Three-Link Snake Robot with a Single Control Input

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Summary. A three-link snake robot is proposed controlled by an internal flywheel installed at the first link. There are spiral springs installed in joints between links. These springs are non-deformed when angles between the links are zero. The robot performs plane-parallel motion over a rough surface. Each link contacts with the surface at a single point. Anisotropic dry friction acts at the contact points. The friction coefficient corresponding to the direction along the link direction is much smaller than that corresponding to the direction transversal to the link. The control torque is applied to the internal flywheel. Parameters of the mechanism and coefficients of the control law are adjusted to ensure irreversible propulsion of the robot along a prescribed direction.

Introduction

Snake robots can be used for works in aggressive areas. Such devices are widely discussed in the literature, and various schemes of construction and control are proposed for them, e.g. [1, 2]. Most of the research in this field assumes that there are motors which control angles between links in each joint. In the current work, we show that the presence of such motors is not necessary to obtain a snake-like motion of a link mechanism. We suppose that the only control applied to the system is associated with the rotation of a single inner flywheel installed at the leading link. Similar approach to the control of a three-link mechanism is applied in [3] for a swimming robot.

Description of the system, statement of the problem

The robot consists of three links *AB*, *BD*, *DE* located in *OXY* plane and connected to each other by cylindrical joints *B* and *D* (fig. 1). Spiral springs with the stiffness *c* are installed in joints. These springs are non-deformed when all links make a single straight line. The inner flywheel with the shaft *A*, the mass m_0, and the central moment of inertia J_0 is installed at the first link. The point *A* is the center of mass of the first link; this link has the mass $(m_1 - m_0)$ and the central moment of inertia J_1. Masses m_2 and m_3 of the links *BD* and *DE* are concentrated in the points *C* and *E*, respectively. The current position of the robot is described by coordinates *x*, *y* of the point *A* and angles $\varphi_i, i = 1, 2, 3$ between links and *OX* axis. The angle of orientation of the inner flywheel is a cyclic coordinate; ω_0 is the absolute angular speed of the flywheel.

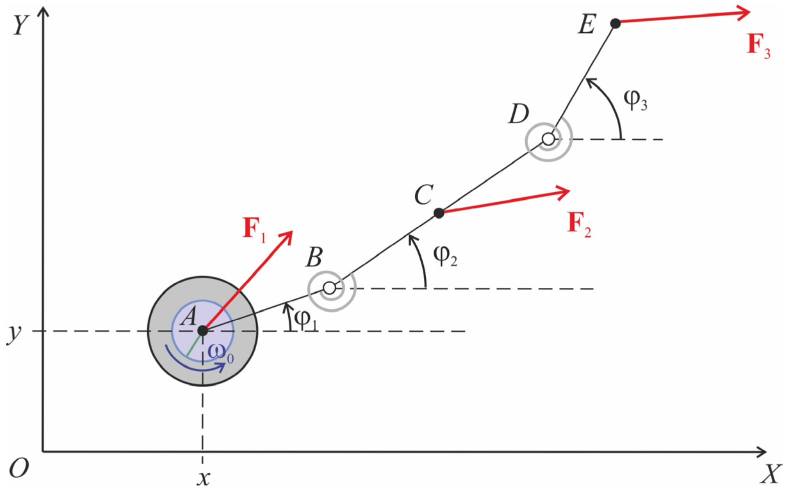

Figure 1: The scheme of the snake robot controlled by a single internal flywheel

Each link contacts the supporting plane in a single point: *A*, *C*, *E*. Anisotropic dry friction $\mathbf{F}_i$ is applied at each contact point; the corresponding friction model is taken from [4]. The friction coefficient $\mu_{i\xi}$ corresponding to the motion along the link is much smaller than $\mu_{i\eta}$ corresponding to the motion in the transversal direction. $V_{i\xi}, V_{i\eta}$ are projections of the speed $\mathbf{V}_i$ of the contact point on the axis directed along the link and the axis orthogonal to it. The following relation holds:

$$\begin{pmatrix} F_{i\xi} \\ F_{i\eta} \end{pmatrix} = -\begin{pmatrix} \mu_{i\xi} & 0 \\ 0 & \mu_{i\eta} \end{pmatrix} \begin{pmatrix} V_{i\xi} \\ V_{i\eta} \end{pmatrix} \frac{m_i g}{V_i}.$$

The single control torque *U* is applied to the shaft of the flywheel. It is limited in the absolute value by the constant $U_{\max}$. The goal of the control is to ensure the existence of an attracting self-sustained irreversible mode of propulsion of the robot opposite to *OX* axis. This means that the projection V_x of the velocity **V** of the center of mass on *OX* shouldn't change sign and the average value of projection V_y of **V** on *OY* should be zero.

