friction forces in the supports. In the final motion, the body has a variable non-zero longitudinal velocity. The time dependence of the longitudinal velocity $\dot{x}_{G_1}$ of the center of mass in the steady regime of motion is shown in Fig. 1b for $F_2 = 0.00199, \mu = 0.002$. Such a regime, in particular, is possible due to the fact that the rear supporting point A_1 from time to time makes an instantaneous stop, and then slides in the opposite direction.

Conclusions

Plane motion of a heavy body supported by two elastic supports on a rough inclined plane is considered. The equations of motion are obtained, which are the dynamic system of variable structure. The isolated equilibrium position is found. It is numerically shown that from the state of rest, the center of mass of the system, can obtain a non-zero longitudinal velocity even if the angle of inclination of the supporting plane is less than the critical one required to start sliding of a rigid (inelastic) body.

The work is partially supported by Russian Science Foundation (project 22-21-00303).

References

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