Methods and main results

Equations of motion are derived basing the Lagrange formalism. Generalized forces are associated with dry friction forces, corresponding generalized torques Q_i, and the control torque U applied to the flywheel:

$$\frac{d}{dt}\left(\frac{dK}{d\dot{x}}\right)=F_{1x}+F_{2x}+F_{3x},\quad \frac{d}{dt}\left(\frac{dK}{d\dot{y}}\right)=F_{1y}+F_{2y}+F_{3y},\quad \frac{d}{dt}\left(\frac{dK}{d\dot{\varphi}_i}\right)-\frac{dK}{d\varphi_i}+\frac{dP}{d\varphi_i}=Q_i\quad (i=1,2,3),\quad \frac{d}{dt}\left(\frac{dK}{d\omega_0}\right)=U;$$

$$F_{ix}=F_{i\xi}\cos\varphi_i-F_{i\eta}\sin\varphi_i,\quad F_{iy}=F_{i\xi}\sin\varphi_i+F_{i\eta}\cos\varphi_i\quad (i=1,2,3);$$

$$Q_1=r(F_{2y}+F_{3y})\cos\varphi_1 - r(F_{2x}+F_{3x})\sin\varphi_1,\quad Q_2=r(F_{2y}+2F_{3y})\cos\varphi_2 - r(F_{2x}+2F_{3x})\sin\varphi_2,\quad Q_3=r(F_{3y}\cos\varphi_3 - F_{3x}\sin\varphi_3).$$

Here K and P are kinetic and potential energies, respectively. The control torque U is as follows:

$$U=\begin{cases} U_p, & |U_p|\le U_{\max}, \\ \mathrm{signum}(U_p)\cdot U_{\max}, & |U_p|>U_{\max}, \end{cases}\qquad U_p=-\left(a_0\,\mathrm{signum}\left(\sin(2\pi w_0 t)\right)-k_0 y-k_1\varphi_1\right).$$

The first term of the U_p represents a periodic excitation aimed to ensure oscillations of the inner flywheel. The other two terms are required to ensure zero average shift of links from the axis *OX* during the self-sustained propulsion.

Analysis of motion equation was performed by direct numerical integration with different geometrical and mass parameters, coefficients of the control law and initial component $\dot{x}(0)$ of the velocity of the point *A*. Initial values of other variables were zero. The total mass of the system was supposed to be equal to 0.5 kg, moments of inertia: $J_0=0.0005$ kgm^2, $J_1=0.002$ kgm^2. Lengths of links were not varied: *AB*=*BC*=*CD*=*DE*=0.05 m. The maximum value of the control torque was fixed: $U_{\max}$=0.1 N. Friction coefficients were fixed: $\mu_{i\xi}=0.03$, $\mu_{i\eta}=0.9\ (i=1,2,3)$.

Parameter continuation was used to find the set of parameters providing higher speed of motion at the program mode.

It was shown that an attracting regime of irreversible propulsion exists in a rather wide range of parameters. In particular, high speed of propulsion is achieved for the following set: $m_1=m_3=0.2$ kg, $m_2=0.1$ kg, $c=0.15$ kgm^{2}s^{-2}, $a_0=0.1$ Nm, $w_0=1.2$ s^{-1}, $k_0=-0.1$ N, $k_1=0.1$ Nm. The transition process with initial value $\dot{x}(0)=0.1$ m/s is shown in the fig. 2: initially robot was pushed in the direction opposite to the target direction; at first, the center of mass decelerates, then it starts accelerating and reaches the attracting self-sustained motion in the prescribed direction, i.e. against *OX*. At the attracting regime, the *x*-component V_x of the speed of the center of mass is always negative. Thus, the motion is irreversible. The average value $\langle V_x\rangle$ of the component V_x is about -0.37 m/s, and the average value of V_y is zero.

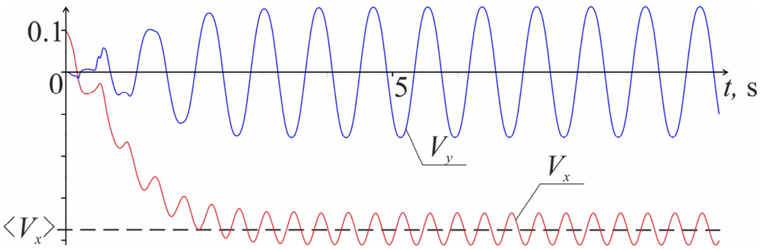

Figure 2: Components of the speed of the center of mass during the transition to the program mode (an example)

The control ensures transition of the system to the program regime of motion from a wide range of initial conditions. The program motion is irreversible with respect to the axis *OX*. It should be noticed, that the presence of springs in joints as well as the anisotropy of friction are necessary conditions for the existence of the desired irreversible mode of propulsion of the proposed robotic scheme.

Conclusions

A new scheme of the snake robot is proposed without any control inputs applied in inter-link joints. The only control is the torque applied to the inner flywheel installed at the first link. The robot performs a plane-parallel motion over a rough surface. An anisotropic dry friction acts in the points of contact between the robot and the supporting surface. The parameters of the construction and coefficients of the control law are adjusted to ensure existence of attracting propulsion mode of motion that is irreversible with respect to the preferable direction of propulsion.

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